2. Use the factor theorem to determine whether the binomial is a factor of the given polynomial.

a.) $x+1$; $P(x) = x^7 - x^5 + x^3 - x$

-1	1	0	-1	0	1	0	-1	0
	↓	-1	1	0	0	-1	1	0
	1	-1	0	0	1	-1	0	0

X + 1 is a factor of P(x).

b.) $y+1$; $P(y) = y^5 + y^4 + y^3 + y^2 + y + 1$

-1	1	1	1	1	1	1
	↓	-1	0	-1	0	-1
	1	0	1	0	1	0

y+1 is a factor of P(y)

c.) $z+2$; $P(z) = z^5 + 2z^4 + z^3 + 2z^2 + z + 2$

-2	1	2	1	2	1	2
	↓	-2	0	-2	0	-2
	1	0	1	0	1	0

Z+2 is a factor of P(z)

3. A root (solution) of the equation is given. Solve the equation.

a.) $x^3 + 3x^2 - 3x - 9 = 0$; -3

-3	1	3	-3	-9
	↓	-3	0	9
	1	0	-3	0
	a	b	c	

$\sqrt{x^2 - 3 = 0}$ OR $x = \frac{0 \pm \sqrt{0-4(1)(-3)}}{2}$

$\sqrt{x^2} = \sqrt{3}$

$x = \pm\sqrt{3}$

$x = \frac{0 \pm \sqrt{12}}{2}$

$x = \frac{0 \pm 2\sqrt{3}}{2}$

$x = \pm\sqrt{3}$

$\{-3, \sqrt{3}, -\sqrt{3}\}$

b.) $2x^3 + 9x^2 + 7x - 6 = 0$; -2

-2	2	9	7	-6
	↓	-4	-10	6
	2	5	-3	0
	a	b	c	

$x = \frac{-5 \pm \sqrt{25 - 4(2)(-3)}}{2(2)}$

$x = \frac{-5 \pm \sqrt{49}}{4}$

$x = \frac{-5+7}{4}$ $x = \frac{-5-7}{4}$

$x = 1/2$ $x = -3$

$\{-3, -2, 1/2\}$

The Rational Root Theorem

In this section we will use an extension of the factor theorem, the graphing calculator/computer, and synthetic substitution to factor higher degree polynomials that have all rational roots.

Change

Rational Root Theorem: $(ax - b)$ is a factor of $P(x)$ if and only if $P\left(\frac{b}{a}\right) = 0$.
Note: The maximum number of roots is equal to the degree of the polynomial.

Break for Practice:

1. Factor $2x^3 - 3x^2 - 8x - 3$

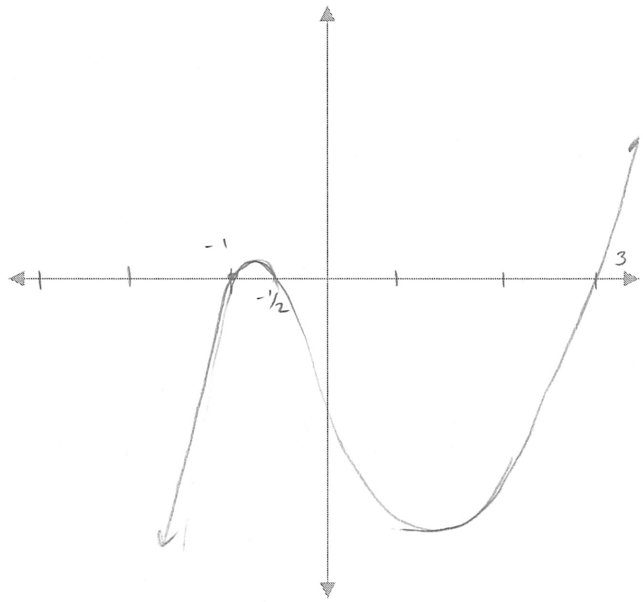
   a) How many factors should we expect?

   3

   b) List all of the possible rational roots.

   $\frac{\pm 1 \pm 3}{\pm 1 \pm 2} \rightarrow \pm 1 \ \pm \frac{1}{2} \ \pm 3 \ \pm \frac{3}{2}$

   c) Sketch a graph.



   d) Write the list of factors.

Roots: $x = -1$

$x = 3$

$x = -1/2$

$-1 \rfloor$ 2 -3 -8 -3
↓ -2 5 3
$3 \rfloor$ 2 -5 -3 L0✓
↓ 6 3
$-1/2 \rfloor$ 2 1 L0✓
↓ -1
2 L0✓

Factors: $(x+1)(x-3)(2x-1)$

2. Factor $x^4 - 5x^2 + 4$

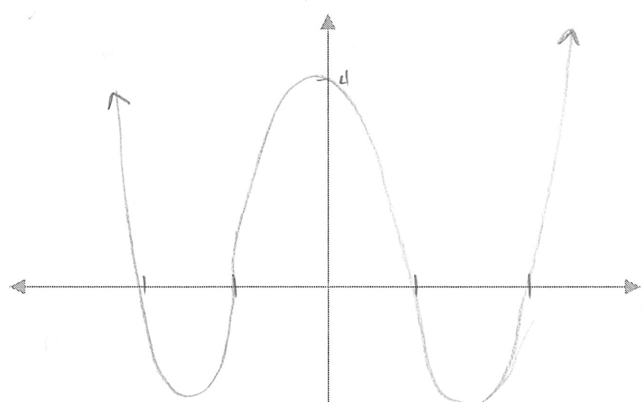
   a) How many factors should we expect?

   4

   b) List all of the possible rational roots.

   $\frac{\pm 1 \pm 2 \pm 4}{\pm 1} \rightarrow \pm 1 \ \pm 2 \ \pm 4$

   c) Sketch a graph.



   d) Write the list of factors.

Roots: $x = -1$

$x = -2$

$x = 1$

$x = 2$

$-1 \rfloor$ 1 0 -5 0 4
↓ -1 1 4 -4
$-2 \rfloor$ 1 -1 -4 4 L0✓
↓ -2 6 -4
$1 \rfloor$ 1 -3 +2 L0✓
↓ 1 -2

$2 \rfloor$ 1 -2
↓ 2
1 L0✓

Factors: $(x+1)(x+2)(x-1)(x-2)$

Extended Practice:

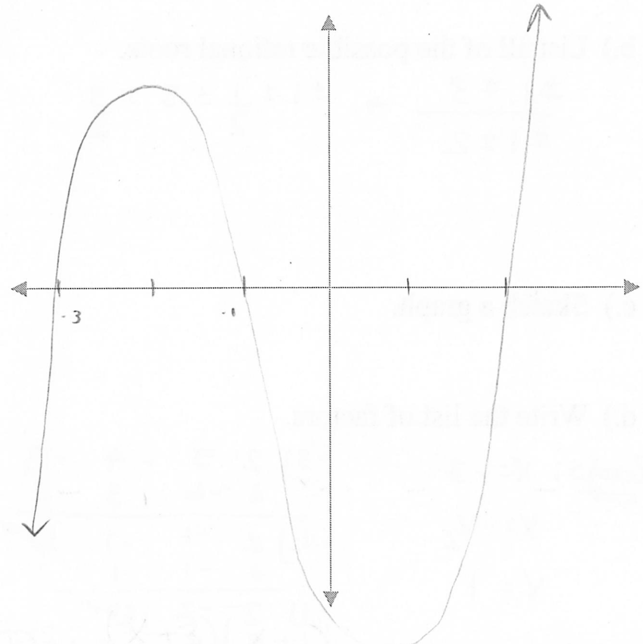
1. Factor $x^3 + 2x^2 - 5x - 6$

   a.) How many factors should we expect? 3

   b.) List all of the possible rational roots.

