**Algebra II**

**Unit 1:**

**Solving Equations and Inequalities in One Variable**

**Priority Standards:** A.CED.1: Create equations and inequalities in one variable and use them to solve problems.

**Unit “I can” statements:**

1. I can solve certain equations in one variable.
2. I can translate word phrases and sentences into algebraic expressions and equations.
3. I can solve word problems by using an equation in one variable.
4. I can solve simple inequalities in one variable.
5. I can solve simple conjunctions and disjunctions.
6. I can solve word problems by using inequalities in one variable.
7. I can solve open sentences involving absolute value.

Common Core State Standards that are addressed in this unit include: A-CED.1, A-CED.2,

A-CED.4, A-REI.3, A-SSE.1b. For more information see [www.corestandards.org/Math/](http://www.corestandards.org/Math/)

**Solving Equations in One Variable**

This unit will begin with a review of solving simple equations in one variable. Do you remember how to solve an equation like this?

 $4x-3=29$

**Keys for Solving Simple Equations**:

1. Simplify either side of an equation if possible. (Get rid of parentheses and combine like terms.)
2. Add or subtract the same amount from both sides of the equation.
3. Multiply or divide both sides of the equation by the same non-zero number.

**Break for Practice**: Solve (if possible) each equation and check the answer.

 1. $48-6x=2x$ 2. $4\left(x-3\right)=2x-6$ 3. $\frac{1}{2}x-4=-2$

 4. $7-\frac{1}{5}y=-2$ 5. $3x-\left(x-5\right)=2(x+4)$ 6. $3x-\left(x-8\right)=2(x+4)$

7. $0.4\left(2r+3\right)=0.6r+3.6$

Let’s review the different types of solutions we saw in the Break for Practice.

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| **What it looks like when solving for** **one-variable…** | **Leads to what type of solution?** | **Example to Reference** |
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The same techniques that are used to solve simple equations can also be used to manipulate formulas. A skill that is useful in many science, math, and business classes is the ability to manipulate a formula and isolate any variable that you choose.

**Break for Practice**: Solve for the given variable. (For fun, try to identify what each formula is used for.)

1. $C=πd \rightarrow Solve for d.$ 2. $I=prt\rightarrow Solve for r$.

3. $P=2l+2w\rightarrow Solve for l.$ 4. $A=\frac{1}{2}h\left(b\_{1}+b\_{2}\right)\rightarrow Solve for h.$

1. $A=\frac{1}{2}h\left(b\_{1}+b\_{2}\right)\rightarrow Solve for b\_{2}.$ 6. $A=πr\left(s+r\right)\rightarrow Find s if A=100π and r=5. $

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**Extended Practice**: Solve, if possible, each equation.

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| 1. $3x-4=5$ | 2. $\frac{2}{3}t-8=0$  |
| 3. $5r=18+2r$ | 4. $3\left(t-1\right)=-\left(t-5\right)$ |
| 5. $2k-1=k-5+3k$ | 6. $-\left(5-x\right)=x+3$ |
| 7. $1.5\left(u+2\right)=7.5$ | 8. $3\left(x-2\right)-x=2(2x+1)$ |
| 9. $\frac{6x-2(x-4)}{3} = 8$ |

**Extended Practice Continued**: Solve each equation for the given variable.

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| 10. $2x-5y=10\rightarrow Solve for y.$ | 11. $A=\frac{1}{2}bh\rightarrow Solve for h.$ |
| 12. $S=-\frac{1}{2}gt^{2}+vt \rightarrow Solve for v.$ | 13. $C=\frac{5}{9}\left(F-32\right) \rightarrow Solve for F.$ |
| 14. $d=\frac{v^{2}}{2g} \rightarrow Find g if d=1000 and v=140. $ |

**Words into Symbols**

One of the main purposes for studying Algebra is to be able to solve applications. In order to solve applications, you need to be able to translate words and sentences into expressions and equations.

**Break for Guided Practice**: Represent each phrase by an algebraic expression.

1. The difference between a number and three.
2. Four less than twice a number.
3. The sum of twice a number and its square.
4. Four less than the product of a number and three.
5. The quotient when eight is divided by twice a number.
6. The square of the sum of a number and three.

