Example: Find an equation in slope-intercept form of the line having slope $\frac{1}{2}$ and y-intercept $\frac{3}{2}$. $(0, 3/2)$

Standard

$y - 3/2 = 1/2(x - 0)$

$y - 3/2 = 1/2x$

$-y \quad -y$

$2(-3/2) = (1/2x - y)2 \Rightarrow -3 = x - 2y$ OR $x - 2y = -3$

**7. Given: a point and the slope**

Example: Find an equation in slope-intercept form of the line containing the point (4, 1) and having slope $-1$. $= m$

Standard

$y - 1 = -1(x - 4)$

$y - 1 = -x + 4$

$+x \quad +x$

$x + y - 1 = 4 \Rightarrow x + y = 5$

$+1 \quad +1$

Example: Find an equation in slope-intercept form of the line containing the point (3, -2) and having slope $-\frac{3}{2}$.

$2(y - (-2)) = (-\frac{3}{2}(x - 3))2$

$2(y + 2) = -3(x - 3)$

$2y + 4 = -3x + 9$

$+3x \quad +3x$

$3x + 2y + 4 = 9 \Rightarrow 3x + 2y = 5$

$-4 \quad -4$

**8. Given: two points**

Example: Find an equation in slope-intercept form (2, -3) and (-1, 2)

$x_1\ y_1 \quad x_2\ y_2$

$m = \frac{2 - (-3)}{-1 - 2}$

$m = \frac{5}{-3}$

$3(y - (2)) = (-\frac{5}{3}(x - (-1)))3$

$3(y - 2) = -5(x + 1)$

$3y - 6 = -5x - 5$

$+5x \quad +5x$

$5x + 3y - 6 = -5 \Rightarrow 5x + 3y = 1$

$+6 \quad +6$

Example: Find an equation in slope-intercept form (-3, 2) and (4, 2)

$x_1\ y_1 \quad x_2\ y_2$

$m = \frac{2 - 2}{4 - (-3)}$

$m = \frac{0}{7}$

$m = 0$

$y - 2 = 0(x - 4)$

$y - 2 = 0$

$+2 \quad +2$

$y = 2$

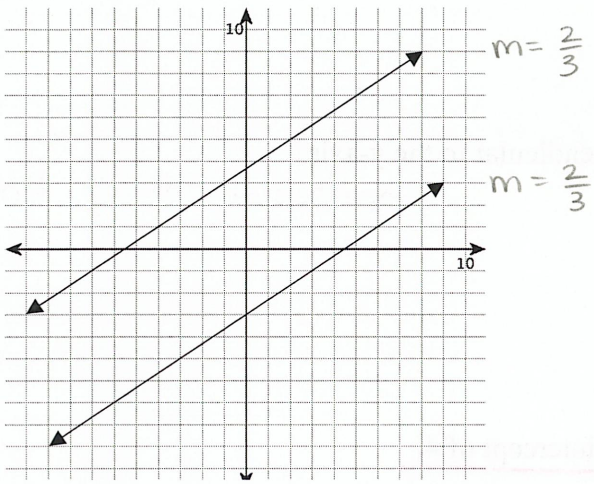
**Extended Practice**: Find the equation in Slope-Intercept Form (odd#'s) and Standard Form (even #'s) for the line with the given information.

| | |
|---|---|
| 1. $m = -1$ and $b = 2$ $y = -x + 2$ | 2. $m = \frac{1}{2}$ and $b = \frac{3}{2}$ $x - 2y = -3$ |
| 3. The line contains the point P(2, 3) and $m = 1$ <br> $y = x + 1$ | 4. The line contains the point P(2, 1) and $m = \frac{2}{3}$ <br> $2x - 3y = 1$ |
| 5. The line contains the point P(-2, -1) and $m = 0$ <br> $y = -1$ | 6. The line contains the points (3, -2) and (-2, 3) <br> $x + y = 1$ |