   $\frac{\pm 1 \pm 2 \pm 3 \pm 6}{\pm 1} \rightarrow \pm 1, \pm 2, \pm 3, \pm 6$

   c.) Sketch a graph.



   d.) Write the list of factors.

Roots: $x = -3$

$x = -1$

$x = 2$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -5 & -6 \\ & \downarrow & -3 & 3 & 6 \\ \hline -1 & 1 & -1 & -2 & 0 \checkmark \\ & \downarrow & -1 & 2 & \\ \hline 2 & 1 & -2 & 0 \checkmark & \\ & \downarrow & 2 & & \\ \hline & 1 & 0 \checkmark & & \end{array}$$

Factors: $(x+3)(x+1)(x-2)$

2. Factor $x^3 - 19x + 30$

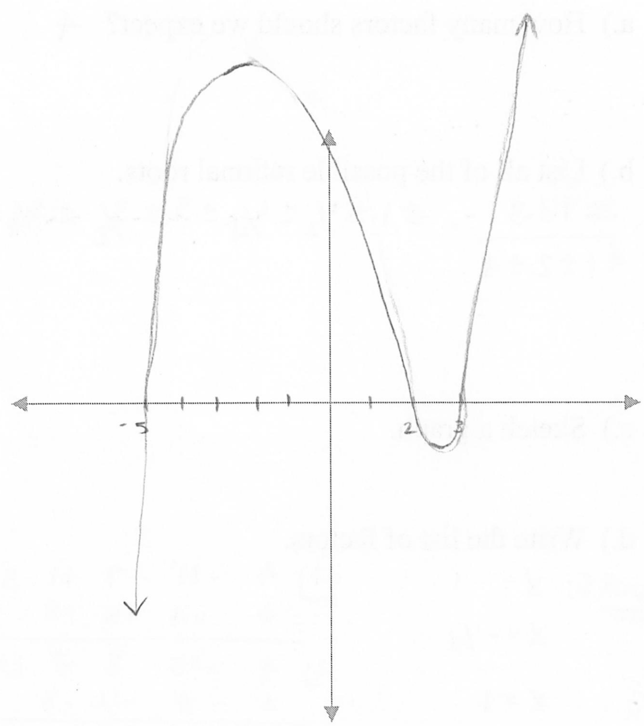
   a.) How many factors should we expect? 3

   b.) List all of the possible rational roots.

   $\frac{\pm 1 \pm 2 \pm 3 \pm 5 \pm 6 \pm 10 \pm 15 \pm 30}{\pm 1}$

   $\pm 1 \pm 2 \pm 3 \pm 5 \pm 6 \pm 10 \pm 15 \pm 30$

   c.) Sketch a graph.



   d.) Write the list of factors.

Roots: $x = -5$

$x = 2$

$x = 3$

$$\begin{array}{r|rrrr} -5 & 1 & 0 & -19 & 30 \\ & \downarrow & -5 & 25 & -30 \\ \hline 2 & 1 & -5 & 6 & 0 \checkmark \\ & \downarrow & 2 & -6 & \\ \hline 3 & 1 & -3 & 0 \checkmark & \\ & \downarrow & 3 & & \\ \hline & 1 & 0 \checkmark & & \end{array}$$

Factors: $(x+5)(x-2)(x-3)$

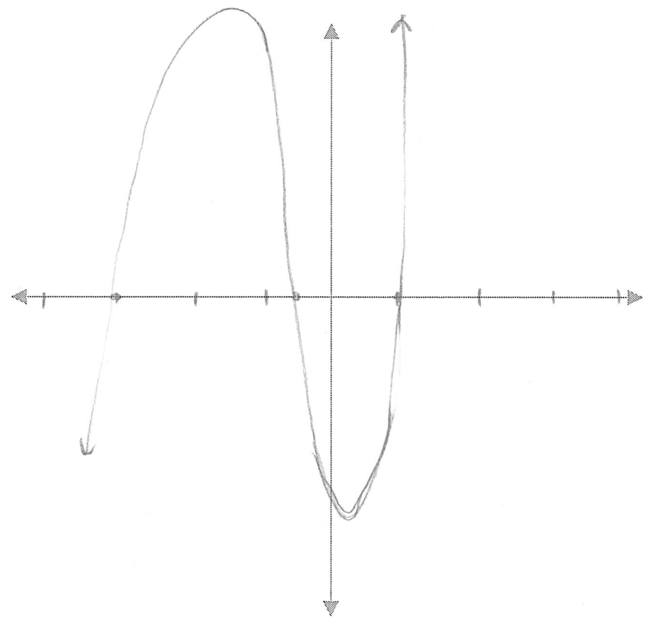
3. Factor $2x^3 + 5x^2 - 4x - 3$

a.) How many factors should we expect? 3

b.) List all of the possible rational roots.

$\frac{\pm 1 \pm 3}{\pm 1 \pm 2} \rightarrow \pm 1 \pm \frac{1}{2} \pm 3 \pm \frac{3}{2}$

c.) Sketch a graph.



d.) Write the list of factors.

Roots: $x = -3$

$x = -1/2$

$x = 1$

$$\begin{array}{r|rrrr} -3 & 2 & 5 & -4 & -3 \\ & \downarrow & -6 & 3 & 3 \\ \hline -1/2 & 2 & -1 & -1 & L0✓ \\ & \downarrow & -1 & 1 & \\ \hline 1 & 2 & -2 & L0✓ & \\ & \downarrow & 2 & & \\ \hline & 2 & L0✓ & & \end{array}$$

Factors: $(x+3)(x-1)(2x+1)$

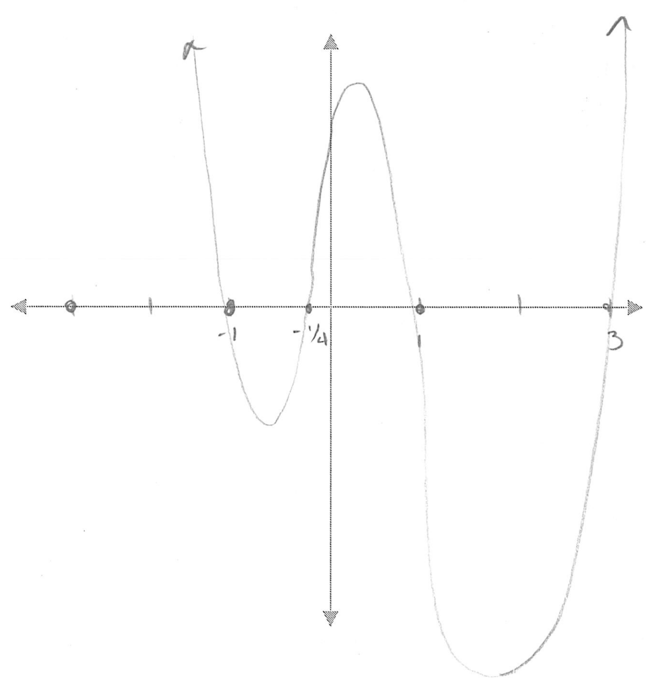
4. Factor $4x^4 - 11x^3 - 7x^2 + 11x + 3$

a.) How many factors should we expect? 4

b.) List all of the possible rational roots.

