Name: ______________________________

Algebra II

Unit 3: Linear Systems

Priority Standard:

Unit "I can" statements:

1. I can solve systems of linear equations by using the graphing method.
2. I can solve systems of linear equations by using the substitution method.
3. I can solve systems of linear equations by using the linear combination (elimination) method.
4. I can solve systems of linear equations by using the Gaussian Elimination method.
5. I can use systems of equations to solve application problems.
6. I can graph systems of linear inequalities.
7. I can solve applications using linear programming.

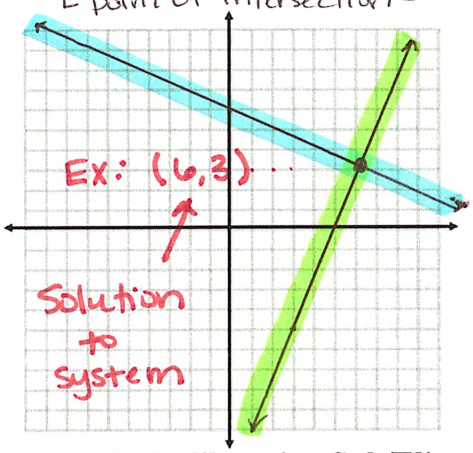
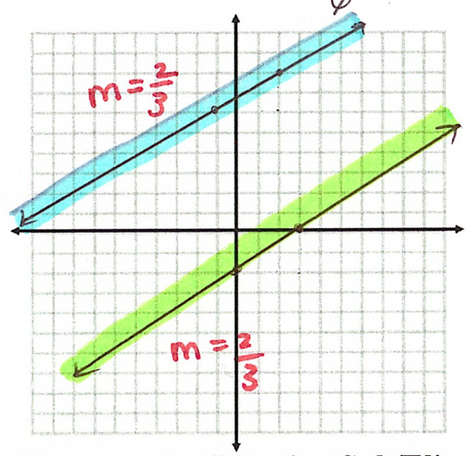
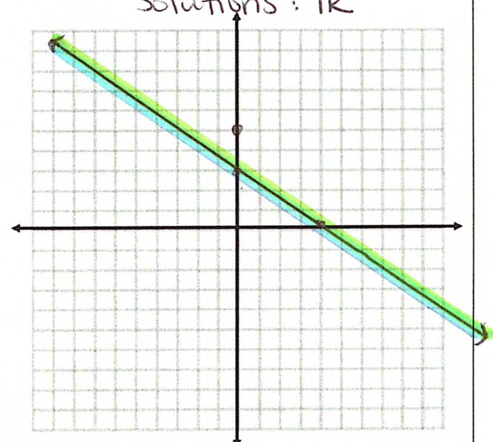
Common Core State Standards that are addressed in this unit include: A.REI .6c, A.REI.8c, A.REI.11d, A.REI.12d
For more information see www.corestandards.org/Math/

Solving Systems Graphically

This unit of study will take what was learned in the last unit on linear equations and inequalities and extend it to working with systems. A system is simply working with more than one equation or inequality at a time. What you are looking for is/are all solutions (points) that make all of the equations or inequalities true. In other words, you are looking for the point(s) that work in all of the equations or inequalities at the same time.

The first method that we will consider is the graphing method. This will allow you to actually see what is taking place. We will start with just systems of two linear equations.

There are three possibilities of what can occur when you have a system of two linear equations.

Option 1:	**Option 2**: ★Same slope★	**Option 3**: ★Same slope★ ★Same y-intercept
The lines intersect.	The lines are parallel.	The lines are the same.
Type of Solution: ordered pair: (x,y) 1 point of intersection	**Type of Solution:** Lines never intersect so... No Solution ∅	**Type of Solution:** Lines Always intersect so... Infinite Solutions: ℝ
		
What is looks like using Sub/Elim ordered pair: (x-value, y-value)	**What is looks like using Sub/Elim** Variables cancel out and you are left with a False Statement Ex. $0 \neq -6$ → No Solution ∅	**What is looks like using Sub/Elim** Variables cancel out and you are left with a True Statement Ex: $0=0$ → Infinite Solutions: ℝ

Break for Practice: Solve each system by graphing. Identify the solution. → point of intersection

