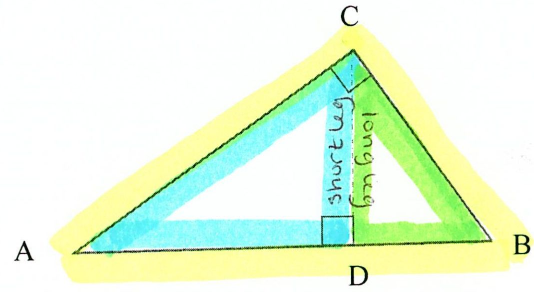
Chapter 7.3: Use Similar Right Triangles

Altitude Similarity Theorem (Theorem 7.5):

If the altitude is drawn to the hypotenuse of a **right triangle**,

then the 2 triangles formed are Similar

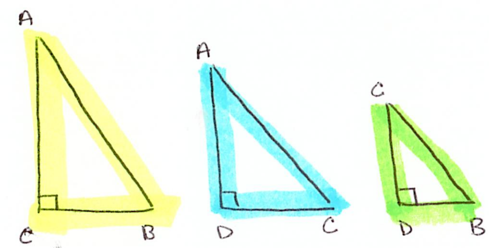
to the original triangle and each other.



$\triangle ACB \sim \triangle ADC$

$\triangle ADC \sim \triangle CDB$

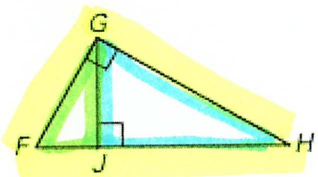
$\triangle CDB \sim \triangle ACB$

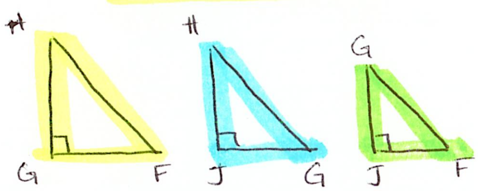


* The altitude (CD) is the short leg of the medium $\triangle$ and the long leg of the small $\triangle$.

Example #1: Identify the similar triangles in the diagram

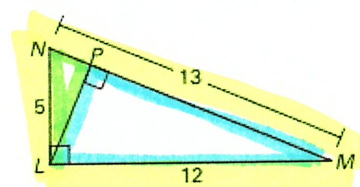
a.)

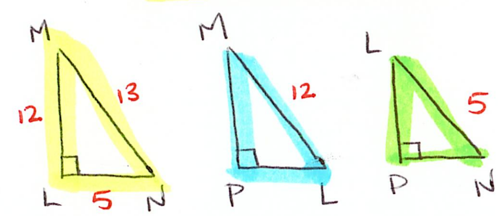




$\triangle HGF \sim \triangle HJG \sim \triangle GJF$

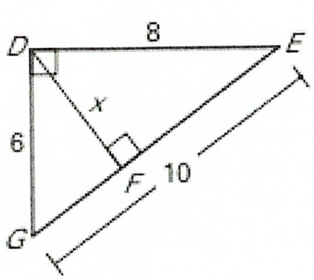
b.)

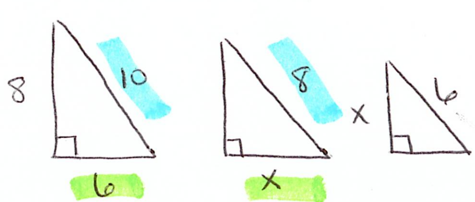




$\triangle MLN \sim \triangle MPL \sim \triangle LPN$

Example #2: Identify the similar triangles. Then find *x*



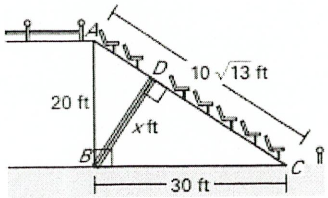


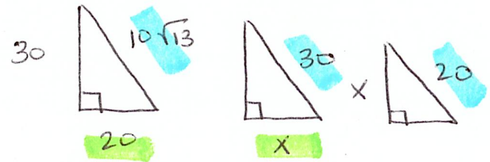
$\frac{\text{hypotenuse}}{\text{short leg}} \Rightarrow \frac{10}{6} = \frac{8}{x}$

$\frac{10x}{10} = \frac{48}{10}$

$x = 4.8$ units

Example #3: A cross section of a group of seats at a stadium shows a drainage pipe $\overline{BD}$ that leads from the seats to the inside of the stadium. What is the length of the pipe?



