Name: Hour 4-A

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Algebra II

Unit 0: Basic Concepts of Real Numbers

Priority Standards: A-SSE.1a: Interpret parts of an expression, such as terms factors and coefficients

Unit "I can" statements:

1. I can graph real numbers on a number line, to compare numbers, and to find their absolute value.
2. I can simplify numerical expressions and evaluate algebraic expressions.
3. I can identify and apply properties of equality of real numbers and properties for adding and multiplying real numbers.
4. I can correctly apply the rules for adding, subtracting, multiplying, and dividing real numbers.

Common Core State Standards that are addressed in this unit include: A.SSE.1a, N-RN.B.3
For more information see www.corestandards.org/Math/

Real Numbers and Their Graphs

This first chapter will serve as a review of some necessary skills and information that you learned in Algebra I. Some material may be new, but most of it should at least contain a familiar foundation. To begin with, let's look at some of the sets of numbers that can be used for classifying numbers.

No negatives

Digits - the symbols used to write **ALL** other numbers

Examples: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Natural Numbers – the counting numbers

Examples: 1, 2, 3, 4, 5 ... goes on forever

No fractions/decimals

Whole Numbers – the counting numbers and zero

Examples: 0, 1, 2, 3, 4, 5, ...

Integers – positive and negative whole numbers

Examples: ..., −3, −2, −1, 0, 1, 2, 3, ...

Rational Numbers – any number that can be written as a fraction

Examples: 0, −24, 7, $5/2 \to 2.5$, $4/3 \to 1.\overline{3}$, $\sqrt{4}=2$

Any # that is a terminating (ends) decimal or repeating decimal

Irrational Numbers – any number that cannot be written as a fraction

Examples: π, $\sqrt{2}$, $-\sqrt{3}$, 2.3223222 3...

Any decimal that never ends or repeats.

Real Numbers – any number on the number line

Examples: All the above examples

Imaginary Numbers – the square root of a negative number

the negative is under the square root sign.

Examples: $\sqrt{-6}$, $\sqrt{-4}$

Break for Practice: To which set(s) does each number belong?

1. 15 Real, Rational, Integer, Whole, Natural

0.8

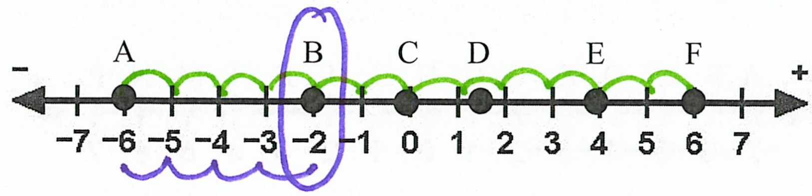
2. $\frac{4}{5}$ Real, Rational

3. $\sqrt{7}$ Real, Irrational

4. $\sqrt{-7}$ Imaginary

5. 4 Real, Rational, Integar, Whole, Natural, Digit

Since real numbers can be graphed, we should review the terminology and practice. Consider the number line shown below:



The Coordinate (#) of A is -6. The graph (letter) of 4 is E. The graph of 0 is C, and this point is called the origin.

Break for Practice: Use the number line above to answer the following questions.

1. What is the coordinate of B? -2
2. What is the coordinate of D? 1.5 $1\frac{1}{2}$ $\frac{3}{2}$
3. What is the graph of 6? F
4. What is point is 1/3 of the way from A to F? Distance $= \frac{12}{3} = 4$ $-6 + 4 = -2$ $\boxed{B(-2)}$

Numbers can be compared with inequalities. Rewrite each statement into symbols.

Less than $<$

Greater than $>$

Less than or equal to $\leq$

Greater than or equal to $\geq$

1. Five is greater than negative three. $5 > -3$
2. Negative four thirds is less than negative two thirds. $-\frac{4}{3} < -\frac{2}{3}$

Another concept that should be reviewed is that of absolute value.

Absolute Value: The distance that a # is from zero (It is ALWAYS positive)

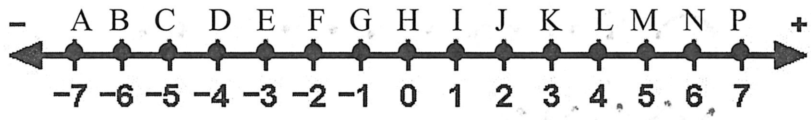
Examples: $|7| =$ 7 $|-7| =$ 7 $|0| =$ 0

$-|-8| = -8$

Extended Practice:

