Extended Practice: Solve each of the following problems if possible.

1. Amy has \$8 less than Maria. Together they have \$30. How much money does each girl have?

Maria = \$19, Amy = \$11

2. At the homecoming football game, the Senior Class officers sold slices of pizza for \$0.75 each and hamburgers for \$1.35 each. They sold 40 more slices of pizza than hamburgers, and sales totaled \$292.50. How many slices of pizza did they sell?

165 slices of pizza

3. If one side of a square is increased by 8 cm and an adjacent side decreased by 2 cm, a rectangle is formed whose perimeter is 40 cm. Find the length of a side of the square.

Side length = 7cm

4. The degree measures of the angles of a pentagon are consecutive even integers. Find the measure of the largest angle. (Hint: The sum of the measures of the angles of a pentagon is 540°.)

Largest $\angle = 112°$

5. At 10:30 am two planes leave Houston, one flying east at 560 km/h and the other flying west at 640 km/h. At what time will they be 2100 km apart?

They will be 2,100 Km apart @ 12:15 pm

6. Two planes leave Wichita at noon. One plane flies east 30 mi/h faster than the other plane, which is flying west. At what time will they be 1200 mi apart?

Not Enough Information

7. A collection of 30 coins worth $5.50 consists of nickels, dimes, and quarters. There are twice as many dimes as nickels. How many quarters are there?

18 Quarters

Solving Inequalities in One Variable

In this section, a comparison between solving equations and solving inequalities will be made.

Consider the inequality $2 < 5$ Is this true? Yes

Now add 4 to both sides. $6 < 9$ Is this still true? Yes

Now add -3 to both sides. $-1 < 2$ Is this still true? Yes

Conclusion: Adding the same amount to both sides of an inequality will still keep a true inequality.

Now multiply both sides by 3. $6 < 15$ Is this still true? Yes

Now multiply both sides by -3. $-6 < -15$ Is this still true? No

Conclusion: If you multiply both sides of a true inequality by a...
1. positive # will keep a true inequality
2. negative # will create a false inequality ⇒ The inequality signed should be flipped.

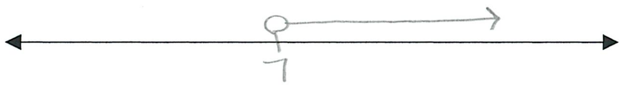
Summary: Solving inequalities is similar to solving equations. The only difference is when multiplying or dividing by a negative number, you should turn the inequality sign around.

Now we will use these ideas to solve an inequality.

Solve and graph: $x - 4 > 3$

$+4 \quad +4$

$x > 7$



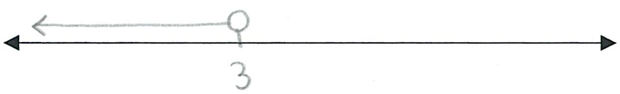
Note for Graphing:
Use an open point for $<$ or $>$
Use a closed point for $\leq$ or $\geq$

Break for Practice: Solve and Graph each inequality.

1. $-5q > -15$

$\overline{-5} \quad \overline{-5}$

$q < 3$



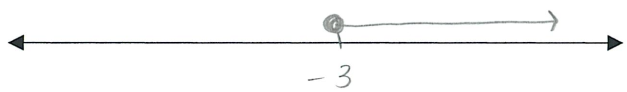
2. $5 - 2c \leq 11$

$-5 \quad -5$

$-2c \leq 6$

$\overline{-2} \quad \overline{-2}$

$c \geq -3$



3. $-10y < -2(5y - 3)$

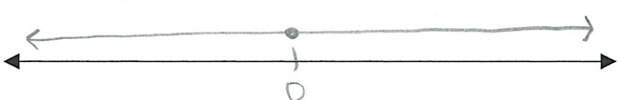
$-10y < -10y + 6$

$+10y \quad +10y$

$0 < 6$

True statement

$\mathbb{R}$



4. $2(x - 1) < 2x - 3$

$2x - 2 < 2x - 3$

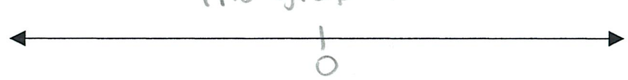
$-2x \quad -2x$

$-2 < -3$

False statement

$\emptyset$

(no graph)



