**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Algebra II**

**Unit 3: Linear Systems**

**Priority Standard:**

**Unit “I can” statements:**

1. I can solve systems of linear equations by using the graphing method.
2. I can solve systems of linear equations by using the substitution method.
3. I can solve systems of linear equations by using the linear combination (elimination) method.
4. I can solve systems of linear equations by using the Gaussian Elimination method.
5. I can use systems of equations to solve application problems.
6. I can graph systems of linear inequalities.
7. I can solve applications using linear programming.

Common Core State Standards that are addressed in this unit include: A.REI .6c, A.REI.8c, A.REI.11d, A.REI.12d

For more information see [www.corestandards.org/Math/](http://www.corestandards.org/Math/)

**Solving Systems Graphically**

This unit of study will take what was learned in the last unit on linear equations and inequalities and extend it to working with systems. A system is simply working with more than one equation or inequality at a time. What you are looking for is/are all solutions (points) that make all of the equations or inequalities true. In other words, you are looking for the point(s) that work in all of the equations or inequalities at the same time.

The first method that we will consider is the graphing method. This will allow you to actually see what is taking place. We will start with just systems of two linear equations.

There are three possibilities of what can occur when you have a system of two linear equations.

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| **Option 1**: The lines \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. **http://www.algebra-class.com/images/blank-graph.gifType of Solution:** **What is looks like using Sub/Elim** | **Option 2**: The lines \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. **http://www.algebra-class.com/images/blank-graph.gifType of Solution:** **What is looks like using Sub/Elim** | **Option 3**: The lines \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.**http://www.algebra-class.com/images/blank-graph.gifType of Solution:** **What is looks like using Sub/Elim** |

**Break for Practice**: Solve each system by graphing. Identify the solution.



1. $3x+2y=6$

$$-x+2y=-2$$

1. $2x-y=10$

$$x+3y=-9$$



1. $x-2y=-12$

$$3x+y=-1$$



1. $2x-4y=8$

$$x-2y=-2$$



**Extended Practice**: Solve each system by graphing. Identify the solution.

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| http://www.algebra-class.com/images/blank-graph.gif1. $2x-y=7$ $x+3y=0$ |
| http://www.algebra-class.com/images/blank-graph.gif2. $3x+2y=6$$$ -4x+y=14$$ |
| http://www.algebra-class.com/images/blank-graph.gif3. $4x-2y=4$ $3x+y=-7$ |
| http://www.algebra-class.com/images/blank-graph.gif4. $ x=3$ $y=4$ |
| http://www.algebra-class.com/images/blank-graph.gif5. $3x-5y=15$ $-6x+10y=-30$ |
| 6. $x+2y=-6$http://www.algebra-class.com/images/blank-graph.gif $y=-\frac{1}{2}x+4$ |

**Solving Systems with Substitution**

 The next method for solving systems is called substitution. The goal is to solve for one of the variables and then substitute to find the other variable. This method is good for systems of linear equations as well as for other types of equations. We will work with the linear systems.

**Steps for Substitution:**

1. Choose an \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ and solve for one of the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ its value into the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ equation.
3. Solve for the remaining \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.
4. Substitute the known \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ into one of the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ equations.
5. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Break for Practice**: Solve each system using the substitution method.

 1. $x+y=4$ 2. $x+y=4$ 3. $-3y=x$

 $3x+2y=9$ $3x+2y=9$ $-y+6x=-38$

**Extended Practice**: Solve each system using the substitution method.

|  |  |  |
| --- | --- | --- |
| 1. $y=6x-11$ $-2x-3y=-7$ | 2. $2x-3y=-1$ $y=x-1$ | 3. $y=5x-7$ $-3x-2y=-12$ |
| 4. $x+2y=1$ $y=-\frac{1}{2}x+4$ | 5. $x+y=6$ $-x+y=2$ |  6. $2x+5y=41$ $2x+y=13$ |
| 7. $x+y=4\left(y+2\right)$ $x-y=2(y+4)$ | 8. $-3x+5y=-6$ $-10y+6x=12$ | 9. $-2x+6y=6$ $-7x+8y=-5$ |

**Solving Linear Systems with Elimination (Linear Combination) 2x2**

 Another method for solving systems of linear equations is called linear combination. In this method, we want to add the equations to get one of the variables to fall out. This may require multiplying one or both equations by constants.

