Algebra II – Mrs. Tilus

Unit 8 Review

Name: ____________________

1. Divide. You may use long **or** synthetic division.

a) $\dfrac{2x^3+3x-5}{x+2}$

$2x^2-4x+11$

$x+2\ \overline{)\,2x^3+0x^2+3x-5}$

$-\ 2x^3 \mp 4x^2$

$0 - 4x^2+3x$

$\pm 4x^2 \mp 8x$

$0 + 11x - 5$

$-\ 11x \mp 22$

$\boxed{-27}$

or

$-2\,|\ 2 \quad 0 \quad 3 \quad -5$

$\downarrow \quad -4 \quad 8 \quad -22$

$2 \quad -4 \quad 11 \quad \boxed{-27}$

$\boxed{2x^2-4x+11+\frac{-27}{x+2}}$

b) $\dfrac{10x^2+x-3}{5x+3}$ $5(x+3/5)$

$2x-1$

$5x+3\ \overline{)\,10x^2+x-3}$

$-\ 10x^2 \mp 6x$

$0 \quad -5x-3$

$\pm 5x \pm 3$

$0+0$

$-3/5\,|\ 10 \quad 1 \quad -3$

$\downarrow \quad -6 \quad 3$

$10 \quad -5 \quad \llcorner 0$

$\frac{10x}{5} - \frac{5}{5}$

$2x-1$

$\boxed{2x-1}$

c) $\dfrac{3x^4+x^3-2x+7}{x^2-x+1}$

$3x^2+4x+1$

$x^2-x+1\ \overline{)\,3x^4+x^3+0x^2-2x+7}$

$-\ 3x^4 \mp 3x^3 \pm 3x^2$

$4x^3-3x^2-2x$

$-\ 4x^3 \mp 4x^2 \pm 4x$

$0 + x^2 - 6x + 7$

$-\ x^2 \mp x \pm 1$

$0 - 5x + 6$

$\boxed{3x^2+4x+1+\frac{-5x+6}{x^2-x+1}}$

2. **Use synthetic substitution** to find P(c) for the given polynomial P(x) and the given number c.

$P(x) = x^3 + 2x^2 - 6x - 4$; $c = -2$

$-2\,|\ 1 \quad 2 \quad -6 \quad -4$

$\downarrow \quad -2 \quad 0 \quad 12$

$1 \quad 0 \quad -6 \quad 8$

P(-2)= 8

3. Use the factor theorem to determine whether x + 1 is a factor of P(x). Show your work to receive full credit, and circle the answer.

$P(x) = x^{12} - 3x^8 - 4x - 2$

Circle one: [✓ Factor] or Not a Factor

-1	1	0	0	0	-3	0	0	0	0	0	0	-4	-2
	↓	-1	1	-1	1	2	-2	2	-2	2	-2	2	2
	1	-1	1	-1	-2	2	-2	2	-2	2	-2	-2	L0

4. Consider the polynomial $2x^3 - 5x^2 - 4x + 3$.

a) State the number of possible factors. 3

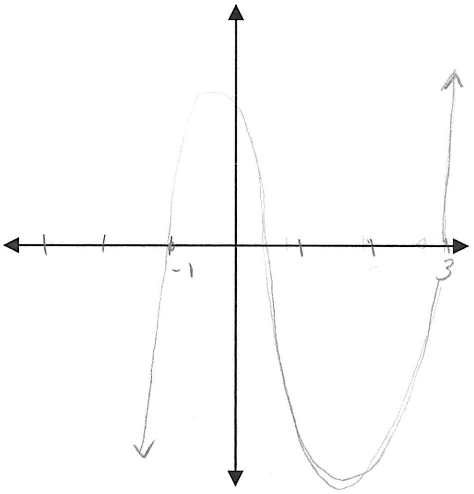
b) State all of the possible roots by using the rational root theorem.

h: ±1±3

K: ±1±2

$\Rightarrow \boxed{\pm 1 \pm 1/2 \pm 3 \pm 3/2}$

c) Draw a graph of the polynomial.
(Include tick marks for x-axis)



d) By using the calculator and/or synthetic division, write the polynomial in factored form.

-1	2	-5	-4	3
	↓	-2	7	-3
3	2	-7	3	L0
	↓	6	-3	
1/2	2	-1	L0	
	↓	1		
	2	L0 ✓		

Roots	→ Factors
X = -1	X + 1
X = 3	X - 3
X = 1/2	2X - 1

Factors: { (X+1)(X-3)(2X-1) }

5. Write a third-degree equation which has solutions of $x = -3i$, and $x = 5$. X = 3i

imaginary roots come in pairs

$x = -3i \longrightarrow (x+3i)$

$X = 3i \longrightarrow (X-3i)$

$X = 5 \longrightarrow (X-5)$

$y = (x+3i)(X-3i)(x-5)$

$y = (x^2 \cancel{-3ix} + \cancel{3ix} - 9\overset{-1}{i^2})(x-5)$

$y = (x^2+9)(x-5)$

$\boxed{y = x^3 - 5x + 9x - 45}$

6. Answer each question.

a) What is true about the tails of an even degree function?

the go the same direction

b) What is the maximum number of "bumps" in a 6th degree polynomial?