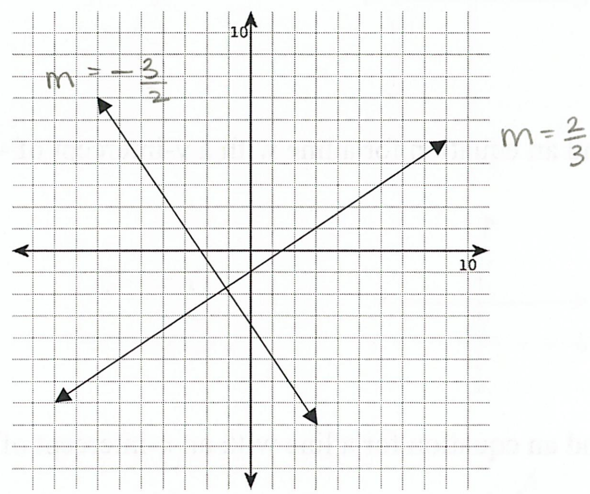
7. The line contains the points (4, -5) and (1, -4)

$y = -\frac{1}{3}x - \frac{11}{3}$

Look at the two parallel lines shown, and the two perpendicular lines shown. What is true about their slopes?




Parallel Lines have the exact same slope




Perpendicular Lines have slopes that are opposite (+/-) signs and reciprocals (flipped fraction)

**Break for Practice**: Find equations in standard form of the lines through P that are

a) parallel to L
b) perpendicular to L

$y = mx + b$

1. P(2, -3) L: $2x + 7y = 14$ $m = -\frac{A}{B} \Rightarrow m = -\frac{2}{7}$

// $m = -\frac{2}{7}$ (same slope as L)

$-3 = -\frac{2}{7}(2) + b$

$-3 = -\frac{4}{7} + b$

$-\frac{21}{7}$ $+\frac{4}{7}$ $+\frac{4}{7}$

$-\frac{17}{7} = b \Rightarrow y = -\frac{2}{7}x - \frac{17}{7}$

⊥ $m = \frac{7}{2}$ (opposite/reciprocal)

$-3 = \frac{7}{2}(2) + b$

$-3 = 7 + b$

$-7$ $-7$

$-10 = b \Rightarrow y = \frac{7}{2}x - 10$

standard form

2. P(-5, -1)   L: $x - 3y = 6$   $m = -\frac{A}{B} \Rightarrow m = \frac{-1}{-3} = \frac{1}{3}$

a.) $\parallel m = \frac{1}{3}$

$3(y - (-1)) = \left(\frac{1}{3}(x - (-5))\right)3$

$3(y+1) = x + 5$

$3y + 3 = x + 5$

$-3y \qquad -3y$

$3 = x - 3y + 5$

$-5 \qquad -5$

$\Rightarrow x - 3y = -2$

b.) $\perp m = -3$

$y - (-1) = -3(x - (-5))$

$y + 1 = -3(x+5)$

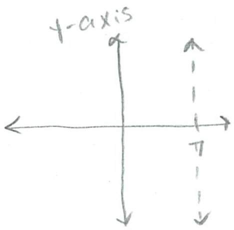
$y + 1 = -3x - 15$

$+3x \qquad +3x$

$3x + y + 1 = -15 \Rightarrow 3x + y = -16$

$-1 \qquad -1$

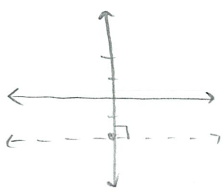
3. Find an equation for a line with an x-intercept of 7 and parallel to the y-axis. (does not cross y-axis)




$x = 7$

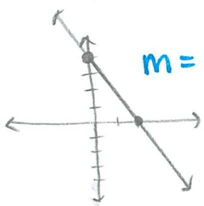
4. Find an equation for a line with a y-intercept of -2 and perpendicular to the y-axis.

