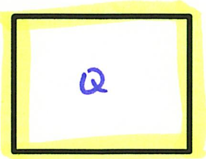
Stability in Structures:

A diagonal support form triangles with fixed side lengths. By SSS, these triangles cannot change shape. A structure without a diagonal support is not stable because there are many possible quadrilaterals with the given side length. ☆ Look for all triangles... no quadrilaterals ☆

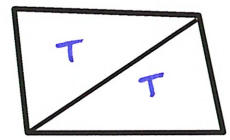
Example #4: Determine whether the figure is stable.

a.)



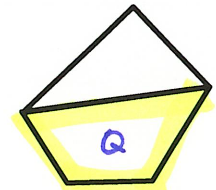
Not Stable

b.)



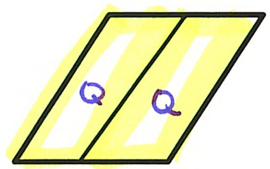
Stable

c.)



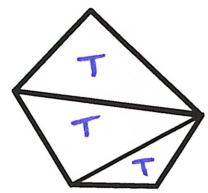
Not Stable

d.)



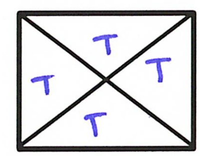
Not Stable

e.)



Stable

f.)



Stable

Chapter 4.4: Prove Triangles Congruent by SAS and HL

Side-Angle-Side (SAS) Congruence Postulate **(Postulate 20):** → in between two ≅ sides

If two sides and the ___included___ angles of one triangle are congruent to two sides and the included of a second triangle, then the two triangles are ___Congruent___

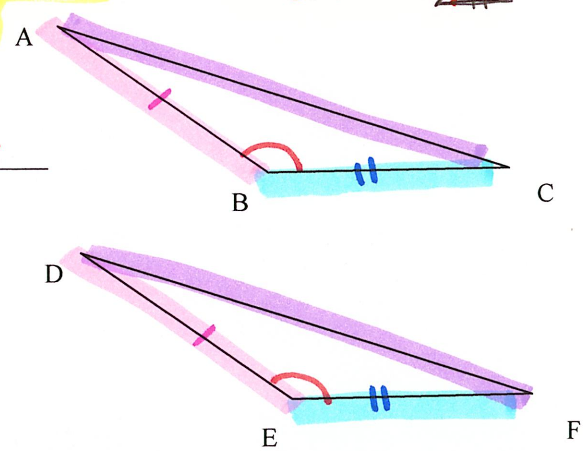
If ___S: $\overline{AB} \cong \overline{DE}$___

___A: $\angle B \cong \angle E$___

___S: $\overline{BC} \cong \overline{EF}$___

Then ___$\triangle ABC \cong \triangle DEF$___

congruence statement



Example #1: Use the diagram to name the included angle between the given pair of sides.

a.) $\overline{AB}$ and $\overline{BC}$

$\angle ABC$

Not just $\angle B$ because there are 3 different angles in that spot. ($\angle ABC$, $\angle ABD$, $\angle CBD$)

b.) $\overline{BC}$ and $\overline{CD}$

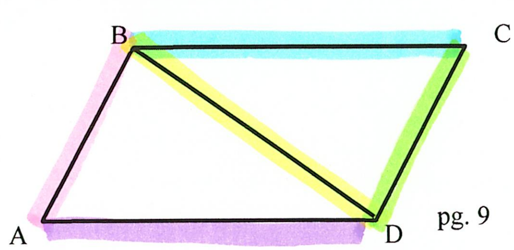
$\angle C$ or $\angle BCD$

c.) $\overline{AB}$ and $\overline{BD}$

$\angle ABD$

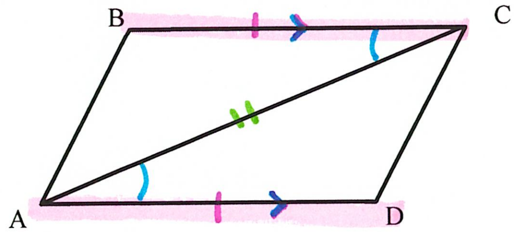
d.) $\overline{BD}$ and $\overline{DA}$

$\angle BDA$



Example #2: Prove.