$\frac{\pm 1 \pm 3}{\pm 1 \pm 2 \pm 4} = \pm 1 \pm 1/2 \pm 1/4 \pm 3 \pm 3/2 \pm 3/4$

c.) Sketch a graph.



d.) Write the list of factors.

Roots: $x = -1$

$x = -1/4$

$x = 1$

$x = 3$

$$\begin{array}{r|rrrrr} -1 & 4 & -11 & -7 & 11 & 3 \\ & \downarrow & -4 & 15 & -8 & -3 \\ \hline 1 & 4 & -15 & 8 & 3 & L0✓ \\ & \downarrow & 4 & -11 & -3 & \\ \hline 3 & 4 & -11 & -3 & L0✓ & \\ & \downarrow & 12 & 3 & & \\ \hline -1/4 & 4 & 1 & L0✓ & & \\ & \downarrow & -1 & & & \\ \hline & 4 & L0✓ & & & \end{array}$$

Factors: $(x-3)(x-1)(x+1)(4x+1)$

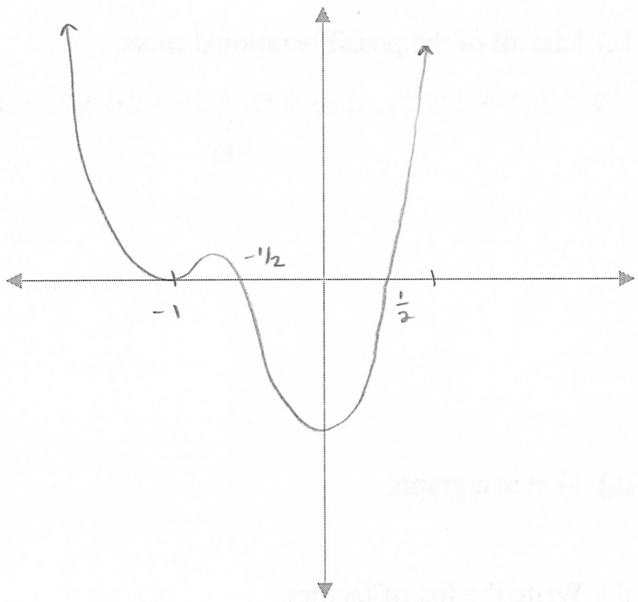
5. Factor $4x^4 + 8x^3 + 3x^2 - 2x - 1$

a.) How many factors should we expect? 4

b.) List all of the possible rational roots.

$\frac{\pm 1}{\pm 1 \pm 2 \pm 4} \rightarrow \pm 1 \; \pm \frac{1}{2} \; \pm \frac{1}{4}$

c.) Sketch a graph.



d.) Write the list of factors.

-1| 4 8 3 -2 -1
↓ -4 -4 1 1

-1| 4 4 -1 -1 LO✓
↓ -4 0 1

1/2| 4 0 -1 LO✓
↓ 2 1

4 2 LO

-1/2| 4 2
↓ -2

4 LO✓

Factors: $(x+1)(x+1)(2x+1)(2x-1)$

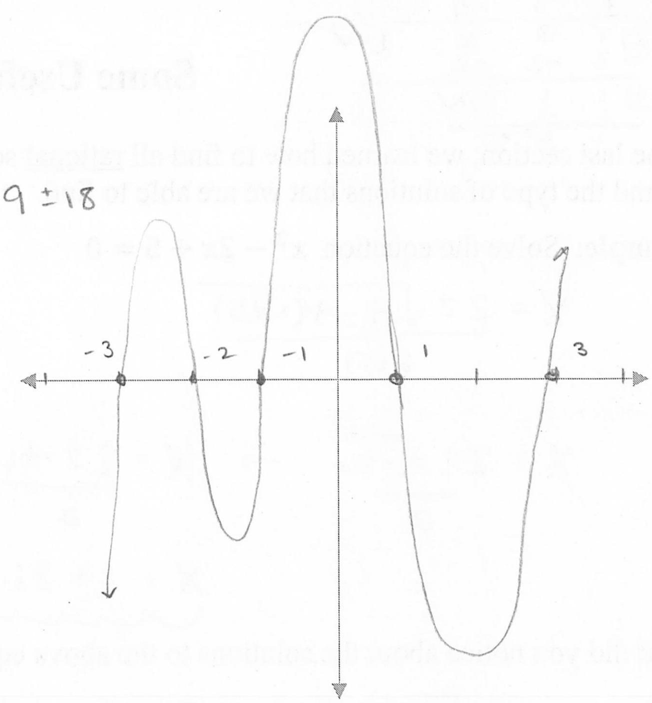
6. $x^5 + 2x^4 - 10x^3 - 20x^2 + 9x + 18$

a.) How many factors should we expect? 5

b.) List all of the possible rational roots.

$\frac{\pm 1 \pm 2 \pm 3 \pm 6 \pm 9 \pm 18}{\pm 1} \rightarrow \pm 1 \pm 2 \pm 3 \pm 6 \pm 9 \pm 18$

c.) Sketch a graph.



d.) Write the list of factors.

-3| 1 2 -10 -20 9 18
↓ -3 3 21 -3 -18

-2| 1 -1 -7 1 6 LO✓
↓ -2 6 2 -6

-1| 1 -3 -1 3 LO✓
↓ -1 4 -3

1| 1 -4 3 LO✓
↓ 1 -3

3| 1 -3 LO✓
↓ 3

1 LO✓

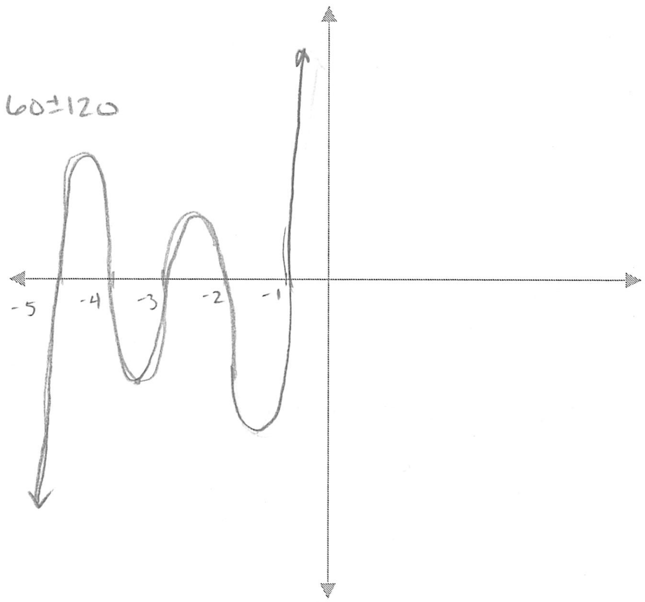
Factors: $(x+3)(x+2)(x+1)(x-1)(x-3)$

5. Factor $x^5 + 15x^4 + 85x^3 + 225x^2 + 274x + 120$

a.) How many factors should we expect? 5

b.) List all of the possible rational roots.