(crosses y-axis @ 90°)



$y = -2$

5. Find an equation for a line with an x-intercept of 2 and a y-intercept of 4.



$m = -\frac{4}{2} \rightarrow m = -2$

$y = -2x + 4$

**Extended Practice**: Find equations in standard form of the lines through P that are
a) parallel to L
b) perpendicular to L

$y = mx + b$

1. P(0, 3)   L: $x + y = 5$

a.) $y = -x + 3$

b.) $y = x + 3$

Standard Form

2. P(-1, 2) L: $x - 3y = -2$

a.) $x - 3y = -7$ b.) $3x + y = -1$

**Extended Practice Continued**:

| | |
|---|---|
| 3. Find an equation for the line having a y-intercept of 6 and parallel to the x-axis.<br>$y = 6$ | 4. Find an equation for the line having an x-intercept of -4 and parallel to the y-axis.<br>$x = -4$ |
| 5. Find an equation for the line having an x-intercept of -3 and a y-intercept of 3.<br>$y = x + 3$ | 6. Find an equation for the line passing through the points (1, 4) and (-3, 4).<br>$y = 4$ |

# Graphing Linear Inequalities

Now that you know how to graph linear equations, you can learn how to graph linear inequalities. In order to do this, you will want to use the following steps.

**Steps**:

1. Graph the boundary line.
   Use a solid line if $\leq$ *or* $\geq$.
   Use a dashed line if < or >

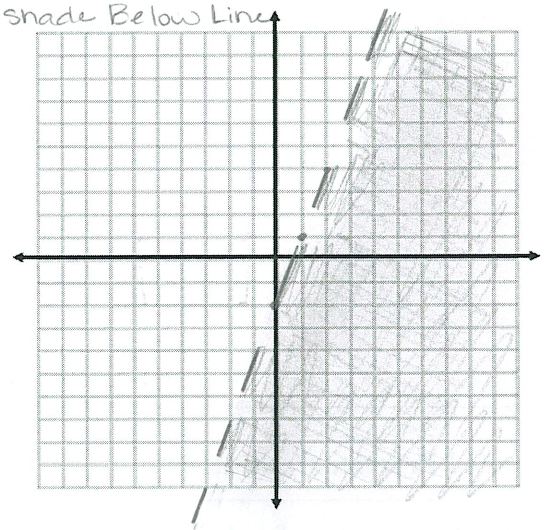
2. Test a point to determine which side of the line to shade. Shade the side that gives a true inequality.

**Example**: $y < 3x - 2$

no = means dashed line
< means shade Below Line

$m = \frac{3}{1}$ or $m = \frac{-3}{-1}$

$b = -2$ ← graph 1st



**Break for Practice**: Graph each inequality.

1. $3x + 4y \geq 8$

$-3x \quad -3x$

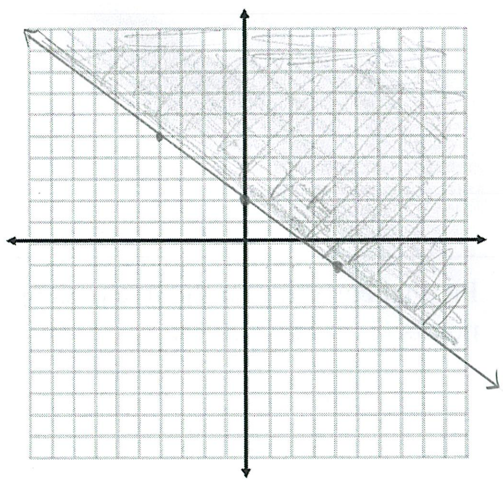
$\frac{4y}{4} \geq \frac{-3x}{4} + \frac{8}{4} \Rightarrow y \geq -\frac{3}{4}x + 2$

= Solid Line
> Shade Above Line

$m = -\frac{3}{4}$ OR

$m = \frac{3}{-4}$

$b = 2$

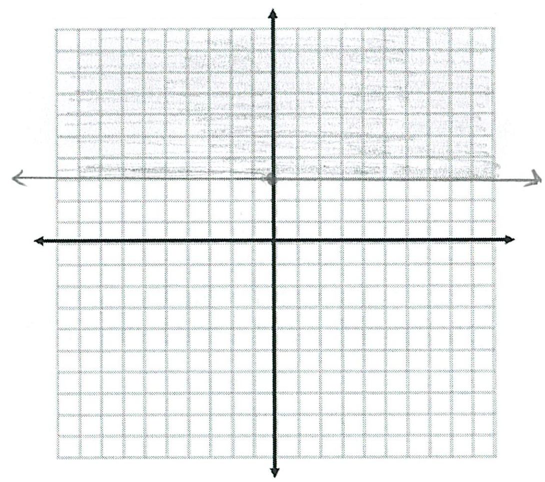


2. $-y + 3 \leq 0$

$-3 \quad -3$

$\frac{-y}{-1} \leq \frac{-3}{-1} \rightarrow y \geq 3$ (horizontal line)

