

Unit 10 Review

“We like good quotes.” ELKS

1. Identify the following as arithmetic, geometric, or neither. Also find the next 2 terms.

$$a.) \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \underline{\frac{5}{6}}, \underline{\frac{6}{7}}$$

0.5 0.6 0.75 0.8

Circle one: Arithmetic Geometric Neither

Adding 1 to the numerator and denominator

$$b.) 3, -6, 9, -12, \underline{15}, \underline{-18}$$

Circle one: Arithmetic Geometric Neither

Adding by 3 then alternating signs

$$c.) 729, 243, 81, 27, \underline{9}, \underline{3}$$

Circle one: Arithmetic Geometric Neither

Dividing by 3

2. Write the formula for the following sequence.

$$-2, 4, 10, 16, \dots$$

+6

$$t_n = -2 + (n-1)6 \quad \text{OR} \quad t_n = -2 + 6n - 6$$

$$t_1 = -2$$

$$d = 6$$

$$t_n = 6n - 8$$

3. Find the specified term of the arithmetic sequence.

$$20, 17, 14, \dots; t_{20}$$

-3

$$t_{20} = 20 + (20-1)(-3)$$

$$t_{20} = 20 + 19(-3)$$

$$t_{20} = -37$$

$$t_1 = 20$$

$$d = -3$$

$$n = 20$$

4. Find the position, n, of the underlined term.

$$-5, \underline{5}, 6, 11.5, \dots, \underline{127}$$

$$127 = -5 + (n-1)5.5$$

+5 +5

$$132 = 5.5(n-1)$$

5.5 5.5

$$24 = n-1$$

+1 +1

$$n = 25$$

$$t_1 = -5$$

$$d = 5.5$$

$$t_n = 127$$

5. Insert three arithmetic means between 12 and 2.

$$12, \quad \text{---}, \quad \text{---}, \quad \text{---}, \quad 2$$

$$t_1 \qquad \qquad \qquad t_5$$

$$2 = 12 + (5-1)d$$

$$-12 \quad -12$$

$$-10 = 4d$$

$$\frac{-10}{4} = \frac{4d}{4}$$

$$\underline{-2.5 = d}$$

$$t_1 = 12$$

$$t_5 = 2$$

$$n = 5$$

5.) 12, 9.5, 7, 4.5, 2

6. Write the formula for the following sequence.

$$-6, -12, -24, \dots$$

$$\underbrace{\quad \quad \quad}_{\times 2}$$

$$t_n = \underline{-6 \cdot 2^{n-1}}$$

$$t_1 = -6$$

$$r = 2$$

7. Find the specified term of the geometric sequence.

$$\frac{1}{9}, \frac{-1}{3}, 1, -3, \dots; t_{15}$$

$$\underbrace{\quad \quad \quad}_{\times -3}$$

$$t_{15} = \frac{1}{9} \cdot (-3)^{15-1}$$

$$t_{15} = \frac{1}{9} \cdot 4182969$$

$$t_{15} = \underline{531441}$$

$$t_1 = \frac{1}{9} \quad n = 15$$

$$r = -3$$

8. Find the position, n, of the underlined term.

$$4, 16, 64, 256, \dots, \underline{16777216}$$

$$\underbrace{\quad \quad \quad}_{\times 4}$$

$$t_1 = 4$$

$$r = 4$$

$$t_n = 16777216$$

$$\frac{16777216}{4} = \frac{4 \cdot 4^{n-1}}{4}$$

$$4194304 = 4^{n-1}$$

$$\log_4 4194304 = n-1$$

$$n-1 = \frac{\log 4194304}{\log 4}$$

$$n-1 = 11$$

$$+1 \quad +1$$

$$\underline{n = 12}$$

9. Insert three geometric means between 81 and 1.

$$81, \quad \text{---}, \quad \text{---}, \quad \text{---}, \quad 1$$

$$t_1 \qquad \qquad \qquad t_5$$

$$1 = 81 \cdot r^{5-1}$$

$$\frac{1}{81} = \frac{81 \cdot r^4}{81}$$

$$\sqrt[4]{\frac{1}{81}} = \sqrt[4]{r^4}$$

$$\frac{1}{3} = r$$

$$t_1 = 81$$

$$t_5 = 1$$

$$n = 5$$

9.) 81, 27, 9, 3, 1

10. Write the series in expanded form, and find its sum.

a.) $\sum_{k=1}^5 4k-3 = 1 + 5 + 9 + 13 + 17$ Sum = 45

$t_1 = 4(1)-3$ $t_2 = 4(2)-3$ $t_3 = 4(3)-3$ $t_4 = 4(4)-3$ $t_5 = 4(5)-3$
 $t_1 = 1$ $t_2 = 5$ $t_3 = 9$ $t_4 = 13$ $t_5 = 17$

b.) $\sum_{k=2}^6 4(2)^{(k-1)} = 8 + 16 + 32 + 64 + 128$ Sum = 248

$t_2 = 4 \cdot 2^{2-1}$ $t_3 = 4 \cdot 2^{3-1}$ $t_4 = 4 \cdot 2^{4-1}$ $t_5 = 4 \cdot 2^{5-1}$ $t_6 = 4 \cdot 2^{6-1}$
 $t_2 = 8$ $t_3 = 16$ $t_4 = 32$ $t_5 = 64$ $t_6 = 128$

11. Rewrite the series into sigma notation.

a.) $6 + 12 + 24 + \dots + 98304$

$\xrightarrow{\times 2}$

$t_1 = 6$
 $r = 2$
 $t_n = 98304$

$98304 = \frac{6 \cdot 2^n}{6}$
 $16384 = 2^{n-1}$
 $\log_2 16384 = n-1$
 $n-1 = \frac{\log 16384}{\log 2}$
 $n-1 = 14$
 $+1$
 $n = 15$

\times Geometric
 $t_n = t_1 \cdot r^{n-1}$

$$\sum_{n=1}^{15} 6 \cdot 2^{n-1}$$

b.) $5 + 8 + 11 + 14 + \dots \infty$ \leftarrow no limit

$\xrightarrow{+3}$

$t_1 = 5$ $t_n = 5 + (n-1)3$
 $d = 3$ $t_n = 5 + 3n - 3$
 \star Arithmetic
 $t_n = t_1 + (n-1)d$ $t_n = 3n + 2$

$$\sum_{n=1}^{\infty} 3n + 2$$

12. A wealthy mother gives her daughter \$5 on her first birthday, \$10 on her second birthday, \$20 on her third birthday, and \$40 on her fourth birthday. If this pattern continues, what will be the gift on her 28th birthday?

5, 10, 20, 40

$\xrightarrow{\times 2}$

$t_{28} = 5 \cdot 2^{28-1}$
 $t_{28} = \$671,088,640$

$t_1 = 5$

$r = 2$

$n = 28$

\star Geometric
 $t_n = t_1 \cdot r^{n-1}$