$\pm 1 \; \pm 2 \; \pm 3 \; \pm 4 \; \pm 5 \; \pm 6 \; \pm 10 \; \pm 12 \; \pm 15 \; \pm 20 \; \pm 24 \; \pm 30 \; \pm 40 \; \pm 60 \; \pm 120$



c.) Sketch a graph.

d.) Write the list of factors.

-5	1	15	85	225	274	120
	↓	-5	-50	-175	-250	-120
-4	1	10	35	50	24	0 ✓
	↓	-4	-24	-44	-24	
-3	1	6	11	6	0 ✓	
	↓	-3	-9	-6		
-2	1	3	2	0 ✓		
	↓	-2	-2			
-1	1	1	0 ✓			
	↓	-1				
	1	0 ✓				

Factors: $(x+5)(x+4)(x+3)(x+2)(x+1)$

Some Useful Theorems

In the last section, we learned how to find all rational solutions to polynomial equations. In this section we shall expand the type of solutions that we are able to find.

Example: Solve the equation $x^2 - 2x + 5 = 0$

$$X = \frac{2 \pm \sqrt{4 - 4(1)(5)}}{2(1)}$$

$$X = \frac{2 \pm \sqrt{-16}}{2} \rightarrow X = \frac{2 \pm 4i}{2}$$

$$X = 1 \pm 2i$$

What did you notice about the solutions to the above equation?

Imaginary Roots → Conjugate complex #'s

Theorem: If a polynomial has real coefficients, then if there is a Complex solution, there will actually be a pair of Complex solutions, and they will be Complex Conjugates.

There is another important theorem that states:

Theorem: A Polynomial equation of degree n , will have n solutions. (These include real, imaginary, complex, and duplicates.)

Break for Practice:

1. Find all of the solutions for $x^3 = 27$.

Set = to zero

$-27 \quad -27$

$x^3 - 27 = 0$

$$\begin{array}{c|cccc} 3 & 1 & 0 & 0 & -27 \\ & \downarrow & 3 & 9 & 27 \\ \hline & 1 & 3 & 9 & \boxed{0} \\ & a & b & c & \end{array}$$

$$x = \frac{-3 \pm \sqrt{9 - 4(1)(9)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{-27}}{2}$$

$$x = \frac{-3 \pm 3i\sqrt{3}}{2}$$

$$\boxed{\left\{ 3, \frac{-3}{2} + \frac{3}{2}i\sqrt{3}, \frac{-3}{2} - \frac{3}{2}i\sqrt{3} \right\}}$$

2. Find all of the solutions for $x^4 = 16$.

Set = 0

$-16 \quad -16$

$x^4 - 16 = 0$

$$\begin{array}{c|ccccc} 2 & 1 & 0 & 0 & 0 & -16 \\ & \downarrow & 2 & 4 & 8 & 16 \\ \hline -2 & 1 & 2 & 4 & 8 & LO \\ & \downarrow & -2 & 0 & -8 & \\ \hline & 1 & 0 & 4 & LO & \\ & a & b & c & & \end{array}$$

(Short cut when $b=0$) OR

$x^2 + 0x + 4 = 0$

$x^2 + 4 = 0$

$-4 \quad -4$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

$$x = \frac{0 \pm \sqrt{0 - 4(1)(4)}}{2(1)}$$

$$x = \frac{0 \pm \sqrt{-16}}{2}$$

$$x = \frac{\pm 4i}{2}$$

$x = \pm 2i$

$$\boxed{\{ 2, -2, 2i, -2i \}}$$

3. Find a **second degree** equation with solutions of $x = 3$, and $x = -4$.

need 2 roots/factor

① ②

Factors: $x = 3 \rightarrow x - 3$

$x = -4 \rightarrow x + 4$

Roots need to be changed to factors

$y = (x-3)(x+4)$

$y = x^2 + 4x - 3x - 12$

$y = x^2 + x - 12$

4. Find a **third degree** equation with solutions $x = 3i$, and $x = 5$. $x = -3i$

① ② ③

need 3 roots/factors

Any complex # will automatically have it's conjugate

Roots ⇒	Factors
$x = 3i$	$x - 3i$
$x = -3i$	$x + 3i$
$x = 5$	$x - 5$

$(x - 3i)(x + 3i)(x - 5) = y$ ← Can only multiply 2 factors @ a tim

$(x^2 + \cancel{3ix} - \cancel{3ix} - 9i^2)(x-5) = y$

$(x^2 + 9)(x - 5) = y$

$x^3 - 5x^2 + 9x - 45 = y$

Extended Practice:

1. Solve the following equations. factoring, completing the square, quadratic formula

a.) $x^2 + 3x + 5 = 0$	b) $x^2 - 3x + 6 = 0$	c) $-x^2 - 2x - 6 = 0$
$X = \frac{-3 \pm \sqrt{9-4(1)(5)}}{2(1)}$	$X = \frac{3 \pm \sqrt{9-4(1)(6)}}{2(1)}$	$X = \frac{2 \pm \sqrt{4-4(-1)(-6)}}{2(-1)}$
$X = \frac{-3 \pm \sqrt{-11}}{2}$	$X = \frac{3 \pm \sqrt{-15}}{2}$	$X = \frac{2 \pm \sqrt{-20}}{-2}$ $^{-4}_{-5}$
$X = \frac{-3 \pm i\sqrt{11}}{2}$	$X = \frac{3 \pm i\sqrt{15}}{2}$	$X = \frac{2 \pm 2i\sqrt{5}}{2}$
		$X = \frac{2}{2} \pm \frac{2i\sqrt{5}}{2}$
$\boxed{\left\{-\frac{3}{2} + \frac{i\sqrt{11}}{2}, -\frac{3}{2} - \frac{i\sqrt{11}}{2}\right\}}$	$\boxed{\left\{\frac{3}{2} + \frac{i\sqrt{15}}{2}, \frac{3}{2} - \frac{i\sqrt{15}}{2}\right\}}$	$\{1+i\sqrt{5}, 1-i\sqrt{5}\}$

2. Find all of the solutions for the following equations.

a.) $x^3 = 8 \rightarrow X^3 - 8 = 0$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & \downarrow & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & LO \\ & a & b & c & \end{array}$$

$\boxed{\{2, -1+i\sqrt{3}, -1-i\sqrt{3}\}}$

$X = \frac{-2 \pm \sqrt{4-4(1)(4)}}{2(1)}$

$X = \frac{-2 \pm \sqrt{-12}}{2} \rightarrow X = \frac{-2}{2} + \frac{2i\sqrt{3}}{2}$

b.) $x^3 = 125 \rightarrow X^3 - 125 = 0$

$$\begin{array}{r|rrrr} 5 & 1 & 0 & 0 & -125 \\ & \downarrow & 5 & 25 & 125 \\ \hline & 1 & 5 & 25 & LOV \\ & a & b & c & \end{array}$$