1. $3x + 2y = 6$ →
   $-x + 2y = -2$

$+x \quad +x$

$\frac{2y}{2} = \frac{x}{2} - \frac{2}{2}$

$y = \frac{1}{2}x - 1$

$b = -1$

$m = \frac{1}{2}$ or $\frac{-1}{-2}$

$3x + 2y = 6$
$-3x \quad -3x$

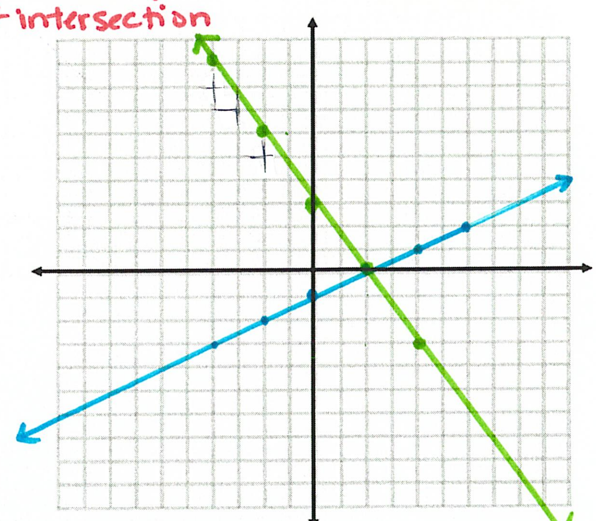
$\frac{2y}{2} = \frac{-3x}{2} + \frac{6}{2}$

$y = -\frac{3}{2}x + 3$

$b = 3$

$m = -\frac{3}{2}$ or $\frac{3}{-2}$

Solution: (2,0)



2. $2x - y = 10$ → $2x - y = 10$

$x + 3y = -9$

$-x \qquad -x$

$\frac{3y}{3} = \frac{-x}{3} - \frac{9}{3}$

$y = -\frac{1}{3}x - 3$

$b = -3$

$m = \frac{-1}{3}$ or $\frac{1}{-3}$

$-2x \qquad -2x$

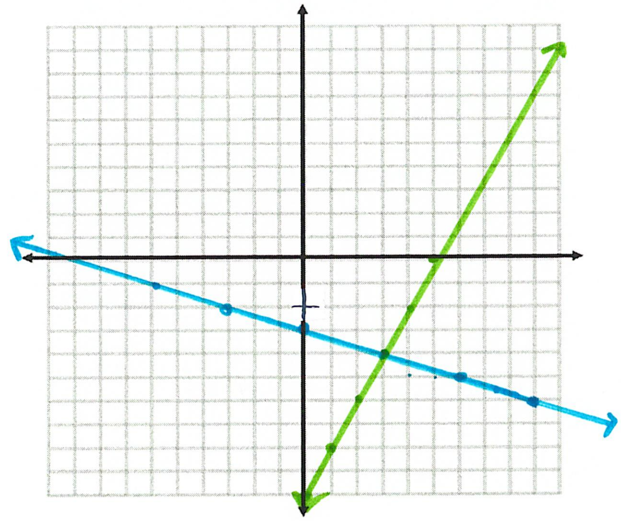
$\frac{-y}{-1} = \frac{-2x}{-1} + \frac{10}{-1}$

$y = 2x - 10$

$b = -10$

$m = \frac{2}{1}$ or $\frac{-2}{-1}$

$\boxed{\text{Solution: } (3,4)}$



3. $x - 2y = -12$ → $x - 2y = -12$

$3x + y = -1$

$-3x \qquad -3x$

$y = -3x - 1$

$b = -1$

$m = -\frac{3}{1}$ or $\frac{3}{-1}$

$-x \qquad -x$

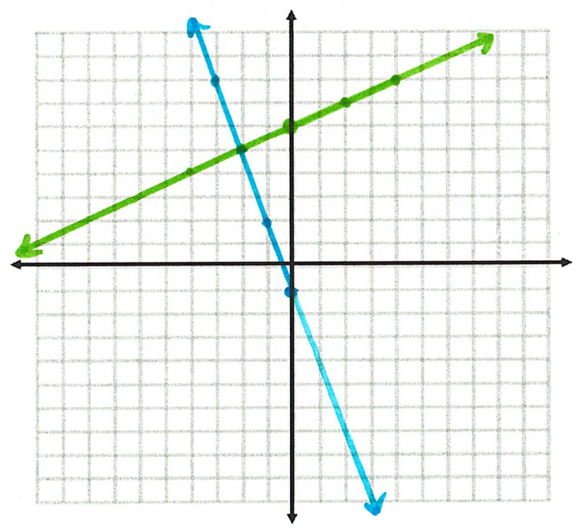
$\frac{-2y}{-2} = \frac{-x}{-2} - \frac{12}{-2}$

