Break for Practice:

1. Match each axiom with its definition and example.

Name	Definition	Example
Commutative Property of Addition * move = order *	G.	N.
Commutative Property of Multiplication	C.	K.
Associative Property of Addition * parentheses = group *	D.	O.
Associative Property of Multiplication	I.	J.
Identity Property of Addition	B.	L.
Identity Property of Multiplication	F.	Q.
Inverse Property of Addition (Property of Opposites)	E.	P.
Inverse Property of Multiplication (Property of Reciprocals)	H.	R.
Distributive Property	A.	M.

Definitions:

A.) Multiplication distributes over addition

B.) Zero can be added to a number without changing the number's identity

C.) Numbers can be multiplied in any order

D.) Numbers can be regrouped when adding

E.) Every number can be added to another number to give a value of zero

F.) 1 can be multiplied by a number without changing the number's identity

G.) Numbers can be added in any order

H.) Every number can be multiplied by another number to give a value of 1

I.) Numbers can be regrouped when multiplying

Examples:

J.) $7(3 \cdot 4) = (7 \cdot 3)4$

K.) $-5(3) = 3(-5)$

L.) $6 + 0 = 6$

M.) $7(9 + 2) = 7(9) + 7(2)$

N.) $3 + 2 = 2 + 3$

O.) $8 + (2 + 7) = (8 + 2) + 7$

P.) $8 + (-8) = 0$

Q.) $8(1) = 8$

R.) $\left(\frac{3}{4}\right)\left(\frac{4}{3}\right) = 1$

Identify the property used in each step. See what changes

2. #1 $x + (x + 5) = (x + x) + 5$ #2 — #1-#2: Associative Prop. (+)

#3 $= (1 \cdot x + 1 \cdot x) + 5$ — #2-#3: Identity Prop (x)

#4 $= (1 + 1)x + 5$ — #3-#4: Distributive Prop.

#5 $= 2x + 5$ — substitution

3. #1 $5 + 2(x + 1) = 5 + (2x + 2 \cdot 1)$ #2 — 1-2: Distributive Prop.

#3 $= 5 + (2x + 2)$ — 2-3: Identity Prop. (•)

#4 $= 5 + (2 + 2x)$ — 3-4: Commutative Prop. (+)

#5 $= (5 + 2) + 2x$ — 4-5: Associative Prop. (+)

$= 7 + 2x$ — substitution

4. $a(b+1)+(-1)a = a(b+1)+a(-1)$ #1 #2 — 1-2: Commutative Prop. (x)

#3 $= a[(b+1)+(-1)]$ — 2-3: Distributive Prop.

#4 $= a[b+(1+(-1))]$ — 3-4: Assoc. Prop.

#5 $= a[b+0]$ — 4-5: Prop. of Opposite (+)

#6 $= ab$ — 5-6 Identity Prop (+)

Simplify by applying the various properties.

5. $\frac{1}{5}(5)+(-2+2)$

$= 1 + 0$

$= 1$

6. $6(2x+3)+(-18)$

$= 12x + 18 + (-18)$

$= 12x + 0$

$= 12x$

7. $\left(\frac{5}{7}a\right)\left(\frac{7}{5}c\right)$

$= \left(\frac{5}{7}\cdot\frac{7}{5}\right)(ac)$

$= 1ac$

$= ac$

Extended Practice: Simplify by applying the various properties.

1. $-6+x+6$ $= x$	2. $\frac{1}{3}(1\cdot 3)+(-3+3)$ $= 1$
3. $\frac{1}{4}(z+4)$ $= \frac{1}{4}z + 1$	4. $\left(\frac{2}{3}a\right)\left(\frac{3}{2}b\right)$ $= ab$
5. $\left(\frac{1}{3}\cdot 3\right)[p+(-1)]+1$ $= p$	6. $(7p)\frac{1}{7}+(-p)$ $= 0$

Extended Practice Continued: Identify the property used in each step.

7. $(2+a)+(-2) = (a+2)+(-2)$	Commutative Prop. (+)
$= a+[2+(-2)]$	Associative Prop. (+)
$= a+0$	Inverse Prop. (+) OR Prop. of Opposites
$= a$	Identity Prop. (+)

8. $\frac{1}{5}[(n+5)+(-n)] = \frac{1}{5}[(5+n)+(-n)]$	Commutative Prop. (+)
$= \frac{1}{5}[5+(n+(-n))]$	Associative Prop. (+)
$= \frac{1}{5}[5+0]$	Inverse Prop (+) OR Prop. of Opposites
$= \frac{1}{5}\cdot 5$	Identity Prop. (+)
$= 1$	Inverse Prop. (*) OR Prop. of Reciprocals

9. $k+(k+2) = (k+k)+2$	Associative Prop (+)
$= (1\cdot k + 1\cdot k)+2$	Identity Prop (*)
$= (1+1)k+2$	Distributive Prop.
$= 2k+2$	Substitution
$= 2k+2\cdot 1$	Identity Prop (*)
$= 2(k+1)$	Distributive Prop.