$X = \frac{-5 \pm \sqrt{25-4(1)(25)}}{2(1)}$

$X = \frac{-5 \pm \sqrt{-75}}{2}$ $^{-25}_{\ 3}$ $\rightarrow X = \frac{-5 \pm 5i\sqrt{3}}{2}$

$\boxed{\left\{5, -\frac{5}{2} + \frac{5i\sqrt{3}}{2}, -\frac{5}{2} - \frac{5i\sqrt{3}}{2}\right\}}$

c.) $x^4 = 16 \rightarrow X^4 - 16 = 0$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 0 & 0 & -16 \\ & \downarrow & 2 & 4 & 8 & 16 \\ \hline -2 & 1 & 2 & 4 & 8 & LOV \\ & \downarrow & -2 & 0 & -8 & \\ \hline & 1 & 0 & 4 & LOV & \end{array}$$

$X^2 + 0X + 4 = 0$

$X^2 + 4 = 0$

$\sqrt{X^2} = \sqrt{-4}$

$\boxed{\{2, -2, 2i, -2i\}}$

$X = \pm 2i$

d.) $x^4 = 81 \rightarrow X^4 - 81 = 0$

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & 0 & 0 & -81 \\ & \downarrow & 3 & 9 & 27 & 81 \\ \hline -3 & 1 & 3 & 9 & 27 & LOV \\ & \downarrow & -3 & 0 & -27 & \\ \hline & 1 & 0 & 9 & LOV & \end{array}$$

$X^2 + 0X + 9 = 0$

$X^2 + 9 = 0$

$\sqrt{X^2} = \sqrt{-9}$

$X = \pm 3i$

$\boxed{\{3, -3, 3i, -3i\}}$

3. Write a second-degree equation which has solutions of x = 5, and x = -2.

Roots ⇒	Factors
$x = 5$	$x - 5$
$x = -2$	$x + 2$

$y = (x-5)(x+2)$

$y = x^2 + 2x - 5x - 10$

$\boxed{y = x^2 - 3x - 10}$

4. Write a third degree equation which includes solutions of x = 4*i*, and x = 5.

Roots ⇒	Factors
$x = 4i$	$x - 4i$
$x = -4i$	$x + 4i$
$x = 5$	$x - 5$

$y = (x-4i)(x+4i)(x-5)$

$y = (x^2 + \cancel{4ix} - \cancel{4ix} - 16i^2)(x-5)$

$y = (x^2 + 16)(x-5)$

$\boxed{y = x^3 - 5x^2 + 16x - 80}$

Graphing Higher Degree Polynomial Functions

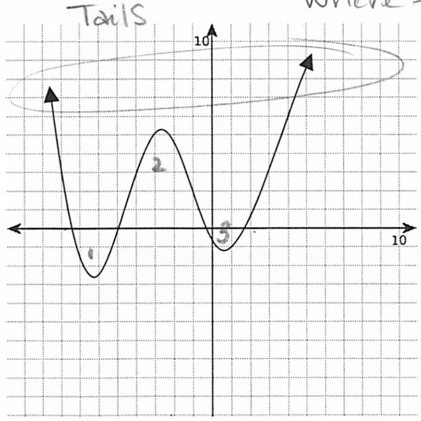
In this section we will be working with the graphs of higher degree polynomial functions. You will be looking for patterns in the shapes of the graphs. You will also learn how to tell how many zeros are real, and how many are imaginary. First you need to learn a little background.

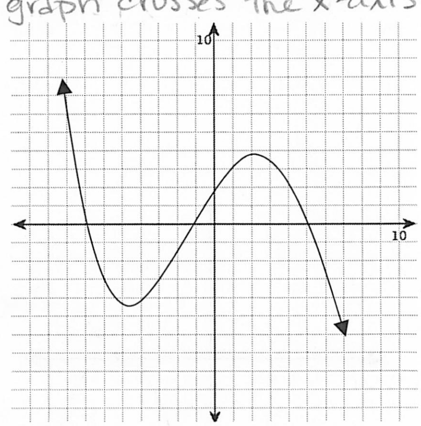
Arrow — where the graph changes direction

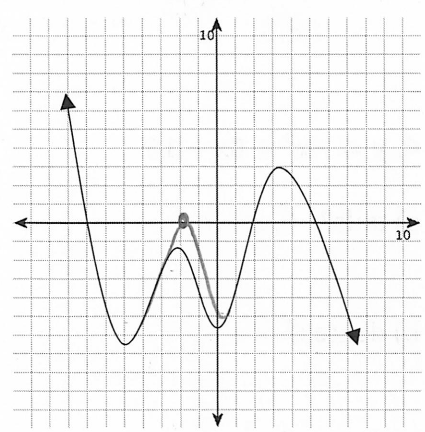
For each graph tell what you notice about the direction of the "tails", how many bumps (relative max and min) that you see, and the number of real zeros that are on the graph.

where the graph crosses the x-axis

Tails







tails: same direction (up)

bumps: 3

real roots: 4

tails: opposite direction

bumps: 2

real roots: 3

tails: opposite direction

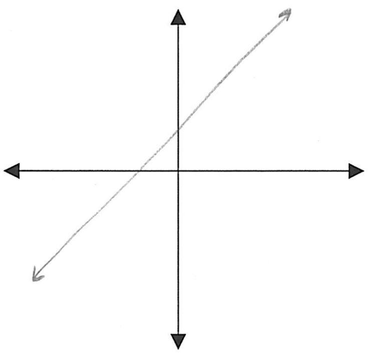
bumps: 4

real roots:

Break for Practice: For each polynomial, draw the graph and write the answers to the following questions.

Questions:
a) What is the direction of the tails?
b) What is the degree of the polynomial?
c) How many bumps (relative max and min) are in the graph?
d) How many real zeros are there? (How many times does it cross the x-axis?)
e) How many imaginary zeros are there? (b − d = e) Degree − Real Roots = Imaginary