5. $3(x - 2) \leq 4 - x$

$3x - 6 \leq 4 - x$

$+x \quad +x$

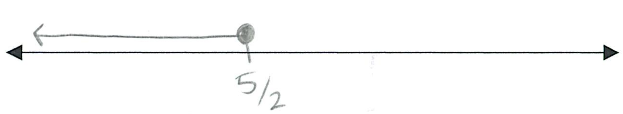
$4x - 6 \leq 4$

$+6 \quad +6$

$4x \leq 10$

$\overline{4} \quad \overline{4}$

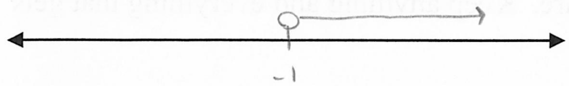
$x \leq 5/2$



Extended Practice: Solve and Graph each inequality.

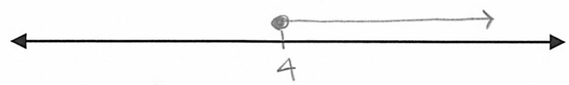
1. $3x - 1 > -4$

$x > -1$



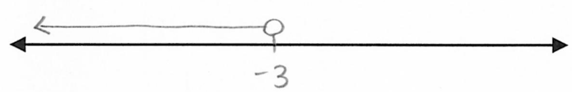
2. $y \leq 7y - 24$

$y \geq 4$



3. $-\frac{t}{2} > \frac{3}{2}$

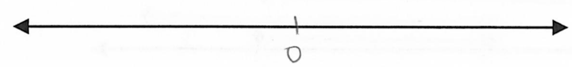
$t < -3$



4. $1 + 2x < 2(x - 1)$

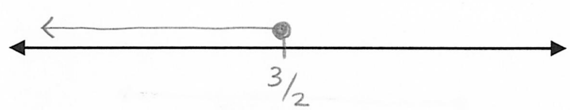
$\varnothing$

(no graph)



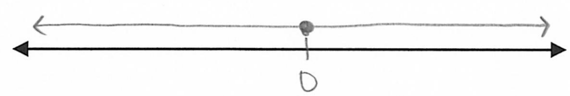
5. $3(x - 2) - 2 \leq x - 5$

$x \leq 3/2$



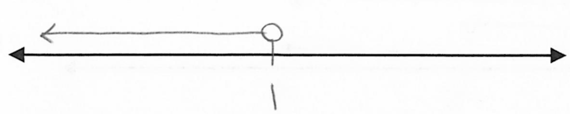
6. $4x + 3(2 - 3x) < 5(2 - x)$

$\mathbb{R}$



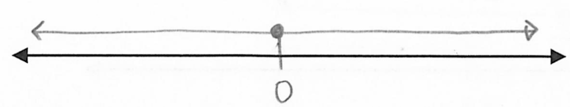
7. $k - 3(2 - 4k) < 7 - (8k - 9 + k)$

$k < 1$



8. $4(y + 2) - 9y \geq y - 3(2y + 1) - 1$

$\mathbb{R}$



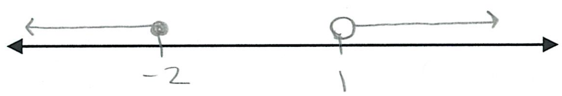
Solving Combined Inequalities

Now the techniques from the previous section can be expanded to combined inequalities. Combined inequalities involve two or more inequalities at once.

A **combined inequality** that is a disjunction is written with "or"

Example #1: $x > 1$ *or* $x \le -2$

It is true if **at least one** of the parts is true. "or" gives the union picture. Keep anything and everything that gets shaded.



A **combined inequality** that is a Conjunction is written with "and"
(This can also be written as $-3 \le x \le 0$)

Example #2: $x \ge -3$ *and* $x \le 0$

It is only true when **both** parts are true. "and" gives the intersection picture. Only keep the part that was shaded both times.

Break for Practice: Solve and graph each combined inequality.