= Solid Line
> Shade Above Line

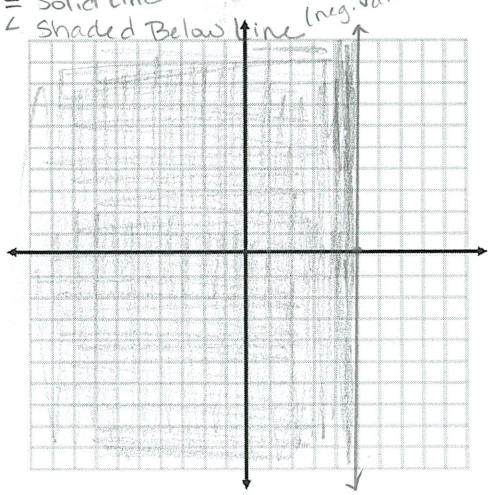


3. $x - 5 \leq 0$

$+5 \quad +5$

$x \leq 5$ (Vertical Line)

= Solid Line
< Shaded Below Line (neg. values)

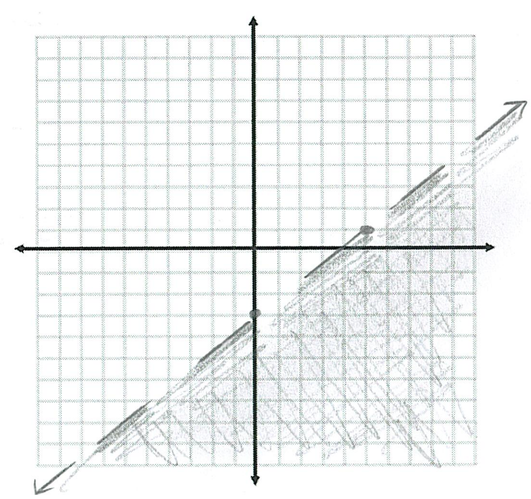


4. $4x - 5y > 15$

$-4x \quad -4x$

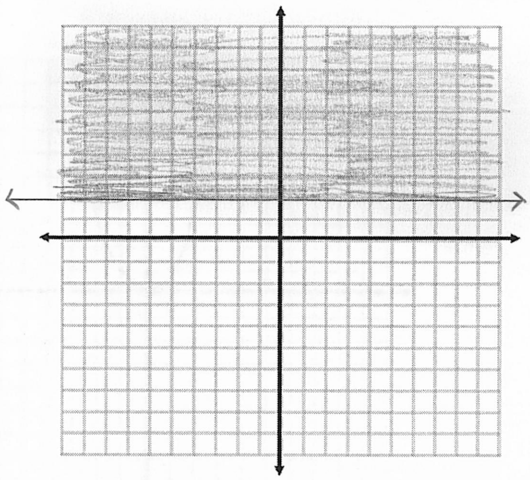
$\frac{-5y}{-5} > \frac{-4x}{-5} + \frac{15}{-5} \Rightarrow y < \frac{4}{5}x - 3$

