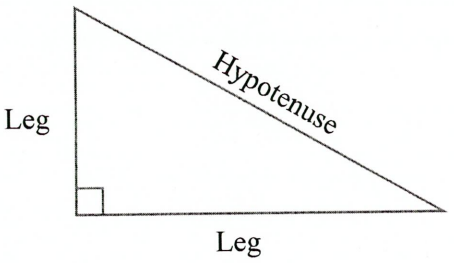
Chapter 7.5: Apply the Tangent Ratio

Definitions:

A **trigonometric ratio** is a ratio of the lengths of two sides in a right triangle. You will use trigonometric ratios to find…

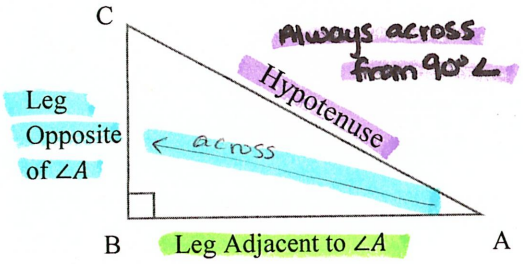
1. Side lengths
2. angle measures (two acute angles)



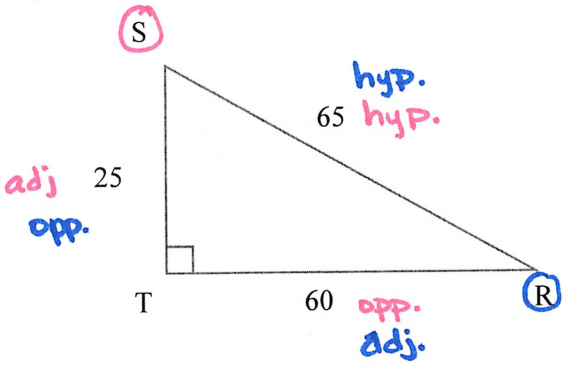
Tangent Ratio: the ratio of the lengths of the legs in a right triangle.

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of of $\angle A$ (written as tan A) is defined as follows:

$$\tan A = \frac{\text{leg opposite of } \angle A}{\text{leg adjacent of } \angle A} = \frac{BC}{AB}$$



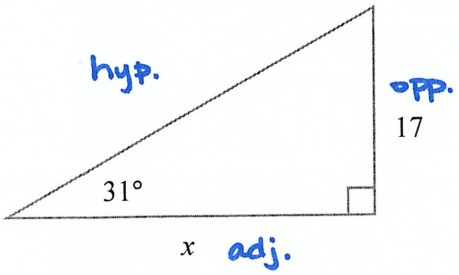
Example #1: Find tan S and tan R. Write each answer as a fraction and as a decimal rounded to four places, if necessary.



$$\tan S = \frac{TR}{ST} = \frac{60}{25} = \frac{12}{5} \approx 2.4$$

$$\tan R = \frac{ST}{TR} = \frac{25}{60} = \frac{5}{12} \approx 0.4167$$

Example #2: Find the value of x.

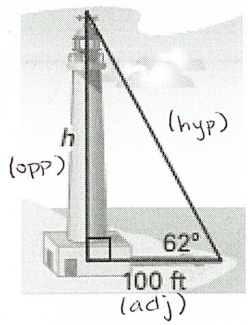


$$x(\tan 31°) = \left(\frac{17}{x}\right)x$$

$$\frac{x \cdot (\tan 31°)}{(\tan 31°)} = \frac{17}{(\tan 31°)}$$

$$x \approx 28.3°$$

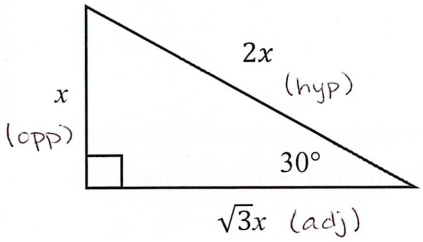
Example #3: Find the height h of the lighthouse to the nearest foot.