1. $3x + 1 \le -2$ *or* $5 - 2x < 1$

$-1 \quad -1 \qquad -5 \quad -5$

$3x \le -3$, $\frac{3x}{3} \le \frac{-3}{3}$ $\qquad$ $-2x < -4$, $\frac{-2x}{-2} \downarrow \frac{-4}{-2}$

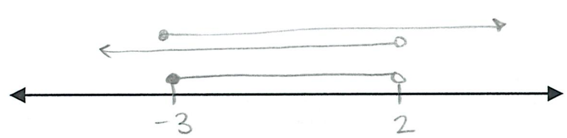
$x \le -1$ or $x > 2$

2. $2x + 3 \ge -3$ *and* $5x - 1 < 9$

$-3 \quad -3 \qquad +1 \quad +1$

$\frac{2x}{2} \ge \frac{-6}{2}$ $\qquad$ $\frac{5x}{5} < \frac{10}{5}$

$x \ge -3$ and $x < 2$ $\Rightarrow$ $-3 \le x < 2$

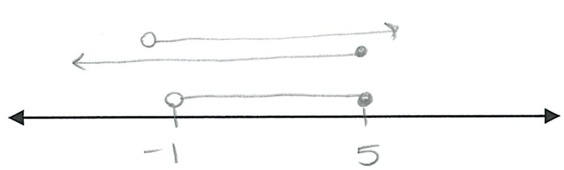


3. $2 < 2x + 4 \le 14$

$-4 \quad -4 \quad -4$

$\frac{-2}{2} < \frac{2x}{2} \le \frac{10}{2}$

$-1 < x \le 5$ $\Rightarrow$ $x > -1$ and $x \le 5$



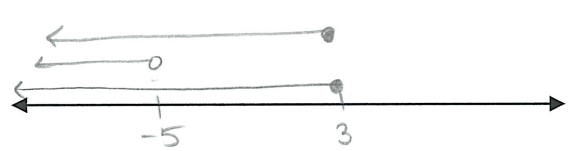
4. $2x + 5 \le 11$ *or* $x + 2 < -3$

$-5 \quad -5 \qquad -2 \quad -2$

$\frac{2x}{2} \le \frac{6}{2}$ $\qquad$ $x < -5$

$x \le 3$ or $x < -5$

$x \le 3$



5. $x + 4 \geq 8$ *and* $3x - 1 \geq 5$

$-4 \quad -4 \qquad\qquad +1 \quad +1$

$x \geq 4 \qquad\qquad \frac{3x}{3} \geq \frac{6}{3}$

$x \geq 2$

$x \geq 4$

6. $x - 6 \leq -11$ *and* $6 - 2x \leq 4$

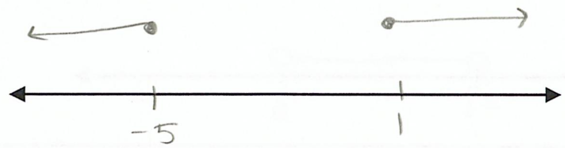
$+6 \quad +6 \qquad\qquad -6 \quad -6$

$x \leq -5 \qquad\qquad \frac{-2x}{-2} \leq \frac{-2}{-2}$ ↓

$x \geq 1$

$\varnothing$

(no over lap)



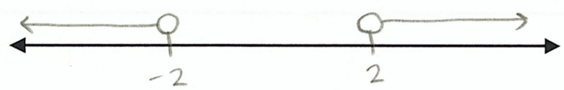
Extended Practice: Solve and graph each combined inequality.