**Steps for Elimination:**

1. Put both equations into \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ one variable: change the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of one

 variable so that you have \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ in a column

 3. \_\_\_\_\_\_\_\_\_ equations together, \_\_\_\_\_\_\_\_\_\_\_\_\_\_ for remaining variable.

 4. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ known value into one of the original equations

 5. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Break for Practice**: Solve each system by using linear combination.

1. $3x+2y=6$ 2. $4x+13y=-7$

$-x+2y=-2$ $6x+11y=15$

 3. $12x-7y=59$ 4. $5x-3y=22$

 $8x+11y=-39$ 6$x-7y=41$

**Extended Practice**: Solve each system using linear combination.

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| --- | --- |
| 1. $2x+y=1$ $2x+3y=7$ | 2. $2x-3y=7$$$ 3x+y=5$$ |
| 3. $5x-6y=9$$$ 2x-3y=3$$ | 4. $4x-3y=6$ $2x-5y=-4$ |
| 5. $6u+5v=-2$$$ 2u+3v=6$$ | 6. $3p+2q=-2$$$ 9p-q=-6$$ |

 Now we will look at more complicated systems and ones that have unique things happen.

**Break for Practice**: Solve each system using linear combination.

1. $8y=6x-3$

$$9x=12y+5$$

 2. $4x+3y=x+6$ 3. $3\left(5-x\right)=y$

 $x+3y-2=2y$ 5$\left(3-x\right)=-2y+1$

**Extended Practice**: Solve each system using linear combination.

|  |  |
| --- | --- |
| 1. $3x-2y=6$

$$ 5x+3y+9=0$$ | 1. $x-y=2x-2$

$$ x+y=2y-2$$ |
| 1. $6x=4y+5$

$$ 6y=9x-5$$ | 1. $x+y=3x-1$

$$x-y=1-x$$ |
| 1. $x+y=4(y+2)$

 $x-y=2(y+4)$ | 1. $2x-3y=2-x$

$$ 3x-2y=-2+y$$ |

**Solving Systems with Elimination (Linear Combination) 3x3**

 So far all of the systems we have worked with have had two equations. Now we will extend our techniques to systems of three equations. The same techniques would also work for even larger systems.

**Break for Practice**: Solve the system using linear combination.

1. $2x-y-z=7$

$$3x+5y+z=-10$$

$$4x-3y+2z=4$$

1. $3x+4y+2z=6$

$$x+3y-5z=-7$$

$$5x+7y-3z=3$$

**Extended Practice**: Solve each system using linear combination.

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| 1. $x-2y+3z=3$

$$2x+y+5z=8$$$$3x-y-3z=-22$$ |
| 1. $3x+2y-z=10$

$$x+4y+2z=3$$$$2x+3y-5z=23$$ |
| 1. $2x+2y+3z=-1$

$$3x-5y-2z=21$$$$7x+3y+5z=10$$ |
| 1. $3x-2y+5z=-17$

$$2x+4y-3z=29$$$$5x-6y-7z=7$$ |

**Solving Systems with Gaussian Elimination 3x3**

 There is one more method for solving systems that we will learn. This is called Gaussian Elimination and it is named for the German mathematician Carl Friederich Gauss. The first example we will try is the same first example that we had for linear combination. If we get the same answer, then we will know that this method truly works.

**Break for Practice**: Solve the system using Gaussian Elimination.

1. $2x-y-z=7$

$$3x+5y+z=-10$$

$$4x-3y+2z=4$$

1. $x-y+3z=-4$

$$-x+2y+z=2$$

$$3x-4y-z=-4$$

**Extended Practice**: Solve each system using Gaussian Elimination.

|  |
| --- |
| 1. $x+2y-3z=11$

$$2x+y-2z=9$$$$4x+3y+z=16$$ |
| 1. $2x-y+3z=19$

$$x+3y-z=-10$$$$3x+5y+2z=3$$ |
| 1. $2x+3y-z=9$

$$x-3y+z=-6$$$$3x+y-4z=31$$ |
| 1. $4x+3y-5z=-26$

$$2x+y-z=-4$$$$x+5y+z=-38$$ |

**Problem Solving Using Linear Systems**

 In this section we will see how systems can be used to solve applications. Some of these problems are similar to problems that we had seen earlier and solved using one variable, now we can learn how to solve them with two variables.