Express each answer in simplest form in terms of the given variable.

1. Carl is *x* years old. His sister Jenny is six more than twice his age. What is the average of their ages?
2. The base and height of a triangle are consecutive odd integers, and the height exceeds the base. If the base is *b* cm, find the area of the triangle.
3. At 2:00 pm a train left a station traveling east at r miles per hour. At 3:00 pm a second train headed west from the station at a rate 20 miles per hour faster than the first. How far apart were the trains at 5:00 pm?
4. The number of dimes and the number of quarters that Margaret has earned in tips are consecutive even integers. She has fewer dimes than quarters. What is the total value of her coins?

**Extended Practice**: Represent each word phrase by an algebraic expression.

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| 1. Five more than a number. | 2. One less than twice a number. |
| 3. Seven more than half of a number. | 4. One more than the square of a number. |

**Extended Practice Continued**: Express each answer in simplest form it terms of the given variable.

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| 5. A rectangular garden that is *w* ft. wide is enclosed by 120 ft. of fencing. How long is the garden? |
| 6. In a basketball game, one team’s score is two points less than half the other team’s score, which is *x*. What is the difference in the scores? |
| 7. The length, width, and height of a rectangular box are consecutive integers, and the largest dimension is *k* cm. Find the volume V of the box. (hint: $V = lwh$) |
| 8. Two jets leave an airport at noon, one flying north at r mi/h, and the other flying south at twice that speed. After 3 h, how far apart are the planes? |
| 9. One angle of a quadrilateral has measure *a*º. Find the average of the measures of the other three angles. |
| 10. The Drama Club sold t students’ tickets at $1.50 each and 100 fewer adults’ tickets at $2.50 each. How much money did the club collect? |

**Problem Solving with Equations**

Now it is time to extend the process from the previous section and actually solve the word problems. First it is time to review the five step plan for approaching story problems.

**5-Step Plan:**

1. Carefully read the problem. Decide what you are looking for and what you are given.
2. Define a variable and use it to describe the number(s) in the problem. Drawing and labeling a sketch, or using a table may help.
3. Reread the problem, and write an equation.
4. Solve the equation and find the required numbers.
5. Check your answer!

Warning! Be careful of extra information or insufficient information.

**Break for Practice**: Solve each of the following problems. If there is not enough information to solve the problem, say so.

1. Cheryl’s weekly allowance is $2.00 more than Emily’s. Together they get $11.00. What is each girl’s weekly allowance?
2. The Junior class sold shirts bearing the school insignia for $12.00 each. An extra $1.00 was charged to have a shirt monogrammed. There were 324 shirts sold, and a total of $4036.00 was collected. Of the shirts sold, 174 were bought by Juniors. How many shirts were not monogrammed?
3. A store sold 40 baseballs and 14 softballs over a two-week period. The sales for these items totaled $200. What was the price of one baseball?
4. The perimeter of an isosceles triangle is 36 cm, and the area is 60 cm2. The length of the base is 3 cm less than the length of a leg. Find the length of each side.
5. The measures of the angles of a quadrilateral are consecutive odd integers. Find the measure of each angle.
6. Kallie has $10.50 in dimes, quarters, and half dollars. She has three times as many quarters as half dollars, and three more half dollars than dimes. How many of each type of coin does she have?

**Extended Practice**: Solve each of the following problems if possible.

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| 1. Amy has $8 less than Maria. Together they have $30. How much money does each girl have? |
| 2. At the homecoming football game, the Senior Class officers sold slices of pizza for $0.75 each and hamburgers for $1.35 each. They sold 40 more slices of pizza than hamburgers, and sales totaled $292.50. How many slices of pizza did they sell? |
| 3. If one side of a square is increased by 8 cm and an adjacent side decreased by 2 cm, a rectangle is formed whose perimeter is 40 cm. Find the length of a side of the square. |
| 4. The degree measures of the angles of a pentagon are consecutive even integers. Find the measure of the largest angle. (Hint: The sum of the measures of the angles of a pentagon is 540º.) |
| 5. At 10:30 am two planes leave Houston, one flying east at 560 km/h and the other flying west at 640 km/h. At what time will they be 2100 km apart? |
| 6. Two planes leave Wichita at noon. One plane flies east 30 mi/h faster than the other plane, which is flying west. At what time will they be 1200 mi apart? |
| 7. A collection of 30 coins worth $5.50 consists of nickels, dimes, and quarters. There are twice as many dimes as nickels. How many quarters are there? |

**Solving Inequalities in One Variable**

In this section, a comparison between solving equations and solving inequalities will be made.