1. $y = 3x + 4$



a) tails? opposite

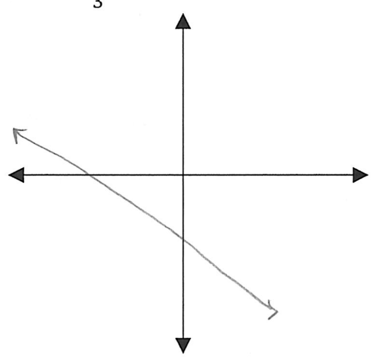
b) degree? 1

c) bumps? None

d) real zeros? 1

e) imaginary zeros? None

2. $y = -\frac{2}{3}x - 5$



a) tails? opposite

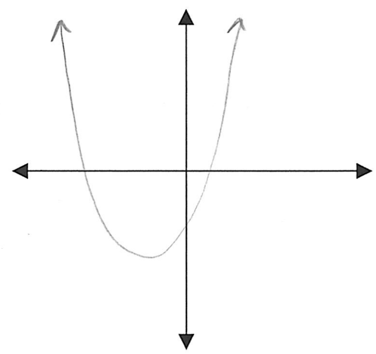
b) degree? 1

c) bumps? None

d) real zeros? One

e) imaginary zeros? None

3. $y = 2x^2 + 5x - 3$



a) tails? Same

b) degree? 2

c) bumps? 1

d) real zeros? 2

e) imaginary zeros? None

4. $y = -x^2 + 8x - 16$

a) tails? Same

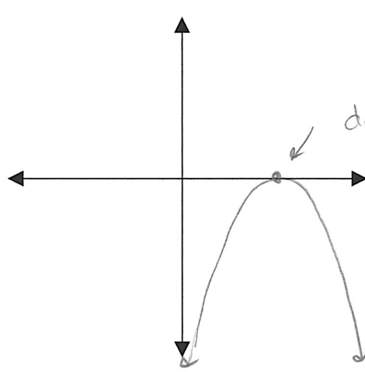
b) degree? 2

c) bumps? 1

d) real zeros? 2

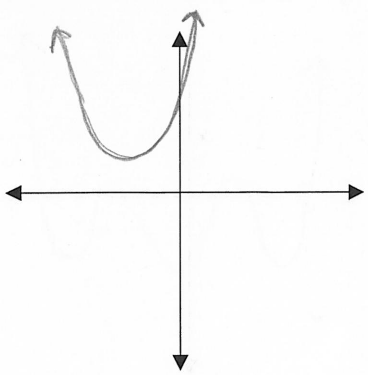
(double root)

e) imaginary zeros? None



double root (HW: 2 distinct real roots)

5. $y = x^2 + 3x + 5$



a) tails? Same

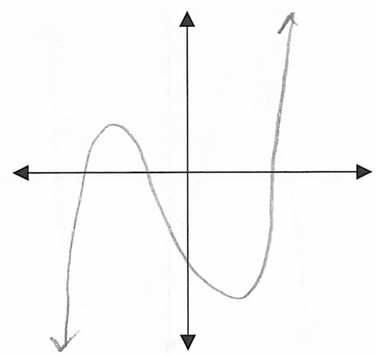
b) degree? 2

c) bumps? 1

d) real zeros? None

e) imaginary zeros? 2

6. $y = x^3 + 2x^2 - 5x - 6$



a) tails? Opposite

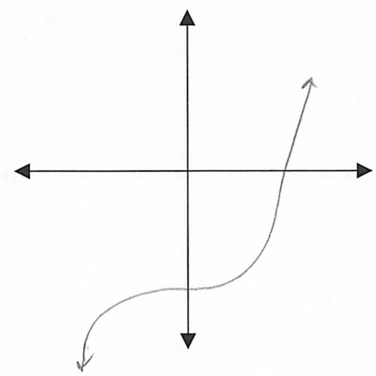
b) degree? 3

c) bumps? 2

d) real zeros? 3

e) imaginary zeros? None

7. $y = x^3 - 3x^2 + 4x - 12$



a) tails? Opposite

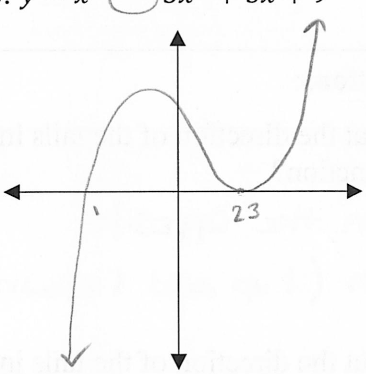
b) degree? 3

c) bumps? 2

d) real zeros? 1

e) imaginary zeros? 2

8. $y = x^3 - -5x^2 + 3x + 9$



a) tails? Opposite

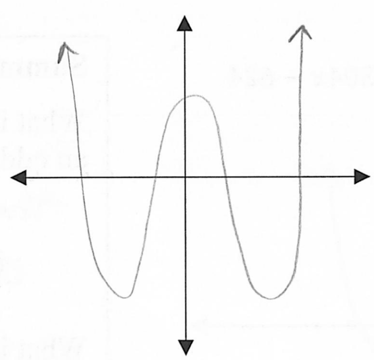
b) degree? 3

c) bumps? 2

d) real zeros? 3

e) imaginary zeros? None

9. $y = x^4 - 2x^3 - 13x^2 + 14x + 24$



a) tails? Same

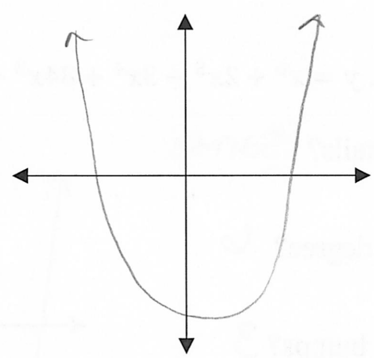
b) degree? 4

c) bumps? 3

d) real zeros? 4

e) imaginary zeros? None

10. $y = x^4 - x^3 + 3x^2 - 9x - 54$



a) tails? Same

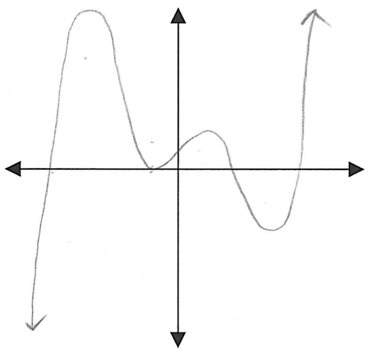
b) degree? 4

c) bumps? 1

d) real zeros? 2

e) imaginary zeros? 2

11. $y = x^5 + 3x^4 - 7x^3 - 11x^2 + 6x + 8$



a) tails? Opposite

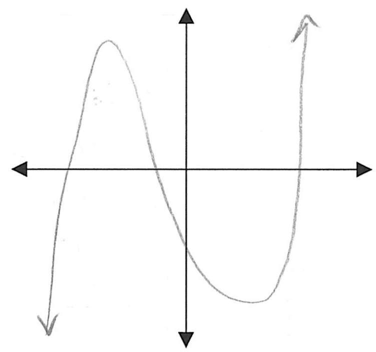
b) degree? 5

c) bumps? 4

d) real zeros? 5

e) imaginary zeros? None

12. $y = x^5 - 2x^4 + 40x^2 - 41x - 78$



a) tails? Opposite

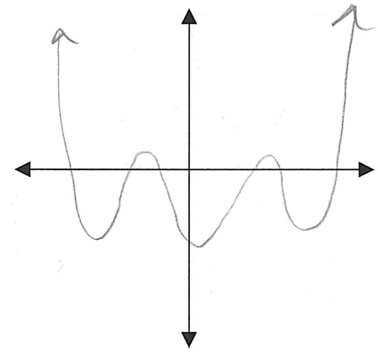
b) degree? 5

c) bumps? 2

d) real zeros? 3

e) imaginary zeros? 2

13. $y = x^6 - 14x^4 + 49x^2 - 36$



a) tails? Same

b) degree? 6

c) bumps? 5

d) real zeros? 6

e) imaginary zeros? None

14. $y = x^6 + 2x^5 - 3x^4 + 84x^3 - 184x^2 + 304x - 624$

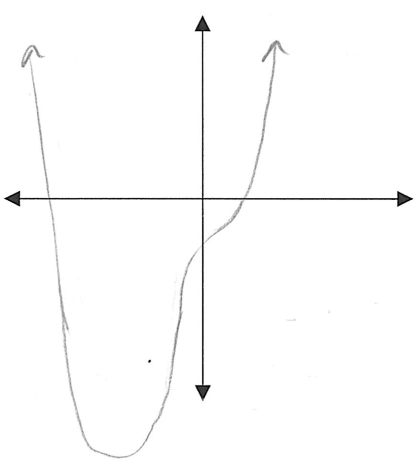
a) tails? Same

b) degree? 6

c) bumps? 3

d) real zeros? 2

e) imaginary zeros? 4



Summary Questions:

What is true about the direction of the tails in an odd degree function?