**Break for Practice**: Solve

1. Tickets for the senior prom cost $25 for a single ticket and $40 for a couple. Ticket sales totaled $3800 and 110 tickets were sold. How many tickets of each type were sold?
2. A cashier had to give Sarah $3.45 in change but he had only quarters and dimes in the cash register. If he gave her 15 coins, how many dimes did she receive?
3. A grocer mixed nuts worth $4 per kilogram with raisins worth $3.25 per kilogram to make 15 kilograms of a mixture worth $3.60 per kilogram. How many kilograms of nuts were used?

**Extended Practice**: Solve

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| 1. Kerry asked a bank teller to cash a $390 check using $20 bills and $50 bills. If the teller gave her a total of 15 bills, how many of each type of bill did she receive? |
| 2. Tickets for the homecoming dance cost$20 for a single ticket or $35 for a couple. Ticket sales totaled $2280, and 128 people attended. How many tickets of each type were sold? |
| 3. On Friday, the With-It Clothiers sold some jeans at $25 a pair and some shirts at $18 each. Receipts for the day totaled $441. On Saturday the store priced both items at $20, sold exactly the same number of each item, and had receipts of $420. How many pairs of jeans, and how many shirts were sold each day? |
| 4. A grain-storage warehouse has a total of 30 bins. Some hold 20 tons of grain each, and the rest hold 15 tons each. How many of each type of bin are there if the capacity of the warehouse is 510 tons? |

**Break for Practice**: Solve

1. With a head wind, a plane travelled 840 miles northward in 2 hours. With the same wind as a tail wind, the return trip southward took 1 hour 45 minutes. Find the plane’s air speed and the wind speed.
2. A garage charges a fixed amount to bring your car to be fixed, and an additional hourly fee for the mechanic to work on your car. Keith brings his car in and received a bill for $160 after the mechanic worked on his car for 4 hours. Mildred had to pay $435 after her car was worked on for 15 hours. What are the hourly and fixed rates?

**Extended Practice**: Solve

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| 1. With a tail wind, a helicopter traveled 300 miles in 1 hour 40 minutes. The return trip against the same wind took 20 minutes longer. Find the wind speed and also the air speed of the helicopter.
 |
| 1. With a head wind, a plane traveled 1000 miles in 4 hours. With the same wind as a tail wind, the return trip took 3 hours 20 minutes. Find the plane’s air speed and the wind speed.
 |
| 1. A caterer’s total cost for catering a party includes a fixed cost, which is the same for every party. In addition, the caterer charges a certain amount for each guest. If it costs $300 to serve 25 guests and $420 to serve 40 guests, find the fixed cost and the cost per guest.
 |
| 1. Davis Rent-A-Car charges a fixed amount per weekly rental plus a charge for each mile driven. A one-week trip of 520 miles’ costs $250, and a two week trip of 800 miles’ costs $440. Find the weekly charge and the charge for each mile driven.
 |

**Graphing Systems of Linear Inequalities**

 Before we begin our final type of application problem in this unit, we need to be able to graph systems of linear inequalities. We worked with single inequalities in Unit 2, so all we need to do now is extend it for systems.

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| **Steps**:1. Graph the boundary lines.

Use a solid line if $\leq or \geq $.Use a dashed line if < or >1. Test a point for each line to determine which side of each line to shade. Shade the side that gives a true inequality.
2. Darken in the region that was shaded by every one of the inequalities.
 | **Example:** $y\geq 2x-3$ $y<-x+3$ | http://www.algebra-class.com/images/blank-graph.gif |

**Break for Practice**: Solve each system.

1. $2x+y\leq 2$

$$x-y>0$$



1. $y\leq 2x+1$

$$-2x+y>-5$$

$$x<-1$$

**Extended Practice**: Solve

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| 1. $y<x+2$

$$1-y<0$$http://www.algebra-class.com/images/blank-graph.gif  |
| 2. $3x+2y\leq 6$$2x+3y\geq 6$http://www.algebra-class.com/images/blank-graph.gif |
| http://www.algebra-class.com/images/blank-graph.gif3. $y-3x<3$ $3y-x>3$ |
| http://www.algebra-class.com/images/blank-graph.gif4. $y-x<3$$$y+x<3$$$$y-1>0$$ |

**Linear Programming**

 The final section in this unit is linear programming. In many organizations, decisions are made by formulating and solving a system of linear inequalities that pertain to the constraints of their situation.

