4. Davis Rent-A-Car charges a fixed amount per weekly rental plus a charge for each mile driven. A one-week trip of 520 miles' costs \$250, and a two week trip of 800 miles' costs \$440. Find the weekly charge and the charge for each mile driven.

Weekly Rate: \$120

Mile Rate: \$0.25

Graphing Systems of Linear Inequalities

Before we begin our final type of application problem in this unit, we need to be able to graph systems of linear inequalities. We worked with single inequalities in Unit 2, so all we need to do now is extend it for systems.

Steps:

1. Graph the boundary lines. Use a solid line if $\leq$ *or* $\geq$. Use a dashed line if $<$ or $>$
2. Test a point for each line to determine which side of each line to shade. Shade the side that gives a true inequality.
3. Darken in the region that was shaded by every one of the inequalities.

Example: $y \geq 2x - 3$

$y < -x + 3$

$b = -3$ $m = \frac{2}{1}$ or $\frac{-2}{-1}$

shade above

$y \geq 2x - 3$

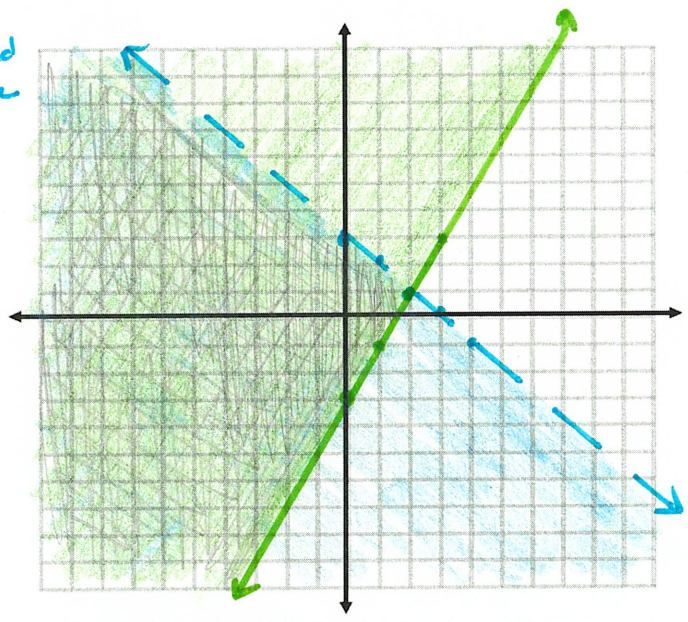
solid line

$b = 3$ $m = \frac{-1}{1}$ or $\frac{1}{-1}$

shade below

$y < -x + 3$

dashed line



Break for Practice: Solve each system.

1. $2x + y \leq 2$
   $x - y > 0$

$2x + y \leq 2$ → $y \leq -2x + 2$ (solid line, shade below)

$-x \quad -x$

$-y > -x + 0$

$-1 \quad -1 \quad -1$

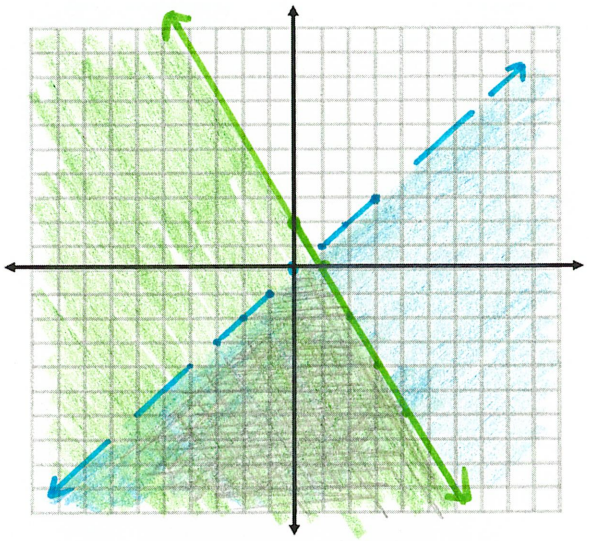
$y < x + 0$ (shade below, dashed line)

$b = 0$

$m = \frac{1}{1}$ or $\frac{-1}{-1}$

$b = 2$

$m = \frac{-2}{1}$ or $\frac{2}{-1}$



2. $y \leq 2x + 1$
   $-2x + y > -5$
   $x < -1$

$y \leq 2x + 1$ (shade below, solid line)