Given: $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$,
Prove: $\triangle ABC \cong \triangle CDA$



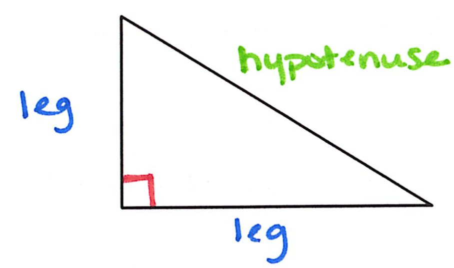
Statement	Reason
1. S: $\overline{BC} \cong \overline{DA}$ $\overline{BC} \parallel \overline{AD}$	1. Given
2. A: $\angle BCA \cong \angle DAC$	2. Alt. Interior Angles.
3. S: $\overline{AC} \cong \overline{CA}$	3. Reflexive Property
4. $\triangle ABC \cong \triangle CDA$	4. SAS

In general, if you know the lengths of two sides and the measure of an angle that is *not included* between them, you can create two different triangles.

Example:

Therefore, SSA is *NOT* a valid method for proving that triangles are congruent, although there is a special case for right triangles.

Right Triangles: In a right triangle the sides adjacent to the right angle are called the legs. The side opposite the right angles is called the hypotenuse



Hypotenuse-Leg (HL) Congruence Theorem (Theorem 4.5):

If the hypotenuse and a leg of a right triangles are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are Congruent.

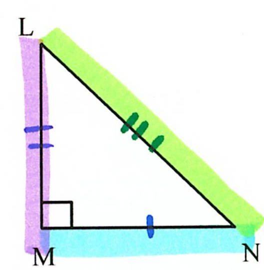
If $\triangle LMN$ and $\triangle OPQ$ are right $\triangle$'s ← this is a must

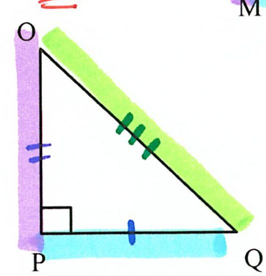
L: $\overline{MN} \cong \overline{PQ}$ (or $\overline{LM} \cong \overline{OP}$)

H: $\overline{LN} \cong \overline{OQ}$

Then $\triangle LMN \cong \triangle OPQ$

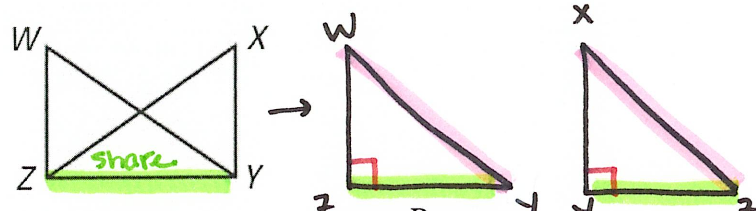
(congruence statement)





Example #3: Prove.

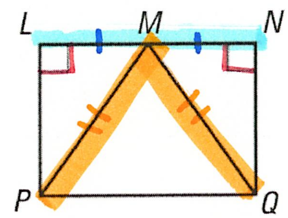
Given: $\overline{WY} \cong \overline{XZ}, \overline{WZ} \perp \overline{ZY}, \overline{XY} \perp \overline{ZY}$
Prove: $\triangle WYZ \cong \triangle XZY$



Statement	Reason
1. H: $\overline{WY} \cong \overline{XZ}$	1. Given
2. $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$	2. Given
3. ∠WZY and ∠XYZ are right ∠'s	3. Def^{n} of perpendicular lines
*4. △WZY and △XYZ are right △'s	4. Def^{n} of right triangles
5. L: $\overline{ZY} \cong \overline{YZ}$	5. Reflexive Property
6. $\triangle WYZ \cong \triangle XZY$	6. HL

Example #4: Prove.

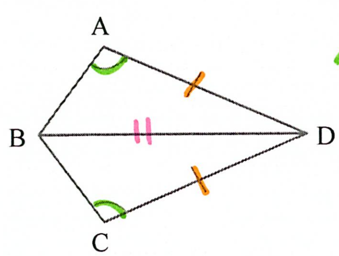
Given: Point M is the midpoint of $\overline{LN}$
$\triangle$PMQ is an isosceles triangle with $\overline{MP} \cong \overline{MQ}$
$\angle L$ and $\angle N$ are right angles
Prove: $\triangle LMP \cong \triangle NMQ$



Statement	Reason
1. ∠L and ∠N are right ∠'s	1. Given
*2. △LMP and △NMQ are right △'s	2. Def^{n} of right triangles
3. Point M is midpt of $\overline{LN}$	3. Given
4. L: $\overline{LM} \cong \overline{NM}$	4. Def^{n} of a midpoint
5. △PMQ is an isosceles triangle → $\overline{MP} \cong \overline{MQ}$: H	5. Given
6. $\triangle LMP \cong \triangle NMQ$	6. HL

Example #5: Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.