**Break for Practice**:

1. Suppose that you go into business raising thoroughbred and Quarter horses. Having studied Linear Programming, you decide to maximize the feasible profit you can make.

 Let x = the number of Thoroughbred horses

 y = the number of Quarter horses

1. Write inequalities expressing each of the following requirements.
2. Your supplier can get you at most 20 Thoroughbreds, and at most 15 Quarter horses.
3. You must raise at least 12 horses, total, each year to make the business worthwhile.
4. A Thoroughbred eats 2 tons of food per year, but a Quarter horse eats 6 tons per year. You can handle no more than 96 tons of food per year.
5. A Thoroughbred requires 1000hours of training per year, and a Quarter horse requires only 250 hours per year. You have enough personnel to do at most 10,000 hours of training per year.
6. Draw a graph of the feasible region.
7. You can make a profit of $500 for each Thoroughbred and $200 for each Quarter horse. What is the maximum feasible profit you can make per year?

**Extended Practice**:

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| 1. Joe P. is the president of Joe’s Corn Chips Inc. His company is divided into two departments which put out two kinds of corn chips, Extra Larges and Really Smalls. Each department has separate regulations concerning the number of bags produced per day.

 a.) Write an inequality for each of the following.1. No more than 20 kilobags of Extra Larges and no more than 30 kilobags of Really Smalls can be put out per day.

 Let x = number of kilobags of Extra Larges  y = number of kilobags of Really Smalls1. No more than 45 kilobags, total, can be manufactured each day.
2. The number of Extra Larges can be no less than ¾ the number of Really Smalls produced each day.
3. At least 300 hours of labor must be used each day to meet union requirements. It takes 10 hours to make a kilobag of Extra Larges and 15 hours to make a kilobag of Really Smalls.

 b.) Graph the feasible region.1. Joe’s Corn Chip company makes a profit of $200 per kilobag of Extra Larges and a profit of $150 per kilobag of Really Smalls. Write an equation expressing profit in terms of the two kinds of chips.
2. How many bags of each kind of chips should be produced each day to give the greatest feasible profit? What is the profit?
3. How many bags of each kind of chips should be produced each day to give the least feasible profit? What is the profit?
 |

**Break for Practice**:

1. Suppose that you are chief mathematician for the Government’s million-dollar Vitamin Research project. Your scientists have been studying the combined effects of vitamins A and B on the human system.
2. Write an equation expressing d in terms of x (vitamin A) and y (vitamin B) if vitamin A costs 0.06 cents per unit (not $0.06), and vitamin B costs 0.05 cents per unit. Write the equation expressing this.
3. Write inequalities to represent the following requirements.
4. The body can tolerate no more than 600 units per day of vitamin A, and no more than 500 units of vitamin B.
5. The total number of units per day of the two vitamins must be between 400 and 1000, inclusive.
6. Due to the combined effects of the two vitamins, the number of units per day of vitamin B must be more than ½ the number of units per day of vitamin A, but less than or equal to three times the number of units per day of vitamin A.
7. Graph the feasible region.
8. Which point in the feasible region represents the minimum cost, optimum point? What is this minimum cost?

**Extended Practice**:

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| 2. The student council decides to raise funds by selling imported clothing. Let x be the number of shirts and y be the number of dresses imported. The import order must follow these parameters. a.) Write an inequality for each of the following parameters.1. The total number of dresses must be at least 10.
2. The number of shirts must be between 5 and 27, inclusive.
3. The number of dresses must be less than or equal to twice the number of shirts, but at least one-fourth the number of shirts.
4. Four times the number of shirts plus three times the number of dresses is at most 135.
5. Graph the feasible region.
6. Shirts cost $10 each and dresses cost $20 each. Write an equation expressing the cost in terms of dresses and shirts.
7. We wish to minimize the cost. Find the point and the minimal feasible cost.
8. Shirts can be sold for $70 and dresses for $50 each. Write an equation expressing the revenue in terms of dresses and shirts.
9. We wish to maximize the revenue. Find the point and the maximum feasible revenue.
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