5 (one less then the degree)

c) Can an even degree polynomial have no x-intercepts?

Yes.

d) Can an odd degree polynomial have no x-intercepts?

No.

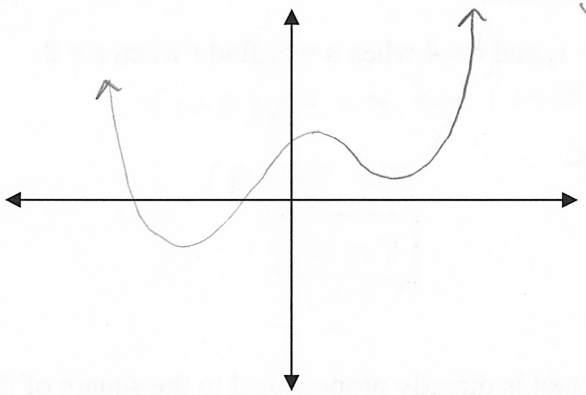
e) Can a polynomial with real coefficients have only one imaginary root?

No.

7. Draw a graph for a fourth-degree polynomial equation that has two real roots.

4th Degree:

- Max: Crosses x-axis 4 times
- Max: 3 bumps
- Tails: Same direction



Crosses x-axis only twice

8. Given the following entries from a table for a function L, use linear interpolation to estimate x to three significant digits if L(x) = 0.525.

x	1.5	1.6	1.7	1.8
L(x)	0.405	0.470	0.531	0.588

X	L(X)
1.6	0.470
X_1	0.525
1.7	0.531

d

0.1

0.055

0.061

$$\frac{d}{0.1} = \frac{0.055}{0.061}$$

$$\frac{0.061d}{0.061} = \frac{0.0055}{0.061}$$

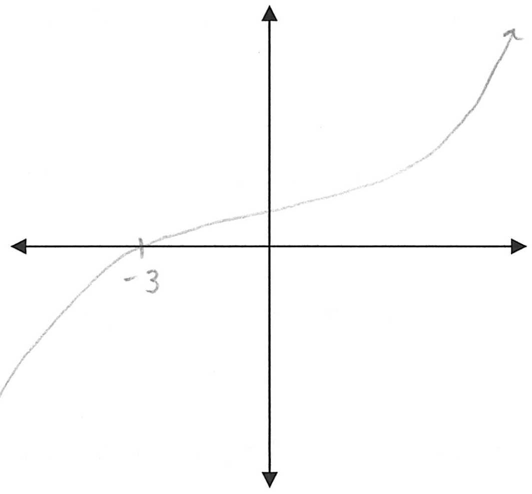
$d = 0.0902$

$X = 1.6 + 0.0902$

$$\boxed{X = 1.6902}$$

9. Consider the function $y = x^3 + 3x^2 + 16x + 48$

a) Draw a graph of the function. **(Include tick marks for x-axis)**



b) Find the values of the **real** zeros of the function.

c) Find the values of the **imaginary** zeros of the function.

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 16 & 48 \\ & \downarrow & -3 & 0 & -48 \\ \hline & 1 & 0 & 16 & \llcorner 0 \end{array}$$

$x^2 + 16 = 0$

$\sqrt{x^2} = \sqrt{-16}$

$\underline{x = \pm 4i}$

Zeros: $\{ -3, 4i, -4i \}$

10. If r is **directly** proportional to s + 1, and (r = 4 when s = 5,) find r when s = 8.

$r = K(s+1) \Rightarrow 4 = K(5+1) \Rightarrow r = 2/3(8+1)$

$\frac{4}{6} = \frac{6K}{6}$ $\qquad$ $r = 2/3(9)$

$\underline{\frac{2}{3} = K}$ $\qquad$ $\boxed{r = 6}$

$K =$ 2/3

$r =$ 6

11. The distance an object falls from rest is directly proportional to the square of the length of time it has fallen. If an object falls 64 feet in 2 seconds, how far will it fall in 3 seconds?

$D = Kt^2 \Rightarrow 64 = K(2^2) \Rightarrow D = 16(3^2)$

$\frac{64}{4} = \frac{4K}{4}$ $\qquad$ $D = 16(9)$

$\underline{16 = K}$ $\qquad$ $\boxed{D = 144 \text{ ft}}$

$K =$ 16

$D =$ 144ft

12. If y varies **inversely** with x, and (y = 5 when x = 4,) find x when y = 10.

$y = \frac{K}{x} \Rightarrow 4 \cdot 5 = \frac{K}{4} \cdot 4 \Rightarrow x \cdot 10 = \frac{20}{x} \cdot x$

$\underline{20 = K}$ $\qquad$ $\frac{10x}{10} = \frac{20}{10}$

$\boxed{x = 2}$

$K =$ 20

$x =$ 2