Consider the inequality $2<5$ Is this true?

 Now add 4 to both sides. Is this still true?

 Now add -3 to both sides. Is this still true?

**Conclusion:**

 Now multiply both sides by 3. Is this still true?

 Now multiply both sides by -3. Is this still true?

**Conclusion:**

**Summary**: Solving inequalities is similar to solving equations. The only difference is

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Now we will use these ideas to solve an inequality.

Solve and graph: $x-4>3$

**Note for Graphing:**

Use an open point for $<$ or $>$

Use a closed point for $\leq $ or $\geq $

**Break for Practice**: Solve and Graph each inequality.

1. $-5q>-15$ 2. $5-2c\leq 11$

 3. $-10y<-2(5y-3)$ 4. $2\left(x-1\right)<2x-3$

5. $3\left(x-2\right)\leq 4-x$

**Extended Practice**: Solve and Graph each inequality.

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| 1. $3x-1>-4$  | 2. $y\leq 7y-24$ |
| 3. $ -\frac{t}{2} > \frac{3}{2}$ | 4. $1+2x<2(x-1)$ |
| 5. $3\left(x-2\right)-2\leq x-5$  | 6. $4x+3\left(2-3x\right)<5\left(2-x\right)$ |
| 7. $k-3\left(2-4k\right)<7-\left(8k-9+k\right)$  | 8. $4\left(y+2\right)-9y\geq y-3\left(2y+1\right)-1$ |

**Solving Combined Inequalities**

Now the techniques from the previous section can be expanded to combined inequalities. Combined inequalities involve two or more inequalities at once.

A **combined inequality** that is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is written with “or”

Example #1: $x>1 or x\leq -2$ It is true if **at least one** of the parts is true. “or” gives the

union picture. Keep anything and everything that gets

shaded.

A **combined inequality** that is a \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ is written with “and”

 (This can also be written as $-3\leq x\leq 0$)

Example #2: $x\geq -3 and x\leq 0$ It is only true when **both** parts are true. “and” gives the

intersection picture. Only keep the part that was shaded both times.

**Break for Practice**: Solve and graph each combined inequality.

1. $3x+1\leq -2 or 5-2x<1$ 2. $2x+3\geq -3 and 5x-1<9$

3. $2<2x+4\leq 14$ 4. $2x+5\leq 11 or x+2<-3$

5. $x+4\geq 8 and 3x-1\geq 5$ 6. $x-6\leq -11 and 6-2x\leq 4$

**Extended Practice**: Solve and graph each combined inequality.

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| 1. $-1\leq 3z+2\leq 8$ | 2. $3k+7<1 or 2k-3>1$ |
| 3. $2t+7\geq 13 or 5t-4<6$  | 4. $2t+7\geq 13 and 5t-4<6$ |
| 5. $-5<1-2k<3$  | 6. $-3<2-\frac{x}{3}\leq -1$ |
| 7. $-\frac{3}{4}x\geq x-1 or -\frac{3}{4}x<x+1$  |

**Problem Solving Using Inequalities**

In this section we will work on translating word problems into inequalities to solve. There are some phrases that you will need to learn how to translate correctly in order to do this.

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| **Phrase** | **Translation** |
| x is at least “a”x is no less than “a” | $$x\geq a$$ |
| x is at most bx is no greater than b | $$x\leq a$$ |
| x is between “a” and bx is between “a” and b, inclusive | $$a<x<b$$$$a\leq x\leq b$$ |

**Break for Practice:** Solve.

1. A summer recreation department charges $45.00 for a season ticket to the town pool. Admission to the pool for one day is $2.75. How many days should you go swimming in order to make the purchase of a season ticket worth while?
2. Two sides of a triangle are consecutive even integers. The other side is 65 cm. If the perimeter is between 215 cm and 230 cm, what are the possible lengths for the first two sides?
3. Ellen’s first three test scores were consecutive odd integers. Her fourth score was 83. She had an average between 80 and 82 inclusive for the four tests. What was her lowest test score?
4. The length of a rectangle is 3 cm more than twice its width. Find the largest possible width if the perimeter is at most 66 cm.