$100(\tan 62°) = \left(\frac{h}{100}\right)100$

$h = 188$ feet

Example #4: Use a **special right triangle** to find the tangent of a 30° angle. ★ $\tan = \frac{\text{opp}}{\text{adj.}}$ ★



$\tan 30° = \frac{*}{\sqrt{3}\cdot *}$

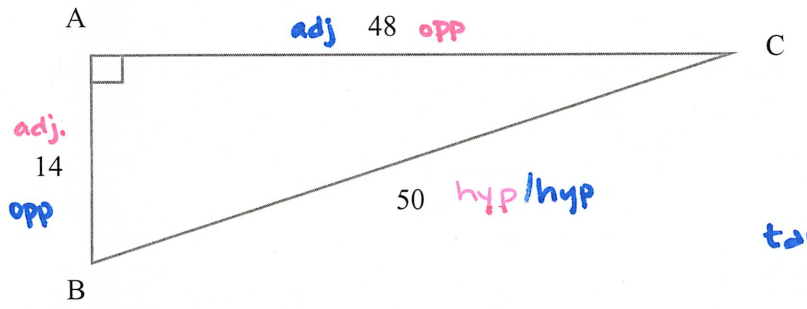
$\tan 30° = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\tan 30° = \frac{\sqrt{3}}{\sqrt{9}}$

$\tan 30° = \frac{\sqrt{3}}{3}$

Checkpoint:

1. Find tan B and tan C. Write each answer as a fraction and as a decimal rounded to four places.



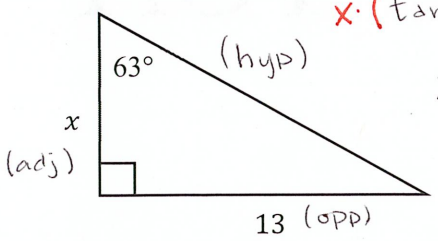
$\tan B = \frac{48}{14}$

$\tan B = \frac{24}{7} \approx 3.4286$

$\tan C = \frac{14}{48} = \frac{7}{24} \approx 0.2917$

Find the value of x. Round to the nearest tenth.

2.

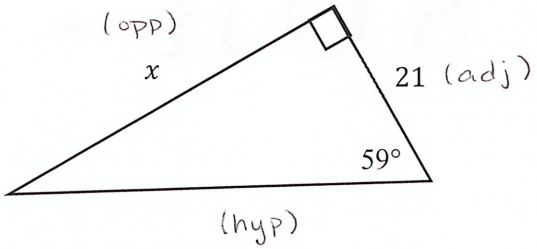


$x \cdot (\tan 63°) = \left(\frac{13}{x}\right)x$

$\frac{x \cdot \tan 63°}{(\tan 63°)} = \frac{13}{(\tan 63°)}$

$x = 6.6$ units

3.



$21(\tan 59°) = \left(\frac{x}{21}\right)21$

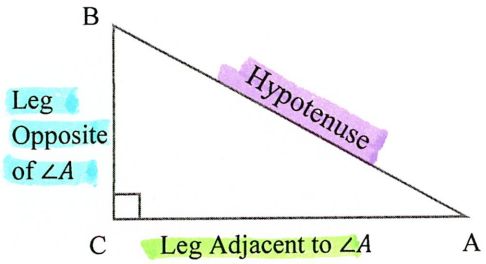
$x = 34.9$ units

Chapter 7.6: Apply the Sine and Cosine Ratios

Definitions:

Sine and Cosine Ratio:

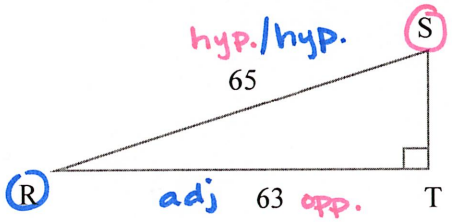
Let $\triangle ABC$ be a right triangle with acute $\angle A$.
The sine and cosine of $\angle A$ (written as sin A and cos A) are defined as follows:



$\sin A = \dfrac{\text{opposite leg of } \angle A}{\text{hypotenuse}} = \dfrac{BC}{AB}$

$\cos A = \dfrac{\text{adjacent leg of } \angle A}{\text{hypotenuse}} = \dfrac{CA}{AB}$

Example #1: Find sin S and sin R. Write each answer as a fraction and as decimal rounded to four places.