$b = 1$

$m = \frac{2}{1}$ or $\frac{-2}{-1}$

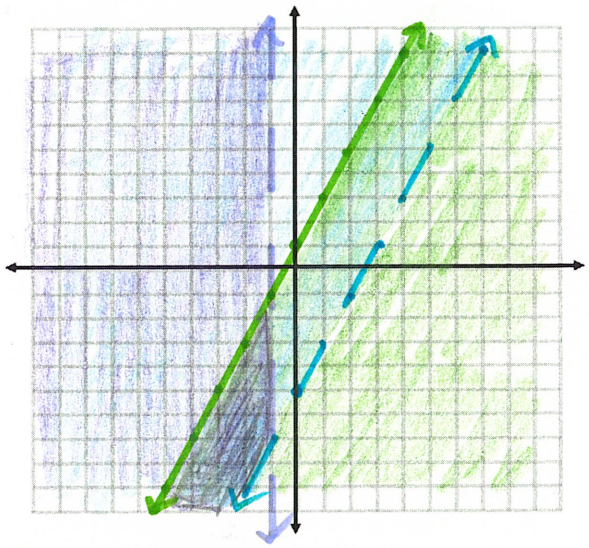
$+2x \quad +2x$

$y > 2x - 5$ (shade above, dashed line)

$b = -5$

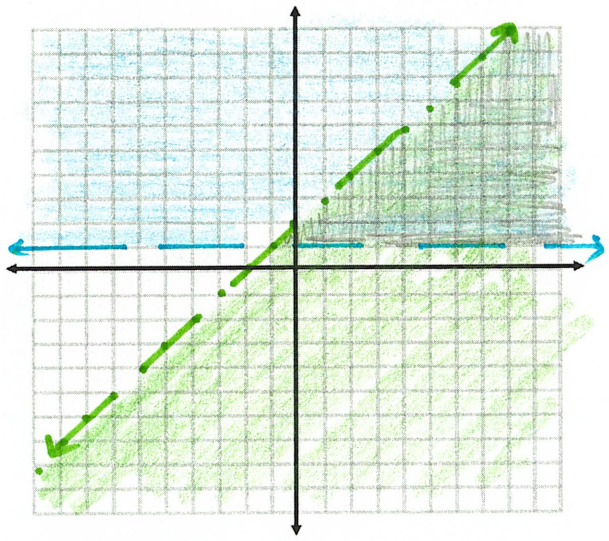
$m = \frac{2}{1}$ or $\frac{-2}{-1}$

$x < -1$ (shade below, dashed line)