Basic Operations with Signed Numbers

The last topic to review in this unit is how to perform the basic operations of addition, subtraction, multiplication and division on signed numbers.

Rules for Adding Signed Numbers:

a) If the two numbers have the same sign, add their absolute values and keep their common sign.
b) If the two numbers have opposite signs, find the difference of their absolute values, and keep the sign of the number with the larger absolute value.

Rule for Subtracting Signed Numbers: To subtract any real number, add its opposite.

Break for Practice: Simplify each problem by applying the rules from above and using your knowledge of absolute value.

1. $-52+17 \rightarrow |-52|-|17|$
$52-17$
-35

2. $-27-14 = |-27|+|-14|$
$= 27+14$
$= -41$

3. $12-(-33) = 45$

4. $-16+(-36) = -52$

5. $3-(5+1)-(1-4)$
$= 3-(6)-(-3)$
$= 0$

6. $[-5+(-17)]-(3-7+2)$
$= -22-(-2)$
$= -20$

7. $|2-6|+|8-20|$
$= |-4|+|-12|$
$= 4+12$
$= 16$

8. $|-21-3|-|8+(-12)|$
$= |-24|-|-4|$
$= 24-4$
$= 20$

Definition: similar/like terms are terms with the exact same variable parts. These can be added and subtracted.

Break for Practice: Simplify by applying the distributive property and/or combining like terms.

9. $19x + 3(8 + x)$

$= 19x + 24 + 3x$

$= 22x + 24$

10. $3(9 - y) + 5(1 - y)$

$= 27 - 3y + 5 - 5y$

$= -8y + 32$

11. $4(3m - 6n) + 2(2m + 5n - 3)$

$= 12m - 24n + 4m + 10n - 6$

$= 16m - 14n - 6$

Rules for Multiplying and Multiplying of Signed Numbers:

a) The product of two positives or the product of two negatives is a positive.

b) The product of a positive and a negative is negative.

c) The absolute value of the product of two or more numbers is the product of their absolute values.

Example: $|-3 \cdot 5| = |-15| = 15$

$|-3| \cdot |5| = 3 \cdot 5 = 15 \; so \; |-3 \cdot 5| = |-3| \cdot |5|$

Break for Practice: Simplify

12. $17 \cdot (-3) = -51$

13. $5 \cdot 8 = 40$

14. $-10 \cdot (-15) = 150$

Multiplicative Property of 0: $a \cdot 0 = 0$

Multiplicative Property of -1: $a(-1) = -a$

Now let's extend these ideas: Simplify

a) $\boxed{(-2)(1)(3)}\boxed{(-4)(-5)}$ -6 20

-120

b) $(7)(-1)(-2)(4)(-2)(-1)$ -7 +14

112

c) $(7)(-1)(-2)(0)(-2)(-1)$

$= 0$

Result: When multiplying a set of numbers, the product is

a) Positive if there are an even number of negative factors. (b)

b) Negative if there are an odd number of negative factors. (a)

c) Zero if any of the factors are zero. (c)

Property of the Opposite of a Product: $-(a \cdot b) = -ab$

Property of the Opposite of a Sum: $-(a + b) = -a - b$

Break for Practice: Simplify

15. $\boxed{\left(-\frac{1}{2}\right)(-6)}\boxed{\left(-\frac{1}{12}\right)(-12)}$

$= (3)(1)$

$= 3$

16. $\boxed{5(2x)}(-3y) \rightarrow (5 \cdot 2 \cdot (-3))(x \cdot y)$

$= \boxed{10x(-3y)}$

$= -30xy$

17. $(4-5)(3+8)$

$= (-1)(11)$

$= -11$

18. $2\boxed{a \cdot 5b}$

$= 2[5ab]$

$= 10ab$

★ Only distribute if there is +/− inside ( )

$2[a+5b]$

19. $\boxed{(-4)^3}\boxed{(-1-1)}(-3)$

$= (-64)(-2)(-3)$

$= -384$

20. $14\boxed{(-1)^8}\boxed{(-3)^2}$

$= 14(1)(9)$

$= 126$

21. $3(x+2y) - 1(2x-4y)$

$= 3x + 6y - 2x + 4y$

$= x + 10y$

22. $m(n+1) - 4(mn+2)$

$= mn + m - 4mn - 8$

$= -3mn + m - 8$

Rules for Division and Division of Signed Numbers:

1. A division problem can be rewritten as multiplication if you multiply by the reciprocal.

Change it

Example: $\frac{8}{5} \div \frac{12}{5}$ $\quad \frac{8}{5} \cdot \frac{5}{12} = \frac{40}{60}$ (Reduce ÷2) $= \frac{2}{3}$