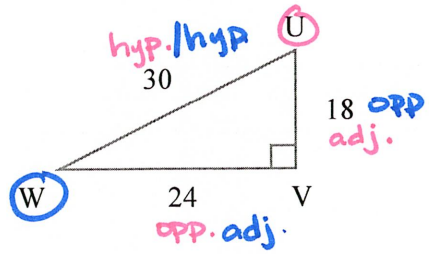


reduced

$\sin S = \frac{63}{65} \approx 0.9692$

$\sin R = \frac{16}{65} \approx 0.2462$

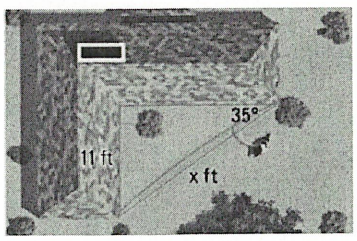
Example #2: Find cos U and cos W. Write each answer as a fraction and as a decimal round to four places.



$\cos U = \frac{18}{30} = \frac{3}{5} \approx 0.6$

$\cos W = \frac{24}{30} = \frac{4}{5} = 0.8$

Example #3: You want to string cable to make a dog run from two corners of a building, as shown in the diagram. Write and solve a proportion using a trigonometric ratio to approximate the length of cable you will need

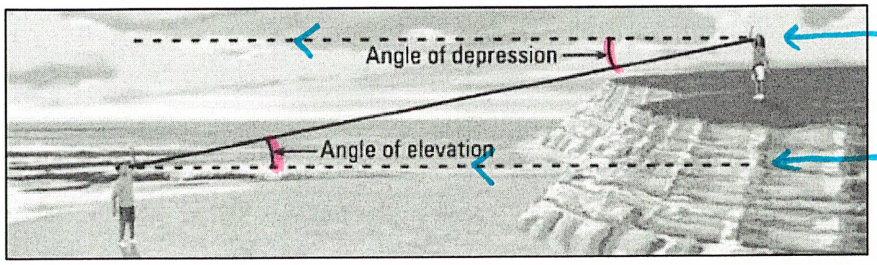


$x \cdot (\sin 35°) = \left(\frac{11}{x}\right) \cdot x$

$\frac{x \cdot (\sin 35°)}{(\sin 35°)} = \frac{11}{(\sin 35°)}$

$x \approx 19.2$ feet

Definitions:



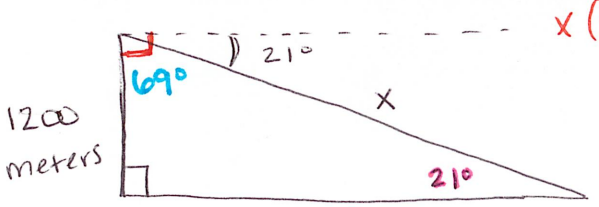
Angle of Elevation: When looking up at an object, the angle your line of sight makes with a horizontal line.

Angle of Depression: When looking down at an object, the angle your line of sight makes with a horizontal line

Angle of elevation and depression are alternate, interior angles

*****These angles are** congruent *******

Example #4: You are skiing on a mountain with an altitude of 1200 meters. The angle of depression is 21°. About how far do you ski down the mountain?



$X(\sin 21°) = \left(\frac{1200}{X}\right)X$

$\frac{X \cdot \sin 21°}{(\sin 21°)} = \frac{1200}{(\sin 21°)}$

$X = 3{,}349$ meters

OR

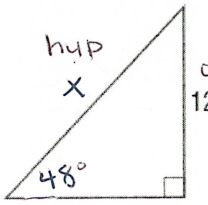
$X(\cos 69°) = \left(\frac{1200}{X}\right)X$

$\frac{X \cdot \cos 69°}{(\cos 69°)} = \frac{1200}{(\cos 69°)}$

$X = 3{,}349$ meters

Example #5: Find *x*. Round answers to the nearest tenth.

a.) Change

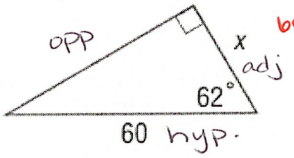


$X(\sin 48°) = \left(\frac{12}{X}\right)X$

$\frac{X \cdot \sin 48}{(\sin 48°)} = \frac{12}{(\sin 48°)}$

$X = 16.1$ units

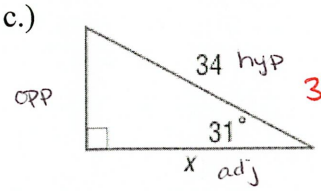
b.)



