Algebra II

Unit 1:
Solving Equations and Inequalities in One Variable

Priority Standards: A.CED.1: Create equations and inequalities in one variable and use them to solve problems.

Unit "I can" statements:

1. I can solve certain equations in one variable.
2. I can translate word phrases and sentences into algebraic expressions and equations.
3. I can solve word problems by using an equation in one variable.
4. I can solve simple inequalities in one variable.
5. I can solve simple conjunctions and disjunctions.
6. I can solve word problems by using inequalities in one variable.
7. I can solve open sentences involving absolute value.

Common Core State Standards that are addressed in this unit include: A-CED.1, A-CED.2, A-CED.4, A-REI.3, A-SSE.1b. For more information see www.corestandards.org/Math/

Solving Equations in One Variable

This unit will begin with a review of solving simple equations in one variable. Do you remember how to solve an equation like this?

$4x - 3 = 29$

$+3 \quad +3$

$\frac{4x}{4} = \frac{32}{4}$

$x = 8$

Keys for Solving Simple Equations:

1. Simplify either side of an equation if possible. (Get rid of parentheses and combine like terms.)
2. Add or subtract the same amount from both sides of the equation.
3. Multiply or divide both sides of the equation by the same non-zero number.

Break for Practice: Solve (if possible) each equation and check the answer.

1. $48 - 6x = 2x$

$+6x \quad +6x$

$\frac{48}{8} = \frac{8x}{8}$

$6 = x$

2. $4(x - 3) = 2x - 6$

$4x - 12 = 2x - 6$

$-2x \quad -2x$

$2x - 12 = -6$

$+12 \quad +12$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

3. $\frac{1}{2}x - 4 = -2$

$+4 \quad +4$

$2 \cdot \left(\frac{1}{2}x\right) = (2) \cdot 2$

$x = 4$

4. $7 - \frac{1}{5}y = -2$

$-7 \quad -7$

$-5 \cdot \left(-\frac{1}{5}y\right) = (-9) \cdot -5$

$y = 45$

5. $3x - (x - 5) = 2(x + 4)$

$3x - x + 5 = 2x + 8$

$2x + 5 = 2x + 8$

$-2x \quad -2x$

$5 = 8$

* False Statement *

No Solution: $\emptyset$

6. $3x - (x - 8) = 2(x + 4)$

$3x - x + 8 = 2x + 8$

$2x + 8 = 2x + 8$

$-2x \quad -2x$

$8 = 8$

* True Statement *

Infinite Solutions: $\mathbb{R}$

7. $0.4(2r + 3) = 0.6r + 3.6$

$0.8r + 1.2 = 0.6r + 3.6$

$-0.6r \quad -0.6r$

$0.2r + 1.2 = 3.6$

$-1.2 \quad -1.2$

$0.2r = 2.4$

$\frac{0.2r}{0.2} = \frac{2.4}{0.2}$

$r = 12$

Let's review the different types of solutions we saw in the Break for Practice.

What it looks like when solving for one-variable…	Leads to what type of solution?	Example to Reference
$x = \#$; $x = 6$	One Solution	Example #1
$\# = \#$; $6 = 6$ No Variable True Statement	Infinite or Many Solutions All Real Number : $\mathbb{R}$	Example #6
$\# \neq \#$; $0 \neq 6$ No variable False Statement	No Solution : $\emptyset$	Example #5

The same techniques that are used to solve simple equations can also be used to manipulate formulas. A skill that is useful in many science, math, and business classes is the ability to manipulate a formula and isolate any variable that you choose.

Break for Practice: Solve for the given variable. (For fun, try to identify what each formula is used for.)

1. $C = \pi d$ → Solve for d.

$\frac{C}{\pi} = \frac{\pi d}{\pi}$

$\frac{C}{\pi} = d$

2. $I = prt$ → Solve for r.

$\frac{I}{pt} = \frac{prt}{pt}$

$\frac{I}{pt} = r$

3. $P = 2l + 2w$ → Solve for l.

$-2w \quad -2w$

$P - 2w = 2l$

$\frac{P-2w}{2} = \frac{2l}{2}$

$\frac{P-2w}{2} = l$ or $l = \frac{P}{2} - w$

4. $A = \left(\frac{1}{2}h(b_1 + b_2)\right)$ → Solve for h.

$2A = 2\left(\frac{1}{2}h(b_1 + b_2)\right)$

$\frac{2A}{(b_1+b_2)} = \frac{h(b_1+b_2)}{(b_1+b_2)}$

$h = \frac{2A}{b_1 + b_2}$

4. $A = \left(\frac{1}{2}h(b_1 + b_2)\right)$ → Solve for b_2.

$2A = 2\left(\frac{1}{2}h(b_1 + b_2)\right)$

$\frac{2A}{h} = \frac{h(b_1+b_2)}{h}$

$\frac{2A}{h} = b_1 + b_2$

$-b_1 \quad -b_1$

$b_2 = \frac{2A}{h} - b_1$

6. $A = \pi r(s + r)$ → Find s if A = 100π and r = 5.