Leave it $\quad$ Flip it

In general: $\frac{a}{b} = a \div b = a \cdot \frac{1}{b}$

2. The quotient of two positives or two negatives is positive.
3. The quotient of a positive and a negative is negative.

Break for Practice: Simplify

23. $\boxed{-72 \div (-6)} \div (-2)$

$= (12) \div (-2)$

$= -6$

24. $-72 \div \boxed{[-6 \div (-2)]}$

$= -72 \div (3)$

$= -24$

25. $\frac{(-9(11)+43)}{(1-(-3)^3)}$ $= \frac{-99+43}{1-(-27)}$

$= \frac{-56}{28}$

$= -2$

26. $24 \div \left(-\frac{2}{3}\right)\left(-\frac{1}{4}\right) \div 27$

$= \frac{\cancel{24}^{6}}{1} \cdot \left(-\frac{\cancel{3}^{1}}{2}\right) \cdot \left(-\frac{1}{\cancel{4}_{1}}\right) \cdot \left(\frac{1}{\cancel{27}_{9}}\right)$

$= \frac{6}{18} = \frac{1}{3}$

Consider the following problem. It can actually be solved two different ways.

Method 1: $\frac{6+10}{2}$ $= \frac{16}{2}$

$= 8$

Method 2: $\frac{6+10}{2}$ $= \frac{6}{2} + \frac{10}{2}$ (Split into separate fractions)

$= 3+5$

$= 8$

Break for Practice: Simplify

27. $\frac{6x^2+(-21)}{-3}$

$= -2x^2+7$

28. $\frac{-12+8x^2+(-4x)}{-4}$

$= \frac{-12}{-4} + \frac{8x^2}{-4} + \frac{(-4x)}{-4}$

$= 3 + (-2x^2) + x$

$= 3 - 2x^2 + x$ or

29. $\frac{15x^2+(-20x)+5}{5}$

$= \frac{15x^2}{5} + \frac{(-20x)}{5} + \frac{5}{5}$

$= 3x^2 - 4x + 1$

Extended Practice: Simplify by applying the rules for operating on signed numbers. No Calculator!

1. $16 + (-50)$ $= -34$	2. $17 - (-26)$ $= 43$
3. $21.2 - 32.3$ $= -11.1$	4. $-23 + 57$ $= 34$
5. $7(-2)(6)(-1)$ $= 84$	6. $\left(\frac{3}{4}\right)(-10)(-8)\left(\frac{1}{5}\right)$ $= 12$
7. $(-a)(-2b)(-3c)$ $= -6abc$	8. $(-2)(1 - 2x - 3x^2)$ $= -2 + 4x + 6x^2$
9. $64 \div (-4) \div (-2)$ $= 8$	10. $64 \div [(-4) \div (-2)]$ $= 32$

11. $\frac{24 \div (-3)}{4(-5) \div 2}$ $= \frac{4}{5}$	12. $\lvert 6-13 \rvert - \lvert 22-(-6) \rvert$ $= -21$
13. $\frac{2}{3}(6r - 9s - 27)$ $= 4r - 6s - 18$	14. $(6x - 5y + 4) + 2(-2x + 3y - 2)$ $= 2x + y$
15. $(3 - 6 - 9) - [8 + (-4) - (-7)]$ $= -23$	16. $(-5)^3\left(-\frac{1}{5}\right)^2$ $= -5$
17. $-3(p - 5) - 7p$ $= -10p + 15$	18. $-c(d + 5) + 6(2 - cd)$ $= -7cd - 5c + 12$
19. $\frac{4^2 - 5^2}{(-4) + (-5)}$ $= 1$	20. $\frac{(-12)\left(-\frac{3}{4} - \frac{1}{2}\right)}{\frac{5}{9} \div (-10)}$ $= -270$

21. $\dfrac{\left[\frac{4}{9}-\left(-\frac{2}{9}\right)\right]\left[\frac{2}{3}-\left(-\frac{2}{3}\right)\right]^2}{\frac{5}{9}\div\left(-\frac{10}{3}\right)}$ $= -\dfrac{64}{9}$

Extended Practice Continued: Evaluate.

22. Evaluate the expression when $x = 2$ $\dfrac{(x^2-4)(x-3)}{x+1}$ $= 0$	23. Evaluate the expression when $b = -2$ $2b^3 + 3b^2 - b + 3$ $= 1$

Extended Practice Continued

24. The highest point in the United States is Mt. McKinley, Alaska, at 20,320 ft. above sea level, and the lowest is Bad Water, California, at 282 ft. below sea level. Find the difference between these elevations.

= 20,602 ft

25. Give a numerical example to show that subtraction is not commutative.

Answers vary