1. Which set of numbers contains all but one of the other sets? Real Numbers Which set is not included in this set? Imaginary	2. Does a decimal such as 2.718 represent a rational or an irrational number? Explain. Rational - Explain why
3. Does a repeating decimal such as $2.\overline{3}$ represent a rational or an irrational number? Explain. Rational - Explain why	4. What real number is neither positive nor negative? 0
5. Write each statement using symbols. a) Zero is greater than negative six. $0 > -6$	b) Negative three is less than negative one. $-3 < -1$

6. Find the coordinate of each point described. Use the number line below.



a) B -6

b) The point halfway between D and J G(-1)

c) The point one fourth of the way from F to N H (0)

7. Write an inequality statement comparing the two numbers.

a) $\frac{1}{2}$ *and* $\frac{-3}{2}$ $\frac{1}{2} > -\frac{3}{2}$ OR $-\frac{3}{2} < \frac{1}{2}$

b) -1.5 and 0.5 $-1.5 < 0.5$ OR $0.5 > -1.5$

8. Find the value of each expression.

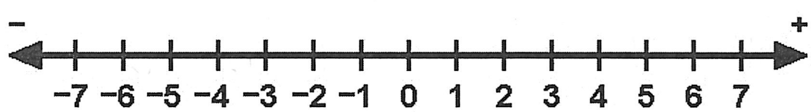
a) $|-5|$ = 5

b) $-|-7|$ = −7

c) $|5| - |-2|$ = 3

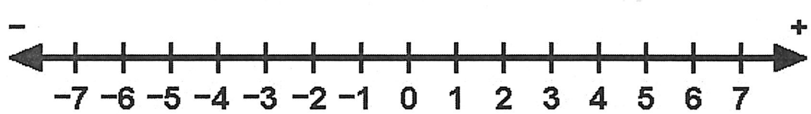
9. Arrange the list of numbers in order from least to greatest and graph the numbers on the number line.

$\frac{5}{2}, -\frac{1}{2}, 2, -\frac{3}{2}, -2$



$-2, -\frac{3}{2}, -\frac{1}{2}, 2, \frac{5}{2}$

10. On a number line, point A has coordinate -5 and point B has coordinate 1. Find the coordinate of each point described.



a) The point 2 units to the left of B

−1

b) The point 1.4 units to the right of A

−3.6

c) Each point that is twice as far from A as from B

−1 and −7

11. Put a check mark in each box for which the number on the left of the chart belongs to the set across the top

	Integer	Digit	Rational	Irrational	Natural	Whole	Real	Imaginary
5	X	X	X		X	X	X	
$-\frac{2}{3}$			X				X	
$\sqrt{-15}$								X
20.62			X				X	
π				X			X	
$-1\frac{1}{2}$			X				X	
$-\sqrt{6}$				X			X	
0	X	X	X			X	X	
1	X	X	X		X	X	X	

Simplifying Expressions

It is important to review the terminology and skills needed to simplify numerical expressions and evaluate algebraic expressions. Some of the terms that should be reviewed are in the tables on pages 6-9 in your book. Use these tables to answer the following questions.

1. What operation should you use when you are asked to find each of the following?

   a) Sum — Addition
   b) Product — Multiplication
   c) Quotient — Division
   d) Difference — Subtraction

2. In the expression 4^3, what is the 4 called? Base

   What is the 3 called? Exponent

3. Give 3 different examples of grouping symbols.

   ( ), [ ], $\frac{\text{(top)}}{\text{(bottom)}}$ ← fraction bar

To simplify an expression, you need to reduce it down to its simplest form. In order to simplify, you need to know the order of operations. Number the following steps 1-4 in the order that they should be performed when simplifying an expression.

__2__ Exponents

__3__ Multiplication and Division- **Work from left to right**

__1__ Parentheses – **Work from inside to outside and from left to right**

__4__ Addition and Subtraction- **Work from left to right**

Break for Practice: Simplify and show all work

1. $18 - (7 + 3) - 1$

$= 18 - 10 - 1$

$= 8 - 1$

$= 7$

2. $6 \cdot (4 + 5 \cdot 2)$

$= 6 \cdot (4 + 10)$

$= 6 \cdot (14)$

$= 84$

3. $(6^2 - 8) \div (2 + 5)$

$= (36 - 8) \div (7)$

$= 24 \div 7$

$= 4$

4. $[4(5 - 2) + 2^3] \div 2$

$= [4(3) + 2^3] \div 2$

$= [4(3) + 8] \div 2$

$= [12 + 8] \div 2$

$= 20 \div 2$

$= 10$

5. $\frac{(4^3+6)}{(4^2-6)} = \frac{64+6}{16-6}$

$= \frac{70}{10}$

$= 7$

6. $\frac{1}{2}\left|\frac{1+9^2}{5^2}\right| = \frac{1}{2}\left|\frac{1+81}{25}\right|$