1. $-1 \leq 3z + 2 \leq 8$

$-1 \leq z \leq 2$

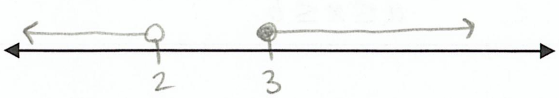
2. $3k + 7 < 1$ *or* $2k - 3 > 1$

$k < -2$ or $k > 2$



3. $2t + 7 \geq 13$ *or* $5t - 4 < 6$

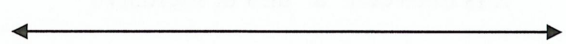
$t \geq 3$ or $t < 2$



4. $2t + 7 \geq 13$ *and* $5t - 4 < 6$

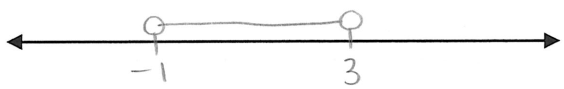
$(t \geq 3$ and $t < 2)$

$\varnothing$



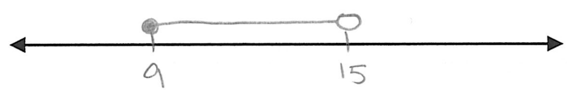
5. $-5 < 1 - 2k < 3$

$3 > K > -1$



6. $-3 < 2 - \frac{x}{3} \leq -1$

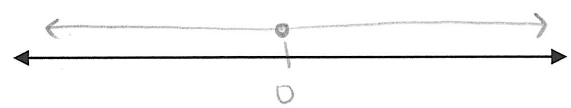
$15 > x \geq 9$



7. $-\frac{3}{4}x \geq x - 1 \ or \ -\frac{3}{4}x < x + 1$

$\left(x \leq \frac{4}{7} \text{ or } x > -\frac{4}{7}\right)$

$\mathbb{R}$



Problem Solving Using Inequalities

In this section we will work on translating word problems into inequalities to solve. There are some phrases that you will need to learn how to translate correctly in order to do this.

Phrase	Translation
x is at least "a" x is no less than "a"	$x \geq a$
x is at most b x is no greater than b	$x \leq a$
x is between "a" and b x is between "a" and b, inclusive	$a < x < b$ $a \leq x \leq b$

Break for Practice: Solve.

1. A summer recreation department charges $45.00 for a season ticket to the town pool. Admission to the pool for one day is $2.75. How many days should you go swimming in order to make the purchase of a season ticket worth while?

x = # of times you go swimming

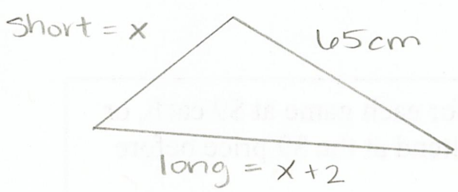
Season pass $\leq$ regular price

$$\frac{45}{2.75} \leq \frac{2.75x}{2.75}$$

$$16.36 \leq x$$

$\boxed{\text{You would need to go swimming @ least 17 times to make purchasing a season pass worth while.}}$

2. Two sides of a triangle are consecutive even integers. The other side is 65 cm. If the perimeter is between 215 cm and 230 cm, what are the possible lengths for the first two sides?



$P = x + x+2 + 65$

$P = 2x+67$

215 < perimeter < 230

$$215 < 2x + 67 < 230$$
$$-67 \quad -67 \quad -67$$

$$\frac{148}{2} < \frac{2x}{2} < \frac{163}{2}$$

$$74 < x < 81.5$$

Possible Lengths

Short	long
76cm	78cm
78cm	80cm

3. Ellen's first three test scores were consecutive odd integers. Her fourth score was 83. She had an average between 80 and 82 inclusive for the four tests. What was her lowest test score?

Test #1 = x ← lowest score

Test #2 = $x+2$

Test #3 = $x+4$

Test #4 = 83

$80 \leq$ average ≤ 82

$$80 \leq \frac{x + x+2 + x+4 + 83}{4} \leq 82$$

$$4(80) \leq 4\left(\frac{3x+89}{4}\right) \leq (82)4$$

$$320 \leq 3x + 89 \leq 328$$
$$-89 \quad -89 \quad -89$$

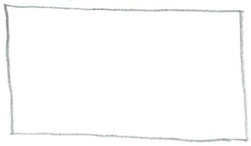
$$\frac{231}{3} \leq \frac{3x}{3} \leq \frac{239}{3}$$

$$77 \leq x \leq 79.\overline{6}$$

$\boxed{\text{Lowest test score was 77 or 79}}$

4. The length of a rectangle is 3 cm more than twice its width. Find the largest possible width if the perimeter is at most 66 cm.

length = 2w+3



width = w

Perimeter ≤ 66

$w + \underbrace{2w+3}_{l} + w + \underbrace{2w+3}_{l} \leq 66$

$6w + 6 \leq 66$

$\quad -6 \quad\; -6$

$6w \leq 60$

$\frac{6w}{6} \leq \frac{60}{6}$

$w \leq 10$

$\boxed{\text{The largest possible width is 10 cm}}$

Extended Practice: Solve

1. For the Hawks' 80 basketball games next year, you can buy separate tickets for each game at $9 each, or you can buy a season ticket for $580. At most how many games could you attend at the $9 price before spending more than the cost of a season ticket?

64 is the maximum number of games.

2. The length of a rectangle is 5 cm more than twice its width. Find the largest possible width if the perimeter is at most 64 cm.

The largest possible width is 9 cm