$100\pi = \pi(5)(s + 5)$

$\frac{100\pi}{5\pi} = \frac{5\pi(s+5)}{5\pi}$

$20 = s + 5$

$-5 \quad -5$

$15 = s$

Extended Practice: Solve, if possible, each equation.

1. $3x - 4 = 5$ $x = 3$	2. $\frac{2}{3}t - 8 = 0$ $t = 12$
3. $5r = 18 + 2r$ $r = 6$	4. $3(t - 1) = -(t - 5)$ $t = 2$
5. $2k - 1 = k - 5 + 3k$ $K = 2$	6. $-(5 - x) = x + 3$ No Solution: $\emptyset$
7. $1.5(u + 2) = 7.5$ $u = 3$	8. $3(x - 2) - x = 2(2x + 1)$ $x = -4$

9. $\frac{6x-2(x-4)}{3} = 8$ $x = 4$

Extended Practice Continued: Solve each equation for the given variable.

10. $2x - 5y = 10 \rightarrow$ Solve for y.

$y = \frac{10-2x}{-5}$ OR $y = -2 + \frac{2}{5}x$

11. $A = \frac{1}{2}bh \rightarrow$ Solve for h.

$h = \frac{2A}{b}$

12. $S = -\frac{1}{2}gt^2 + vt \rightarrow$ Solve for v.

$V = \frac{S + \frac{1}{2}gt^2}{t}$ OR $V = \frac{S}{t} + \frac{1}{2}gt$

13. $C = \frac{5}{9}(F - 32) \rightarrow$ Solve for F.

$F = \frac{9}{5}C + 32$

14. $d = \frac{v^2}{2g} \rightarrow$ Find g if $d = 1000$ and $v = 140$.

$g = 9.8$

Words into Symbols

One of the main purposes for studying Algebra is to be able to solve applications. In order to solve applications, you need to be able to translate words and sentences into expressions and equations.

Break for Guided Practice: Represent each phrase by an algebraic expression.

1. The difference between a number and three. $n-3$

2. Four less than twice a number. $2n-4$

3. The sum of twice a number and its square. $2n+n^2$

4. Four less than the product of a number and three. $3n-4$

5. The quotient when eight is divided by twice a number. $\frac{8}{2n}$ or $8 \div 2n$

6. The square of the sum of a number and three. $(n+3)^2$

Express each answer in simplest form in terms of the given variable.

7. Carl is x years old. His sister Jenny is six more than twice his age. What is the average of their ages?

Carl = x

Jenny = $2x+6$

$\text{Average} = \frac{\text{Sum}}{2} \Rightarrow \frac{\text{Carl} + \text{Jenny}}{2}$

$\text{Average} = \frac{x+2x+6}{2} \Rightarrow \text{Average} = \frac{3x+6}{2}$

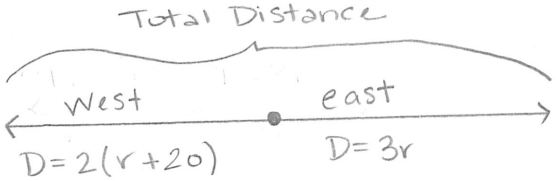
8. The base and height of a triangle are consecutive odd integers, and the height exceeds the base. If the base is b cm, find the area of the triangle. (differ by 2) height is longer than base

$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

$A = \frac{b(b+2)}{2}$

9. At 2:00 pm a train left a station traveling east at r miles per hour. At 3:00 pm a second train headed west from the station at a rate 20 miles per hour faster than the first. How far apart were the trains at 5:00 pm?

Distance = rate x time



Total Distance = West Bound Distance + East Bound Distance

$D = 2(r+20) + 3r$

$D = 2r + 40 + 3r$

$D = 5r + 40$ miles

10. The number of dimes and the number of quarters that Margaret has earned in tips are consecutive even integers. She has fewer dimes than quarters. What is the total value of her coins? (differ by 2)

d = # of dimes

$d+2$ = # of quarters

★ Dime = 0.10

★ Quarter = 0.25

Total Amount = $0.10d + 0.25(d+2)$

($0.10d$: total value of dimes; $0.25(d+2)$: total value of quarters)

Total = $0.10d + 0.25d + 0.50$

Total = $0.35d + 0.50$

Extended Practice: Represent each word phrase by an algebraic expression.

1. Five more than a number. $n+5$	2. One less than twice a number. $2n-1$
3. Seven more than half of a number. $7 + \frac{1}{2}n$ OR $\frac{1}{2}n + 7$	4. One more than the square of a number. n^2+1

Extended Practice Continued: Express each answer in simplest form it terms of the given variable.

5. A rectangular garden that is w ft. wide is enclosed by 120 ft. of fencing. How long is the garden?

$l = 60 - w$ ft

6. In a basketball game, one team's score is two points less than half the other team's score, which is x. What is the difference in the scores?