no = Dashed Line
< Shaded Below Line

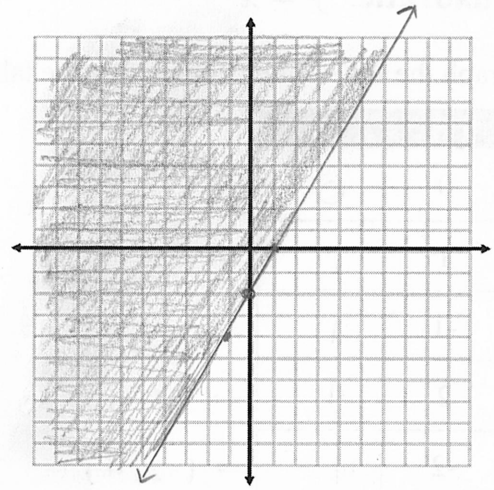


**Extended Practice**: Graph each inequality.

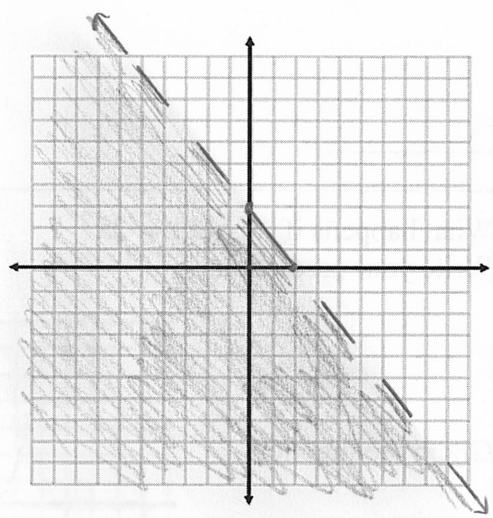
1. $y - 2 \geq 0$



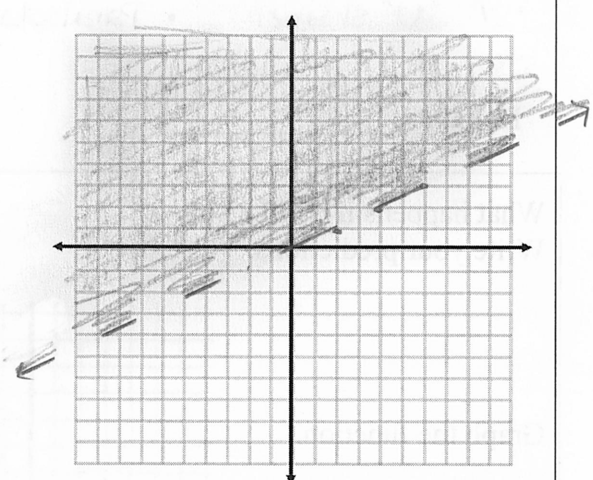
2. $2x - y \leq 2$



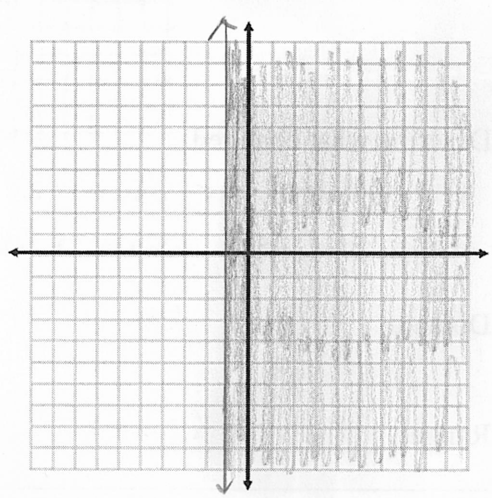
3. $3x + 2y < 6$



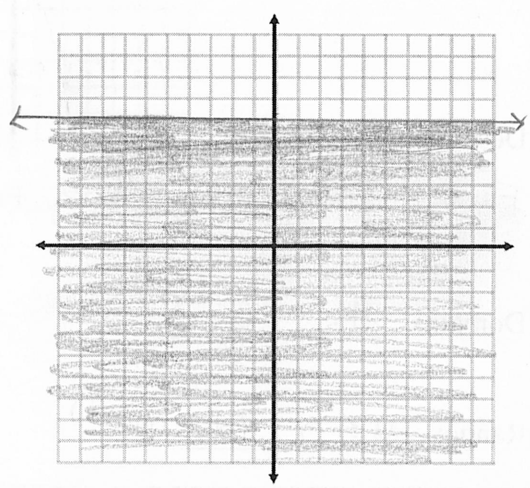
4. $x < 2y$



5. $x + 1 \geq 0$



6. $y \leq 6$



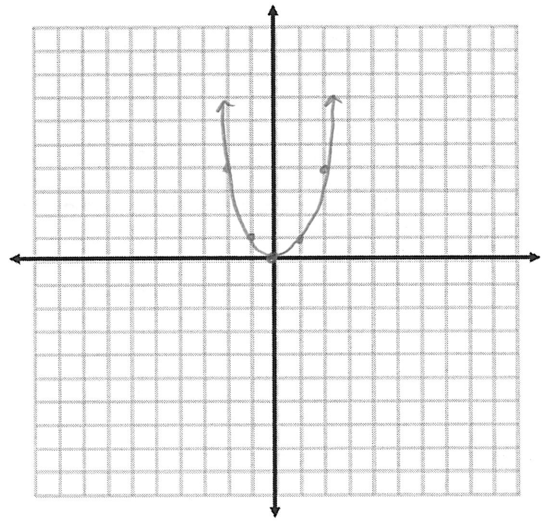
# Common Functions

This unit will end with the study of common functions. You will explore what the basic graph of each looks like, and how minor changes in the equation change the graph.

## Quadratic: $y = x^2$

Graph the function by completing the table of values.

| $x$ | $y$ | |
|---|---|---|
| 0 | 0 | → (0,0) |
| 1 | 1 | → (1,1) |
| -1 | 1 | → (-1,1) |
| 2 | 4 | → (2,4) |
| -2 | 4 | → (-2,4) |



Describe the graph. What does it look like?

- A "U" shape