**Extended Practice**: Solve

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| 1. For the Hawks’ 80 basketball games next year, you can buy separate tickets for each game at $9 each, or you can buy a season ticket for $580. At most how many games could you attend at the $9 price before spending more than the cost of a season ticket?
 |
| 1. The length of a rectangle is 5 cm more than twice its width. Find the largest possible width if the perimeter is at most 64 cm.
 |
| 1. Find all sets of three consecutive even integers whose sum is between 25 and 45.
 |
| 1. Jeannie’s scores on her first four tests were 80, 65, 87, and 75. What will she have to score on her next test to obtain an average of at least 80 for the term?
 |
| 1. The telephone company offers two types of service. With Plan A, you can make an unlimited number of calls per month for $38.50. With Plan B, you pay $26.50 monthly, plus 10 cents for each minute of calls after the first 340 minutes. At least how many minutes would you have to use the telephone each month to make Plan A the better option?
 |

**Absolute Value in Open Sentences**

The last section in this unit explores equations and inequalities involving absolute value. Remember that absolute value refers to the distance a number is from zero. To begin we will look at three very basic examples.

1. $\left|x\right|=5$ What is really being asked is, “What numbers have a distance from zero of 5?”
2. $\left|x\right|\geq 5$ What is really being asked is, “What part(s) of the number line represents numbers

whose distance from zero is greater than or equal to 5 units?”

1. $\left|x\right|\leq 5$ What is really being asked is, “What part(s) of the number line represents numbers

whose distance from zero is less than or equal to 5 units?”

**Summary**:

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| **Absolute Value Statement** | **Rewrite** | **What the graph looks like** |
| $$\left|x\right|=a$$ | $$x=a or x=-a$$ |  |
| $$\left|x\right|\geq a$$ | $$x\geq a or x\leq -a$$ |  |
| $$\left|x\right|\leq a$$ | $$-a\leq x\leq a$$ |  |

**Break for Practice**: First rewrite each inequality then THINK if it’s possible. If possible, solve and graph the inequality. If it’s not possible, say no solution.

1. $\left|3-x\right|=5$
2. $1<\left|2-\frac{x}{3}\right| $
3. $\left|x-1\right|=-4$
4. $\left|2y+1\right|\leq 5$

**Extended Practice**: Solve and Graph

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| --- | --- |
| 1. $\left|2t+5\right|<3$ | 2. $\left|3x+2\right|>4$ |
| 3. $\left|2u-5\right|=0$ | 4. $8=\left|5y+2\right|$ |
| 5. $\left|1-\frac{x}{3}\right|\leq -\frac{2}{3}$  | 6. $\left|1-\frac{p}{2}\right|\leq 2$ |
| 7. $\left|\frac{x-2}{4}\right|>\frac{1}{2}$  | 8. $1>\left|2-0.8n\right|$ |

Some inequalities involving absolute value can be a little harder. It is important to realize that you must isolate the absolute value before you try to rewrite the inequality. Also, don’t forget to think. Don’t be tricked into working harder than you have to.

**Break for Practice**: Solve and Graph

1. $\left|r+1\right|-3<1$

1. $\left|3x+2\right|+4\geq 2$
2. $9-2\left|x+3\right|<3$

**Extended Practice**: Solve and Graph

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| --- | --- |
| 1. $\left|2t-3\right|+2=5$ | 2. $\left|2u-1\right|+3\leq 6$ |
| 3. $4-\left|3k+1\right|<2$ | 4. $7-3\left|4d-7\right|\geq 4$ |
| 5. $4+2\left|\frac{3t-5}{2}\right|>5$ | 6. $2\left|\frac{2t-5}{3}\right|-3\geq 5$ |