They go in the opposite direction (1 up and 1 down)

What is true about the direction of the tails in an even degree function?

They go in the same direction (Both up OR Both down)

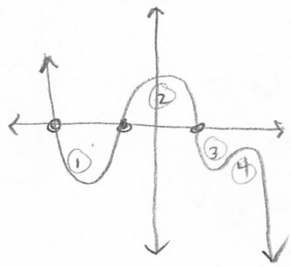
How does the maximum number of bumps (relative max and min) compare to the degree of a function?

One less than the degree

Extended Practice: Use what you have observed about the graphs of higher degree functions to sketch graphs of the functions described.

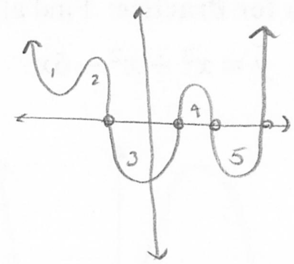
1. Quintic (5th degree) function with exactly 3 real zeros

- Tails - opposite ✓
- Max 4 bumps ✓
- Crosses x-axis ✓ 3 times
- Max - 5 roots



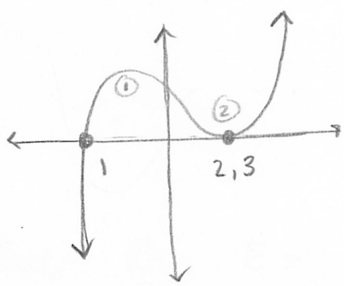
2. Sixth degree function with exactly 4 real zeros

- Tails - Same ✓
- Max 5 bumps ✓
- Crosses x-axis 4 times ✓
- Max 6 roots



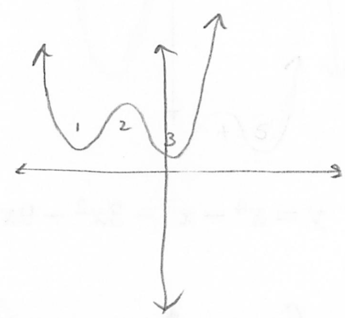
3. Cubic function with exactly two distinct real zeros

- Tails - opposite
- Max 2 bumps
- Crosses x-axis 1 time + touches 1 time
- Max 3 roots



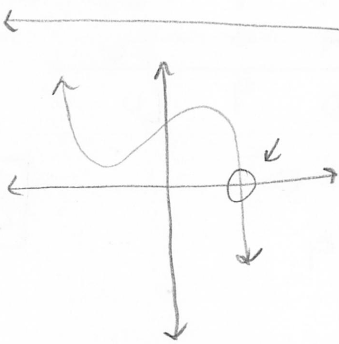
4. Quartic (4th degree) function with no real zeros

- Tails - same
- Max 3 bumps
- Crosses x-axis 0 times
- Max 4 roots



5. Cubic function with no real zeros

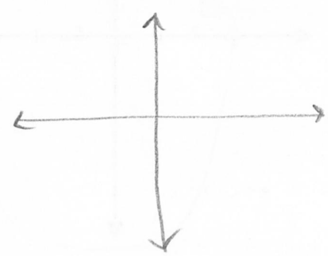
- Tails - opposite
- Max 2 bumps
- Crosses x-axis 0 times
- Max 3 roots



(Not possible)

6. Quartic function with exactly five real zeros

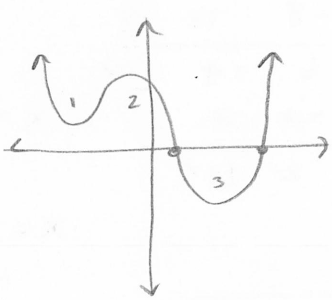
- Tails - same
- Max 3 bumps
- Max 4 roots
- Crosses x-axis 5 times



(Not Possible)

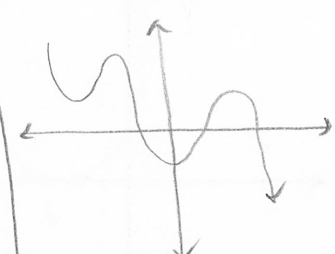
7. Quartic function with exactly 2 real zeros

- Tails - Same ✓
- Max 3 bumps ✓
- Max 4 roots
- Crosses x-axis 2 times ✓



8. Seventh degree function with 4 real zeros

- Tails - opposite
- Max 6 bumps
- Max 7 roots
- Crosses x-axis 4 times



(Not possible)

need to have an even amount of imginary roots.

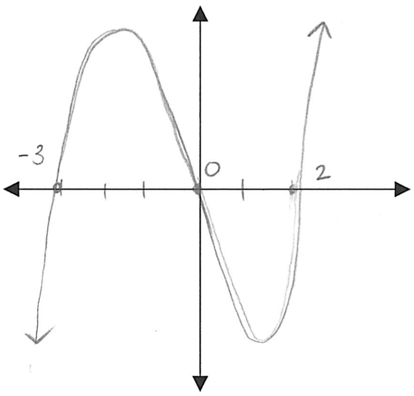
Finding all Rational and Complex Zeros of Polynomial Functions

In this section we will take all that we have previously learned to find all of the rational and complex zeros of a polynomial function.

Roots/Solutions

Break for Practice: Find all of the rational and complex zeros for each polynomial function.

1. $y = x^3 + x^2 - 6x$



$-3\rfloor$ 1   1   -6   0
↓   -3   6   0

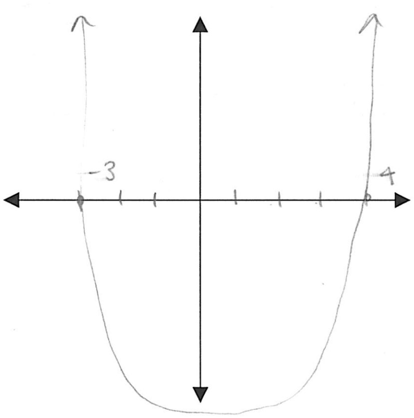
$0\rfloor$ 1   -2   0 $\underline{|0}$ ✓
↓   0   0

