Break for Practice: Solve each system by using linear combination.

1. $3x + 2y = 6$
   $3(-x + 2y = -2)$

$3x + 2y = 6$
$-3x + 6y = -6$

$8y = 0$
$y = 0$

$-x + 2(0) = -2$
$-x = -2$
$x = 2$

Solution: $(2, 0)$

2. $(4x + 13y = -7)$ ×3
   $-2(6x + 11y = 15)$

$12x + 39y = -21$
$-12x - 22y = -30$

$17y = -51$
$y = -3$

$6x + 11(-3) = 15$
$6x - 33 = 15$
$+33 \quad +33$
$6x = 48$
$x = 8$

Solution: $(8, -3)$

3. $(12x - 7y = 59)$
   $-3(8x + 11y = -39)$

$24x - 14y = 118$
$-24x - 33y = 117$

$-47y = 235$
$y = -5$

$8x + 11(-5) = -39$
$8x - 55 = -39$
$+55 \quad +55$
$8x = 16$
$x = 2$

Solution: $(2, -5)$

4. $(5x - 3y = 22)$
   $-5(6x - 7y = 41)$

$30x - 18y = 132$
$-30x + 35y = -205$

$17y = -73$
$y = -\frac{73}{17}$

$7(5x - 3y = 22)$ ← repeat for y.
$-3(6x - 7y = 41)$

$35x - 21y = 154$
$-18x + 21y = -123$

$17x = 31$
$x = \frac{31}{17}$

So too ugly to substitute

Solution: $\left(\frac{31}{17}, -\frac{73}{17}\right)$

Here is an example of a system that would be difficult to solve graphically.

Extended Practice: Solve each system using linear combination.

1. $2x + y = 1$ $2x + 3y = 7$ $(-1, 3)$	2. $2x - 3y = 7$ $3x + y = 5$ $(2, -1)$

3. $5x - 6y = 9$ $2x - 3y = 3$ (3, 1)	4. $4x - 3y = 6$ $2x - 5y = -4$ (3, 2)
5. $6u + 5v = -2$ $2u + 3v = 6$ (−4.5, 5)	6. $3p + 2q = -2$ $9p - q = -6$ $\left(-\frac{2}{3}, 0\right)$

Now we will look at more complicated systems and ones that have unique things happen.

Break for Practice: Solve each system using linear combination.

1. $8y = 6x - 3$
   $9x = 12y + 5$

$8y = 6x - 3$
$-6x \quad -6x$

$-12y \quad -12y$

$3(-6x + 8y = -3) \rightarrow -18x + 24y = -9$

$2(9x - 12y = 5) \rightarrow 18x - 24y = 10$

$0 + 0 = 1$

$0 \neq 1$

False Statement

Solution: No Solution: $\emptyset$ (Parallel lines)

2. $4x + 3y = x + 6$
$x + 3y - 2 = 2v$

$4x+3y = x+6$ (−x, −x)

$-2y \quad +2 \quad -2y \quad +2$

$3x + 3y = 6$

$-3(x + y = 2)$

$3x + 3y = 6$

$-3x - 3y = -6$

$0 + 0 = 0$

$0 = 0$

True Statement

Solution: Infinite Solutions: ℝ

(same line)

3. $3(5 - x) = y$
$5(3 - x) = -2y + 1$

$y = 15 - 3x$ (+3x, +3x)

$15 - 5x = -2y + 1$ (+2y, −15; +2y, −15)

$(3x + y = 15)(-2)$

$-5x + 2y = -14$

$-6x - 2y = -30$

$-5x + 2y = -14$

$-11x = -44$

$-11 \quad -11$

$x = 4$

$y = 15 - 3(4)$

$y = 15 - 12$

$y = 3$

Solution: (4,3)

Extended Practice: Solve each system using linear combination.