Difference = $\frac{1}{2}x + 2$

7. The length, width, and height of a rectangular box are consecutive integers, and the largest dimension is k cm. Find the volume V of the box. (hint: $V = lwh$)

$V = K(K-1)(K-2)$

8. Two jets leave an airport at noon, one flying north at r mi/h, and the other flying south at twice that speed. After 3 h, how far apart are the planes?

Distance = 9r miles

9. One angle of a quadrilateral has measure a°. Find the average of the measures of the other three angles.

$$\text{Average} = \frac{360 - a}{3}$$

10. The Drama Club sold t students' tickets at \$1.50 each and 100 fewer adults' tickets at \$2.50 each. How much money did the club collect?

Total Amount = 4t - 250

Problem Solving with Equations

Now it is time to extend the process from the previous section and actually solve the word problems. First it is time to review the five step plan for approaching story problems.

5-Step Plan:

1. Carefully read the problem. Decide what you are looking for and what you are given.
2. Define a variable and use it to describe the number(s) in the problem. Drawing and labeling a sketch, or using a table may help.
3. Reread the problem, and write an equation.
4. Solve the equation and find the required numbers.
5. Check your answer!

Warning! Be careful of extra information or insufficient information.

Break for Practice: Solve each of the following problems. If there is not enough information to solve the problem, say so.

1. Cheryl's weekly allowance is \$2.00 more than Emily's. Together they get \$11.00. What is each girl's weekly allowance?

X = Emily allowance

$X+2$ = Cheryl allowance

Emily + Cheryl = total

$X + (X+2) = 11$

$2X + 2 = 11$

$-2 \quad -2$

$\frac{2X}{2} = \frac{9}{2}$

$X = \$4.50$

Emily = \$4.50
Cheryl = \$6.50

2. The Junior class sold shirts bearing the school insignia for \$12.00 each. An extra \$1.00 was charged to have a shirt monogrammed. There were 324 shirts sold, and a total of \$4036.00 was collected. Of the shirts sold, 174 were bought by Juniors. How many shirts were not monogrammed?

Money Earned = Cost • # of shirts Sold

X = # of plain shirts sold

$324 - X$ = # of monogrammed shirts sold

Total \$ Earned = $12x$ + $13(324 - x)$

($12x$: value of plain shirts sold; $13(324-x)$: value of mono. shirts sold)

$\$4036 = 12x + 4212 - 13x$

$\$4036 = -x + 4212$

$-4212 \quad -4212$

$-176 = -x$

$\frac{-176}{-1} = \frac{-x}{-1}$

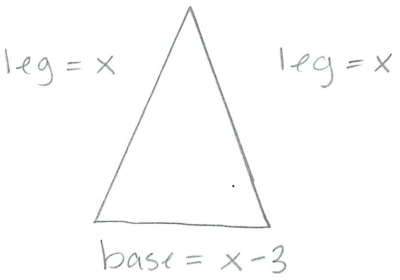
$x = 176$

176 shirts were not monogrammed

3. A store sold 40 baseballs and 14 softballs over a two-week period. The sales for these items totaled $200. What was the price of one baseball?

Not Enough Information

4. The perimeter of an isosceles triangle is 36 cm, and the area is 60 cm^2. The length of the base is 3 cm less than the length of a leg. Find the length of each side.



P = leg + leg + base

$36 = x + x + x - 3$

$36 = 3x - 3$

$+3 \quad +3$

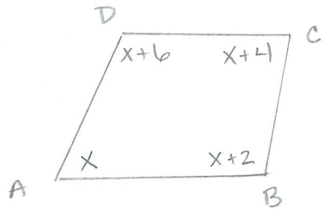
$39 = 3x$

$\frac{39}{3} = \frac{3x}{3}$

$x = 13$ cm

Legs = 13cm
Base = 10cm

5. The measures of the angles of a quadrilateral are consecutive odd integers. Find the measure of each angle.



differ by 2

Sum of the angles = 360°

$x + x+2 + x+4 + x+6 = 360°$

$4x + 12 = 360°$

$-12 \quad -12$

$4x = 348$

$\frac{4x}{4} = \frac{348}{4}$

$x = 87°$

m∠A = 87°
m∠B = 89°
m∠C = 91°
m∠D = 93°

6. Kallie has $10.50 in dimes, quarters, and half dollars. She has three times as many quarters as half dollars, and three more half dollars than dimes. How many of each type of coin does she have?

d = # of dimes

d+3 = # of half dollars

3(d+3) = # of quarters

Dimes = 0.10
Half $ = 0.50
Quarter = 0.25

Total Value = Amount of Dimes + Amount of Half $ + Amount of Quarters

$10.50 = 0.10d + 0.50(d+3) + 0.25(3(d+3))$

$10.50 = 0.10d + 0.50d + 1.5 + 0.75(d+3)$

$10.50 = 0.60d + 1.5 + 0.75d + 2.25$

$10.50 = 1.35d + 3.75$

$-3.75 \quad -3.75$

$\frac{6.75}{1.35} = \frac{1.35d}{1.35}$

$d = 5$

5 Dimes
8 Half Dollars
24 Quarters