$2\rfloor$ 1   -2 $\underline{|0}$ ✓
↓   2

1   $\underline{|0}$ ✓

$\boxed{\{-3, 0, 2\}}$

2. $y = x^4 - x^3 - 3x^2 - 9x - 108$



$-3\rfloor$ 1   -1   -3   -9   -108
↓   -3   12   -27   108

$4\rfloor$ 1   -4   9   -36 $\underline{|0}$
↓   4   0   36

1   0   9 $\underline{|0}$

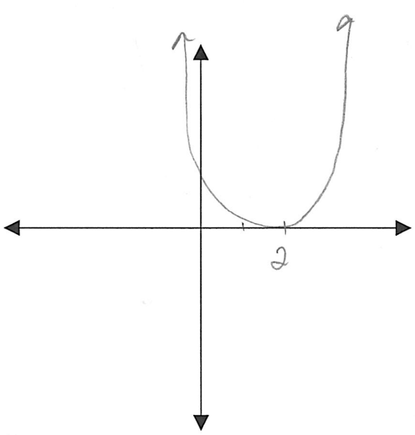
$x^2 + 9 = 0$

$\sqrt{x^2} = \sqrt{-9}$

$x = \pm 3i$

$\boxed{\{-3, 4, 3i, -3i\}}$

3. $y = x^4 - 6x^3 + 14x^2 - 16x + 8$



$2\rfloor$ 1   -6   14   -16   8
↓   2   -8   12   -8

$2\rfloor$ 1   -4   6   -4 $\underline{|0}$ ✓
↓   2   -4   4

1   -2   2 $\underline{|0}$ ✓

$x = \dfrac{2 \pm \sqrt{4 - 4(1)(2)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{-4}}{2}$

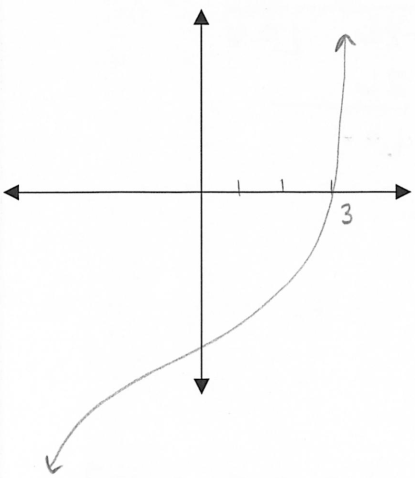
$x = \dfrac{\cancel{2} \pm \cancel{2}i}{\cancel{2}}$

$x = 1 \pm i$

$\boxed{\{2, 2, 1+i, 1-i\}}$

Extended Practice: Find all of the zeros for each polynomial function.

1. $y = x^3 - 3x^2 + 4x - 12$



3	1	-3	4	-12
	↓	3	0	12
	1	0	4	L0✓

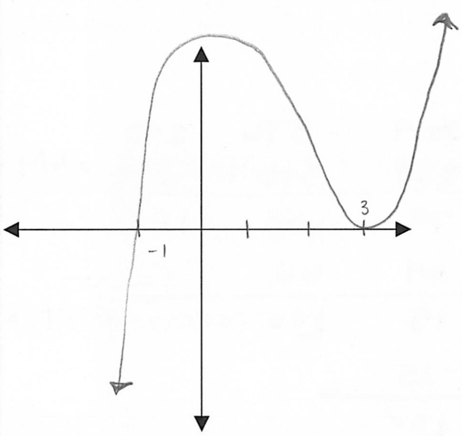
$x^2 + 4 = 0$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

$\{3, 2i, -2i\}$

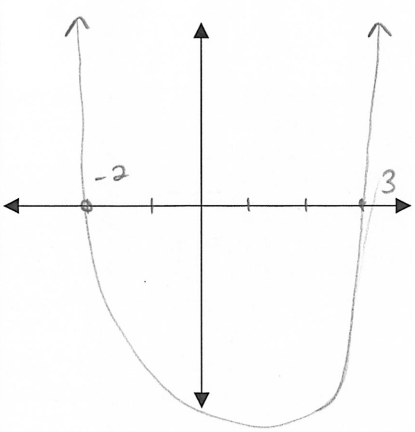
2. $y = x^3 - 5x^2 + 3x + 9$



-1	1	-5	3	9
	↓	-1	6	-9
3	1	-6	9	L0✓
	↓	3	-9	
3	1	-3	L0✓	
	↓	3		
	1	L0✓		

$\{-1, 3, 3\}$

3. $y = x^4 - x^3 + 3x^2 - 9x - 54$



3	1	-1	3	-9	-54
	↓	3	6	27	54
-2	1	2	9	18	L0
	↓	-2	0	-18	
	1	0	9	L0	

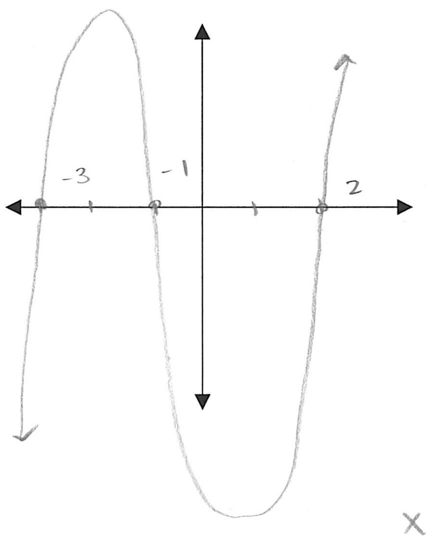
$x^2 + 9 = 0$

$\sqrt{x^2} = \sqrt{-9}$

$x = \pm 3i$

$\{-2, 3, 3i, -3i\}$

4. $y = x^5 - 2x^4 + 40x^2 - 41x - 78$



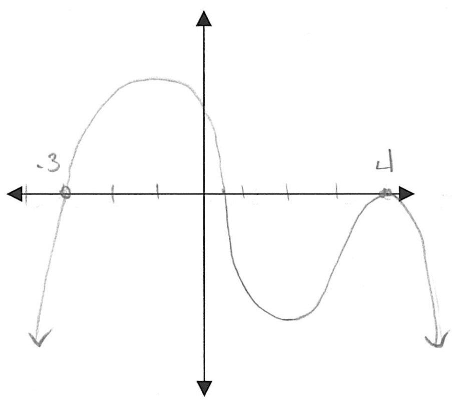
$-3\rfloor$	1	-2	0	40	-41	-78
	↓	-3	15	-45	15	78
$-1\rfloor$	1	-5	15	-5	-26	Lo✓
	↓	-1	6	-21	26	
$2\rfloor$	1	-6	21	-26	Lo✓	
	↓	2	-8	26		
	1	-4	13	Lo✓		
	a	b	c			

$$x = \frac{4 \pm \sqrt{16 - 4(1)(13)}}{2(1)}$$

$$x = \frac{4 \pm 6i}{2} \rightarrow x = 2 \pm 3i \qquad \{-3, -1, 2, 2+3i, 2-3i\}$$

5. $y = -2x^6 + 5x^5 + 34x^4 - 16x^3 - 209x^2 - 376x + 240$

Try a viewing window of x-min = -5, x-max = 5, y-min = -1500, and y-max = 1000



$4\rfloor$	-2	5	34	-16	-209	-376	240
	↓	-8	-12	88	288	316	-240
$4\rfloor$	-2	-3	22	72	79	-60	Lo✓
	↓	-8	-44	-88	-64	60	
$-3\rfloor$	-2	-11	-22	-16	15	Lo✓	
	↓	6	15	21	-15		
$\frac{1}{2}\rfloor$	-2	-5	-7	5	Lo✓		
	↓	-1	-3	-5			
	-2	-6	-10	Lo✓			
	a	b	c				

$$x = \frac{6 \pm \sqrt{36 - (4)(-2)(-10)}}{2(-2)}$$