$= \frac{1}{2}\left|\frac{82}{25}\right|$

$= \frac{1}{2} \cdot \frac{82}{25}$

$= \frac{82}{50}$ (Reduce) (÷2) $\Rightarrow \frac{41}{25}$

7. Evaluate if $x = 2, y = 3, and\ z = 7$.

a) $y|xy - z|$ Substitute Values $\rightarrow$ $3|(2)(3) - 7|$

$= 3|6 - 7|$

$= 3|-1|$

$= 3 \cdot 1$

$= 3$

b) $\frac{2xy}{z^2 - y^3}$ Substitute Values $\rightarrow$ $\frac{2(2)(3)}{(7)^2 - (3)^3}$

$= \frac{4 \cdot 3}{49 - 27}$

$= \frac{12}{22}$ (Reduce ÷2)

$= \frac{6}{11}$ OR $0.\overline{54}$

Extended Practice:

1. Use one of the symbols <, =, or > to make a true statement.

a) $5 \cdot 1$ = $5 \div 1$

b) $\frac{3+2}{3-2}$ > $\frac{4+2}{4-2}$

c) $3^2 \cdot 4^2$ = $(3 \cdot 4)^2$

Extended Practice Continued: Simplify.

2. $11 - 3 + 5 - 2$ = 11	3. $11 - (3 + 5) - 2$ = 1
4. $3 \cdot 8 + 4 \cdot 5$ = 44	5. $3 \cdot (8 + 4) \cdot 5$ = 180
6. $\frac{2^3+1}{2^2-1}$ = 3	7. $\frac{1}{3}\left\lvert\frac{1+7^2}{5^2}\right\rvert$ = $\frac{2}{3}$
8. $[3^3 - (2^3 + 2^2)] \div 5$ = 3	9. $14 - 2[9 - 2(5 - 3)]$ = 4

Extended Practice Continued: Evaluate if $x = 3$, $y = 2, and\ z = 5$.

10. $2x^2 + x - 2$ = 19	11. $(yz - x)^3$ = 343	12. $\frac{x+z}{y} - \frac{x+y}{2z}$ = 3½ OR 3.5

Basic Properties of Real Numbers

Another topic that should be reviewed is the various properties that we apply when working with Algebra. These properties help us to simplify and solve numerical and/or algebraic problems.

The first set of properties are called properties of equality. These are fairly straight forward even if some of the names are not.

Properties of Equality:

Reflexive Property: $a = a$ (anything equals itself)

Symmetric Property: If $a=b$, then $b=a$ (you can switch the 2 sides)

Transitive Property: If $a=b$, and $b=c$, then $a=c$.

Addition Property: If $a=b$, then $a+c=b+c$ Ex. $x-12=20$ $+12$ $+12$ $x=32$

Multiplication Property: If $a=b$, then $a \cdot c = b \cdot c$ Ex. $8 \cdot \frac{x}{8} = 3 \cdot 8$ $x=24$

The next set of properties are called **Field Axiom Properties**.

Name and Description	Example
Commute = move $\boxed{\text{Commut}}$ative Property for Addition – numbers can be added in any order	$3+4=4+3$
Commutative Property for Multiplication – numbers can be multiplied in any order	$-4(3)=3(-4)$
Associative Property for Addition – numbers can be regrouped when adding	$8+(2+6)=(8+2)+6$
Associative Property for Multiplication – numbers can be regrouped when multiplying	* order doesn't change * $(7 \cdot 3) \cdot 2 = 7 \cdot (3 \cdot 2)$
Distributive Property – a single term in front of parentheses can be multiplied by two or more terms inside the parentheses	$7(x+3) = 7 \cdot x + 7 \cdot 3$ $= 7x + 21$ it can go backwar
Identity Property for Addition – zero can be added to any number without changing the number's value	$6 + \underline{0} = 6$
Identity Property for Multiplication – any number can be multiplied by the number one without changing the value of the original number	$-7 \cdot \underline{1} = -7$
Inverse Property for Addition (Property of Opposites) – for each number there is a unique opposite number so that the sum of the two numbers is zero	$-18 + 18 = 0$ opposites
Inverse Property for Multiplication (Property of Reciprocals) – for each number (excluding zero) there is a unique reciprocal so that the product of the two numbers is one	$\frac{5}{1} \cdot \frac{1}{5} = 1$ reciprocals
Closure Property for Addition – a set is closed for addition when any two numbers in a set can be added and the sum still belongs to the same set	4 and 8 are even $4+8=12$ also even
Closure Property for Multiplication – a set is closed for multiplication when any two numbers in a set can be multiplied and the product still belongs to the same set	3 and 5 are odd $3 \cdot 5 = 15$ also odd