$y = \frac{1}{2}x + 6$

$b = 6$

$m = \frac{1}{2}$ or $\frac{-1}{-2}$

$\boxed{\text{Solution: } (-2, 5)}$



4. $2x - 4y = 8$ → $2x - 4y = 8$

$x - 2y = -2$

$-x \qquad -x$

$\frac{-2y}{-2} = \frac{-x}{-2} - \frac{2}{-2}$

$y = \frac{1}{2}x + 1$

$b = 1$

$m = \frac{1}{2}$ or $\frac{-1}{-2}$

$-2x \qquad -2x$

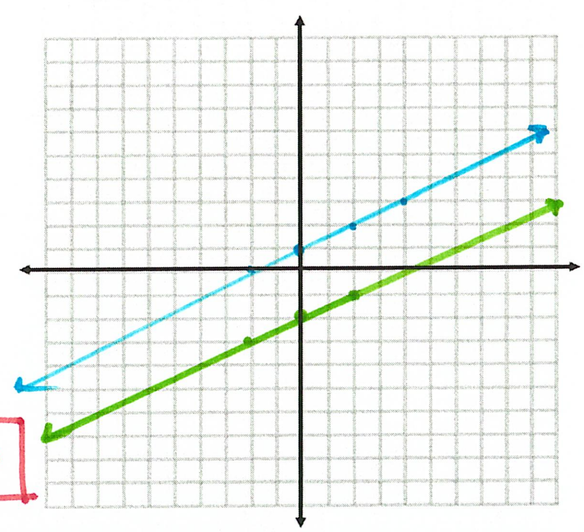
$\frac{-4y}{-4} = \frac{-2x}{-4} + \frac{8}{-4}$

$y = \frac{1}{2}x - 2$

$b = -2$

$m = \frac{1}{2}$ or $\frac{-1}{-2}$

$\boxed{\text{Solution: No Solution}}$

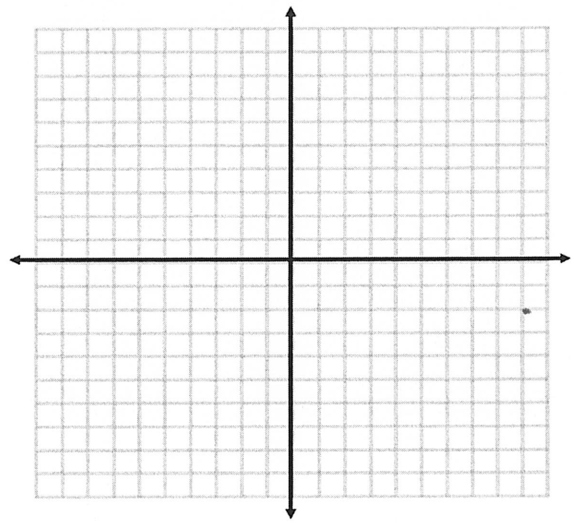


Extended Practice: Solve each system by graphing. Identify the solution.