$(\cos 62°)60 = \left(\frac{X}{60}\right)60$

$X = 28.2$ units

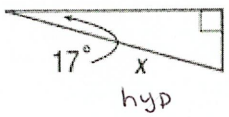
c.)



$34(\cos 31°) = \left(\frac{X}{34}\right)34$

$X = 29.1$ units

d.)



$6.6\,X(\sin 17°) = \left(\frac{6.6}{X}\right)X$

$\frac{X \cdot \sin 17°}{(\sin 17°)} = \frac{6.6}{(\sin 17°)}$

$X = 22.6$ units

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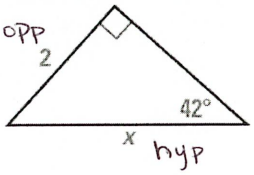
Sine = $\frac{\text{opposite leg}}{\text{hypotenuse}}$ Cosine = $\frac{\text{adjacent leg}}{\text{hypotenuse}}$ Tangent = $\frac{\text{opposite leg}}{\text{adjacent leg}}$

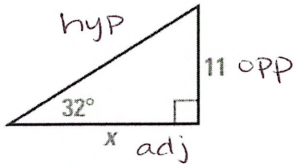
Check Point: Find the measure of the unknown side lengths. Round answers to the nearest tenth.

1. 

$x(\sin 42°) = \left(\frac{2}{x}\right)x$

$\frac{x \cdot \sin 42°}{(\sin 42°)} = \frac{2}{(\sin 42°)}$

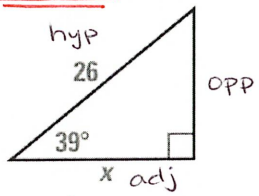
$x = 3$ units

2. 

$x(\tan 32°) = \left(\frac{11}{x}\right)x$

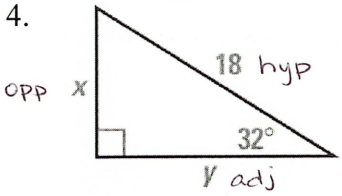
$\frac{x \cdot \tan 32°}{(\tan 32°)} = \frac{11}{(\tan 32°)}$

$x = 17.6$ units

3. 

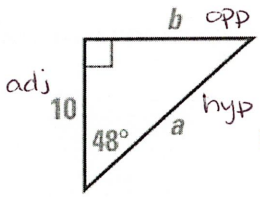
$26(\cos 39°) = \left(\frac{x}{26}\right)26$

$x = 20.2$ units

4. 

$8(\sin 32°) = \left(\frac{x}{18}\right)18$ $18(\cos 32°) = \left(\frac{y}{18}\right)18$

$x = 9.5$ units $y = 15.3$ units

5. 

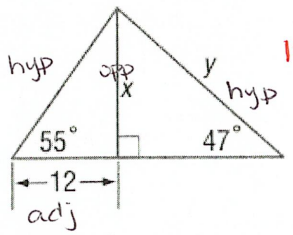
$10(\tan 48°) = \left(\frac{b}{10}\right)10$

$b = 11.1$ units

$a(\cos 48°) = \left(\frac{10}{a}\right)a$

$\frac{a \cdot \cos 48°}{(\cos 48°)} = \frac{10}{(\cos 48°)}$

$a = 15$ units

6. 

$12(\tan 55°) = \left(\frac{x}{12}\right)12$

$x = 17.1$ units

$y(\sin 47°) = \left(\frac{17.1}{y}\right)y$

$\frac{y \cdot \sin 47°}{(\sin 47°)} = \frac{17.1}{(\sin 47°)}$

$y = 23.4$ units