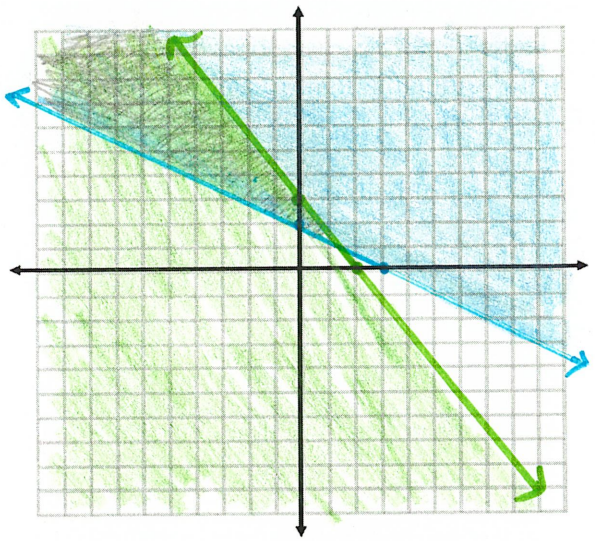


Extended Practice: Solve

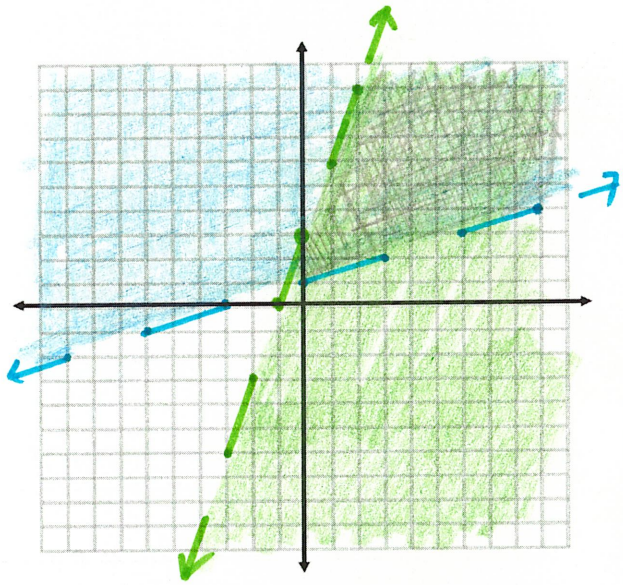
1. $y < x + 2$
   $1 - y < 0$



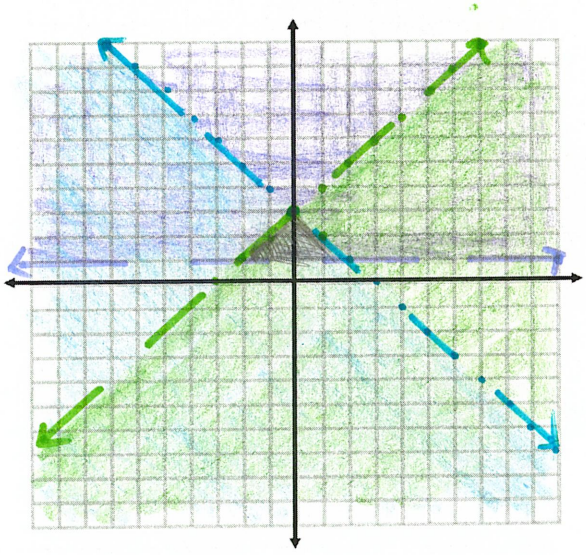
2. $3x + 2y \leq 6$
$2x + 3y \geq 6$



3. $y - 3x < 3$
$3y - x > 3$



4. $y - x < 3$
$y + x < 3$
$y - 1 > 0$



Linear Programming

The final section in this unit is linear programming. In many organizations, decisions are made by formulating and solving a system of linear inequalities that pertain to the constraints of their situation.

Break for Practice:

1. Suppose that you go into business raising thoroughbred and Quarter horses. Having studied Linear Programming, you decide to maximize the feasible profit you can make.

Let x = the number of Thoroughbred horses
y = the number of Quarter horses

★ opposite shading ★
↳ Leaves desired region open (easier to find points of intersection)

a.) Write inequalities expressing each of the following requirements.

i. Your supplier can get you at most 20 Thoroughbreds, and at most 15 Quarter horses.

shade below
opposite: above
$x \leq 20$
↑ Solid line (Vertical)

shade below
opposite: above
$y \leq 15$
↑ Solid line (horizontal)

ii. You must raise at least 12 horses, total, each year to make the business worthwhile.

$x + y \geq 12$
$-x \quad -x$

$b = 12$

$m = -\frac{1}{1}$ or $\frac{1}{-1}$

shade above
opposite: below
$y \geq -x + 12$
↑ Solid line

iii. A Thoroughbred eats 2 tons of food per year, but a Quarter horse eats 6 tons per year. You can handle no more than 96 tons of food per year.

$2x + 6y \leq 96$
$-2x \quad -2x$

$\frac{6y}{6} \leq \frac{-2x}{6} + \frac{96}{6}$

$b = 16$

$m = -\frac{1}{3}$ or $\frac{1}{-3}$

shade below
opposite: above
$y \leq -\frac{1}{3}x + 16$
↑ Solid line

iv. A Thoroughbred requires 1000hours of training per year, and a Quarter horse requires only 250 hours per year. You have enough personnel to do at most 10,000 hours of training per year.

$1000x + 250y \leq 10000$
$-1000x \quad -1000x$

$\frac{250y}{250} \leq \frac{-1000x}{250} + \frac{10000}{250}$

$b = 40$

$m = -\frac{4}{1}$ or $\frac{4}{-1}$

shade below
opposite: above
$y \leq -4x + 40$
↑ Solid line

Example#1: Thoroughbred vs. Quarter Horses