1. $2x - y = 7$

   $x + 3y = 0$

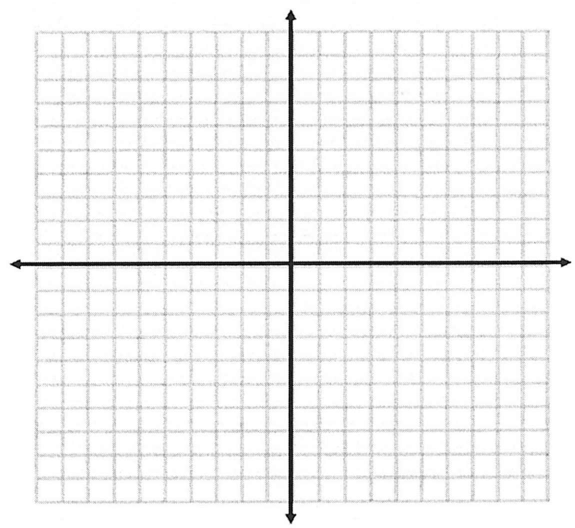
   $(3, -1)$



2. $3x + 2y = 6$

   $-4x + y = 14$

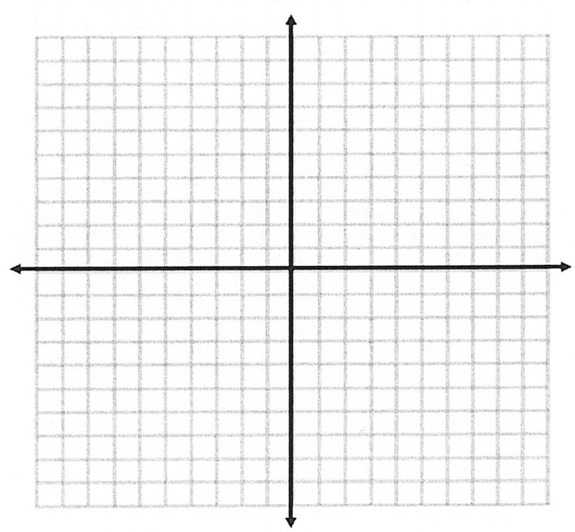
   $(-2, 6)$



3. $4x - 2y = 4$

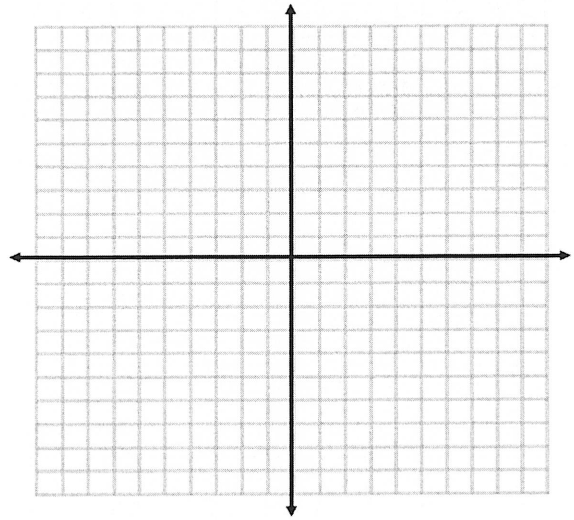
   $3x + y = -7$

   $(-1, -4)$



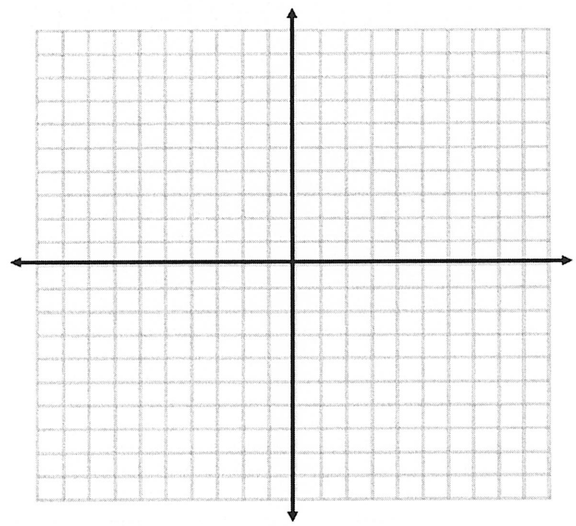
4. $x = 3$

$y = 4$ (3, 4)