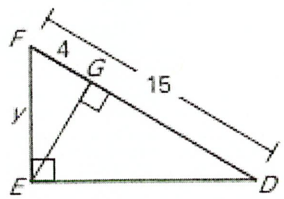


$\frac{\text{hypotenuse}}{\text{short}} \Rightarrow \frac{10\sqrt{13}}{20} = \frac{30}{x}$

$\frac{10\sqrt{13}\,x}{10\sqrt{13}} = \frac{600}{10\sqrt{13}}$

$x = 16.6$ ft

Example #4: Find the value of y. Write your answer in simplest radical form



$\frac{\text{hypotenuse}}{\text{short}} \Rightarrow \frac{15}{y} = \frac{y}{4}$

$\sqrt{y^2} = \sqrt{60} \Rightarrow y = 2\sqrt{3 \cdot 5}$

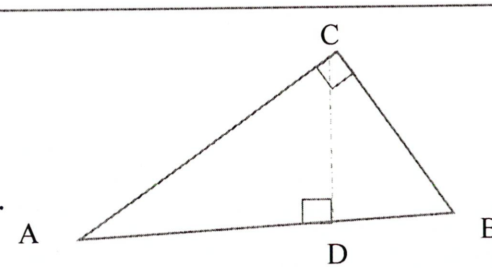
60 → 2, 30 → 2, 15 → 3, 5

$y = 2\sqrt{15}$

Geometric Mean (Altitude) Theorem (Theorem 7.6):

In a right triangle, the altitude from the right angles to the hypotenuse (CD), divides the hypotenuse into two segments.

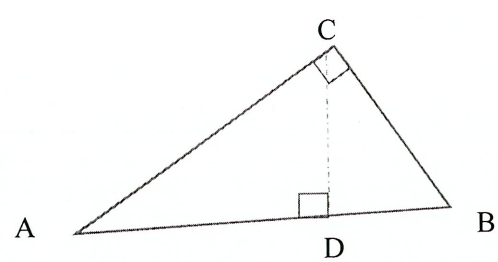
The length of the **altitude** is the geometric mean of the lengths of the two segments.



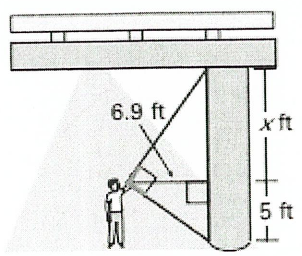
Geometric Mean (Leg) Theorem (Theorem 7.7):

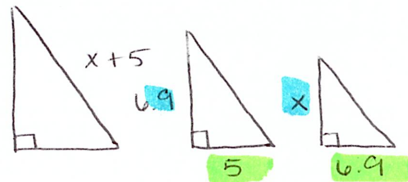
In a right triangle, the altitude from the right angles to the hypotenuse, divides the hypotenuse into two segments

The length of each leg of the right triangle is the geometric mean of the lengths of the Hypotenuse and the segment of the hypotenuse that is adjacent to the leg



Example #5: To find the clearance under an overpass, you need to find the height of a concrete support beam. You use a cardboard square to line up the top and bottom of the beam. Your friend measures the vertical distance from the ground to your eye and the distance from you to the beam. Approximate the height of the beam.



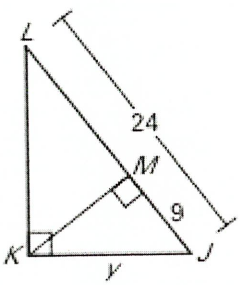


$$\frac{6.9}{5} = \frac{x}{6.9} \Rightarrow \frac{5x}{5} = \frac{47.61}{5}$$

$x = 9.5 \text{ ft}$

$\boxed{\text{Height of beam} = 9.5 + 5 = 14.5 \text{ ft}}$

Example #6: Find the value of y. Write your answer in simplest radical form.



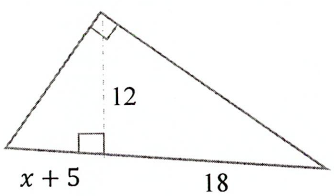
$$\frac{24}{y} = \frac{y}{9} \Rightarrow \sqrt{y^2} = \sqrt{216}$$

216 → 2, 108 → 2, 54 → 2, 27 → 3, 9 → 3, 3

$y = 2 \cdot 3 \sqrt{2 \cdot 3}$

$y = 6\sqrt{6}$ units

Example # 7: Find the value of x.



$$\frac{18}{12} = \frac{12}{x+5} \Rightarrow 144 = 18(x+5)$$

$144 = 18x + 90$

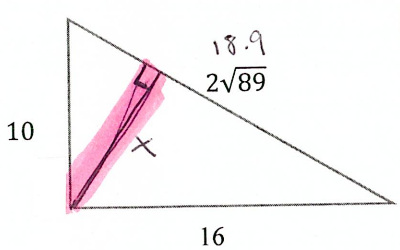
$-90 \qquad -90$

$\frac{54}{18} = \frac{18x}{18}$

$x = 3$ units

use Pythagorean Thrm

Example # 8: Tell whether the triangle is a right triangle. If so, find the length of the altitude to the hypotenuse. Round decimal answer to the nearest hundredth.



$10^2 + 16^2 \stackrel{?}{=} (2\sqrt{89})^2$

$356 = 356$ ✓

$$\frac{2\sqrt{89}}{10} = \frac{16}{x} \Rightarrow \frac{160}{2\sqrt{89}} = \frac{2\sqrt{89}\,x}{2\sqrt{89}}$$

$x = 8.48$ units