1. $3x - 2y = 6$ $5x + 3y + 9 = 0$ (0, −3)	2. $x - y = 2x - 2$ $x + y = 2y - 2$ (0, 2)

3. $6x = 4y + 5$
$6y = 9x - 5$

No Solution: $\emptyset$

4. $x + y = 3x - 1$
$x - y = 1 - x$

Infinite Solutions: $\mathbb{R}$

5. $x + y = 4(y + 2)$
$x - y = 2(y + 4)$

Infinite Solution: $\mathbb{R}$

6. $2x - 3y = 2 - x$
$3x - 2y = -2 + y$

No Solution: $\emptyset$

Solving Systems with Elimination (Linear Combination) 3x3

So far all of the systems we have worked with have had two equations. Now we will extend our techniques to systems of three equations. The same techniques would also work for even larger systems.

Break for Practice: Solve the system using linear combination.

$$\begin{cases} 2x - y - z = 7 \\ 3x + 5y + z = -10 \\ 4x - 3y + 2z = 4 \end{cases}$$

② Eliminate same variable (z) using 2 different equations

$4x - 2y - 2z = 14$

$4x - 3y + 2z = 4$

$4(8x - 5y = 18)$

① Eliminate 1 variable using 2 equations (z)

$2x - y - z = 7$

$3x + 5y + z = -10$

$5(5x + 4y = -3)$

③ Eliminate 1 variable (y) and solve for remaining variable.

$25x + 20y = -15$

$32x - 20y = 72$

$57x = 57$

$\frac{57x}{57} = \frac{57}{57}$

$x = 1$

④ Substitute known variable (x) into one of the cloud equations

$5(1) + 4y = -3$

$5 + 4y = -3$

$-5 \quad -5$

$4y = -8$

$\frac{4y}{4} = \frac{-8}{4}$

$y = -2$

⑤ Substitute 2 known variables (x and y) into one original equation.

$3(1) + 5(-2) + z = -10$

$3 - 10 + z = -10$

$-7 + z = -10$

$+7 \quad +7$

$z = -3$

Solution: $(1, -2, -3)$ (x, y, z)

2. $\begin{cases} 3x + 4y + 2z = 6 \\ x + 3y - 5z = -7 \\ 5x + 7y - 3z = 3 \end{cases}$

② Eliminate same variable (x) using 2 different equations.

$-5x - 15y + 25z = 35$

$5x + 7y - 3z = 3$

$5(-8y + 22z = 38)$

① Eliminate 1 variable using 2 equations (x)

$3x + 4y + 2z = 6$

$-3x - 9y + 15z = 21$

$-8(-5y + 17z = 27)$

③ Eliminate 1 variable (y) and solve for remaining variable

$40y - 136z = -216$

$-40y + 110z = 190$

$-26z = -26$

$\frac{-26z}{-26} = \frac{-26}{-26}$

$z = 1$

④ Substitute known variable (z) into one of the cloud equations

$-5y + 17(1) = 27$

$-5y + 17 = 27$

$-17 \quad -17$

$-5y = 10$

$\frac{-5y}{-5} = \frac{10}{-5}$

$y = -2$

⑤ Substitute 2 known variables (y, z) into one original equation.

$x + 3(-2) - 5(1) = -7$

$x - 6 - 5 = -7$

$x - 11 = -7$

$+11 \quad +11$

$x = 4$

Solution: $(4, -2, 1)$ (x, y, z)

Extended Practice: Solve each system using linear combination.

1. $x - 2y + 3z = 3$
   $2x + y + 5z = 8$
   $3x - y - 3z = -22$

   $(-4, 1, 3)$

2. $3x + 2y - z = 10$
   $x + 4y + 2z = 3$
   $2x + 3y - 5z = 23$

   $(1, 2, -3)$

3. $2x + 2y + 3z = -1$
   $3x - 5y - 2z = 21$
   $7x + 3y + 5z = 10$

$(3, -2, -1)$

4. $3x - 2y + 5z = -17$
   $2x + 4y - 3z = 29$
   $5x - 6y - 7z = 7$

$(2, 4, -3)$