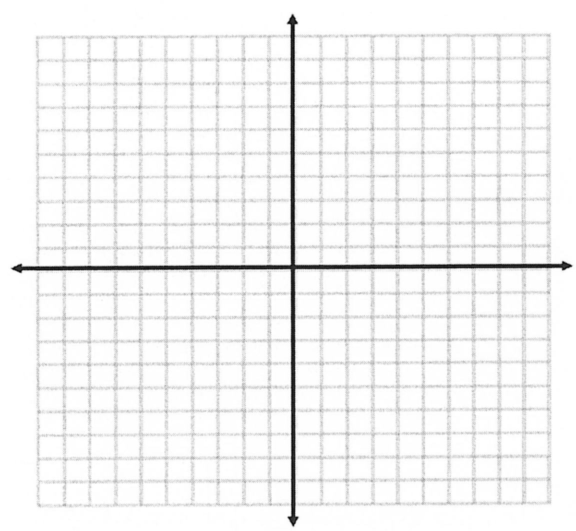
5. $3x - 5y = 15$

$-6x + 10y = -30$ Infinite Solutions : $\mathbb{R}$



6. $x + 2y = -6$

$y = -\frac{1}{2}x + 4$ No Solution : $\emptyset$



Solving Systems with Substitution

The next method for solving systems is called substitution. The goal is to solve for one of the variables and then substitute to find the other variable. This method is good for systems of linear equations as well as for other types of equations. We will work with the linear systems.

Steps for Substitution:

Look for a variable with 1 as the coefficient

1. Choose an equation and solve for one of the variables.
2. Substitute its value into the other equation.
3. Solve for the remaining variable.

Equation from Step #1

4. Substitute the known variable into one of the original equations.
5. Check Answers : Substitute x and y into both equations
   both should be true

Break for Practice: Solve each system using the substitution method.

1. $x + y = 4$
   $3x + 2y = 9$

① $x + y = 4$
$\quad -x \quad\ -x$

$y = -x + 4$

② $3x + 2y = 9$

$3x + 2(-x+4) = 9$

③ $3x - 2x + 8 = 9$

$x + 8 = 9$
$\quad -8 \quad -8$

$x = 1$

④ $y = -x + 4$

$y = -1 + 4$

$y = 3$

$\boxed{\text{Solution: } (1,3)}$

2. $x + y = 4$
   $3x + 2y = 9$

① $x + y = 4$
$\quad -y \quad -y$

$x = 4 - y$

② $3x + 2y = 9$

$3(4-y) + 2y = 9$

③ $12 - 3y + 2y = 9$

$12 - y = 9$
$-12 \quad\ -12$

$\dfrac{-y}{-1} = \dfrac{-3}{-1}$

$y = 3$

④ $x = 4 - y$

$x = 4 - 3$

$x = 1$

$\boxed{\text{Solution: } (1,3)}$

3. ① $-3y = x$
   $-y + 6x = -38$

② $-y + 6x = -38$

$-y + 6(-3y) = -38$

③ $-y - 18y = -38$

$\dfrac{-19y}{-19} = \dfrac{-38}{-19}$

$y = 2$

④ $x = -3y$

$x = -3(2)$

$x = -6$

$\boxed{\text{Solution: } (-6,2)}$

Extended Practice: Solve each system using the substitution method.

1. $y = 6x - 11$ $-2x - 3y = -7$ (2, 1)	2. $2x - 3y = -1$ $y = x - 1$ (4, 3)	3. $y = 5x - 7$ $-3x - 2y = -12$ (2, 3)
4. $x + 2y = 1$ $y = -\frac{1}{2}x + 4$ No Solution: $\emptyset$	5. $x + y = 6$ $-x + y = 2$ (2, 4)	6. $2x + 5y = 41$ $2x + y = 13$ (3, 7)

7. $x + y = 4(y + 2)$ $x - y = 2(y + 4)$ Infinite Solutions: $\mathbb{R}$	8. $-3x + 5y = -6$ $-10y + 6x = 12$ Infinite Solutions: $\mathbb{R}$	9. $-2x + 6y = 6$ $-7x + 8y = -5$ $(3, 2)$

Solving Linear Systems with Elimination (Linear Combination) 2x2

Another method for solving systems of linear equations is called linear combination. In this method, we want to add the equations to get one of the variables to fall out. This may require multiplying one or both equations by constants.

Steps for Elimination:

1. Put both equations into Standard Form $\Rightarrow Ax + By = C$
2. Eliminate one variable: change the Coefficients of one variable so that you have Opposites in a column
   Ex. -6 and 6
3. Add equations together, Solve for remaining variable.
4. Substitute known value into one of the original equations and Solve.
5. Check Answers: substitute x and y into both equations
   both should be true