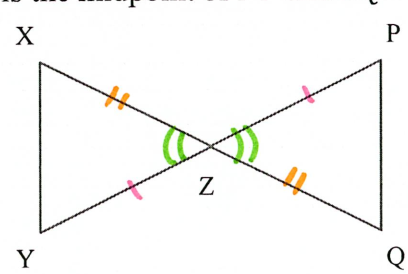
a.)



∠A ≅ ∠C
$\overline{AD} \cong \overline{CD}$
$\overline{BD} \cong \overline{BD}$

Not Congruent; SSA is not a valid method

b.) Z is the midpoint of $\overline{PY}$ and $\overline{XQ}$

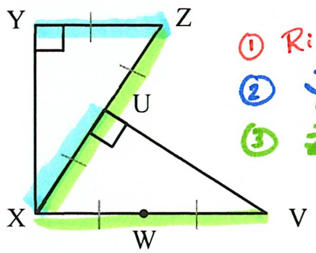


$\overline{PZ} \cong \overline{YZ}$
$\overline{XZ} \cong \overline{QZ}$
∠XZY ≅ ∠QZP

△XZY ≅ △QZP; SAS

Example #5: Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.

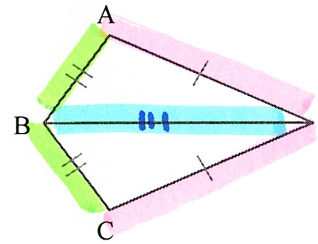
c.)



① Right Δ's
② $\overline{YZ} \cong \overline{UX}$
③ $\overline{ZX} \cong \overline{XV}$

$\Delta YZX \cong \Delta UXV$; HL

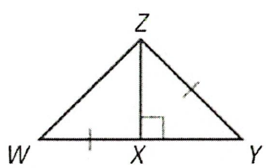
d.)



① $\overline{AD} \cong \overline{CD}$
② $\overline{AB} \cong \overline{CB}$
③ $\overline{BD} \cong \overline{BD}$

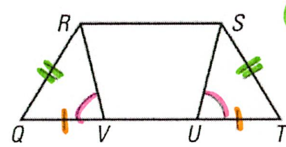
$\Delta ABD \cong \Delta CBD$; SSS

e.)



Not Enough Information

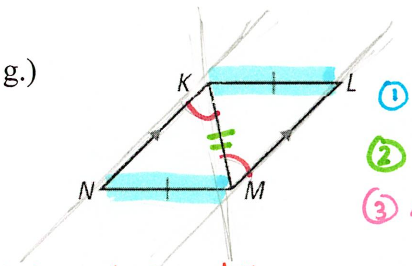
f.)



① $\overline{RQ} \cong \overline{ST}$
$\overline{QV} \cong \overline{TU}$
$\angle QVR \cong \angle TUS$

$\angle QVR$ and $\angle TUS$ are not the included angle → SSA is NOT a valid method

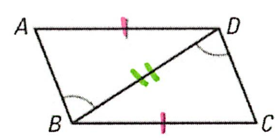
g.)



① $\overline{KL} \cong \overline{MN}$
② $\overline{MK} \cong \overline{KM}$
③ $\angle NKM \cong \angle LMK$

$\angle NKM$ and $\angle LMK$ are not included angles → SSA is not a valid method

h.)



① $\overline{BD} \cong \overline{DB}$
② $\overline{AD} \cong \overline{CB}$
③ $\angle ABD \cong \angle CDB$

$\angle ABD$ and $\angle CDB$ are not included angles → SSA is not a valid method

Chapter 4.5: Prove Triangles Congruent by ASA and AAS

Angle- Side-Angle (ASA) Congruent Postulate **(Postulate 21):**

If two angles and the <u>included</u> side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are <u>Congruent</u>.

If A : $\angle B \cong \angle Y$

S : $\overline{BC} \cong \overline{YZ}$

A : $\angle C \cong \angle Z$

Then $\Delta ABC \cong \Delta XYZ$

Congruence Statement