- Parabola

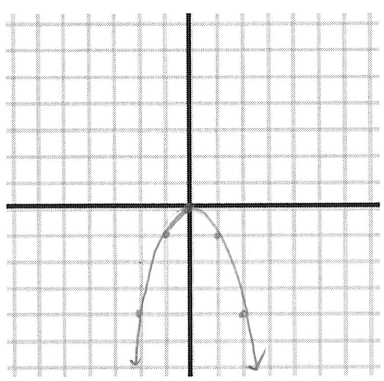
Domain: $\mathbb{R}$

Range: $y \geq 0$

---

What happens if … $h(x) = -x^2$ ?
Write your predictions.

Graph the function.



Describe what changed.

Flipped when x by a negative

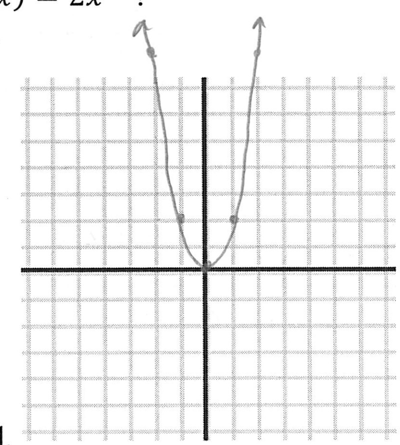
Domain: $\mathbb{R}$ (same)

Range: $y \leq 0$ (sign flipped)

---

What happens if … $g(x) = 2x^2$ ?
Write your predictions.

Graph the function.



Describe what changed.

It got thinner when x2
→ Gets wider when x by fraction

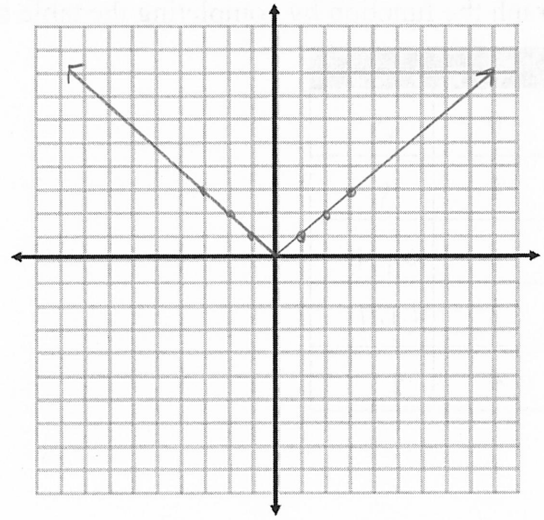
Domain: $\mathbb{R}$ (same)

Range: $y \geq 0$ (same)

## Absolute Value: $y = |x|$

Graph the function by completing the table of values.

| $x$ | $y$ | |
|---|---|---|
| 0 | 0 | → (0,0) |
| 1 | 1 | → (1,1) |
| -1 | 1 | → (-1,1) |
| 2 | 2 | → (2,2) |
| -2 | 2 | → (-2,2) |
| 3 | 3 | → (3,3) |
| -3 | 3 | → (-3,3) |



Describe the graph. What does it look like?

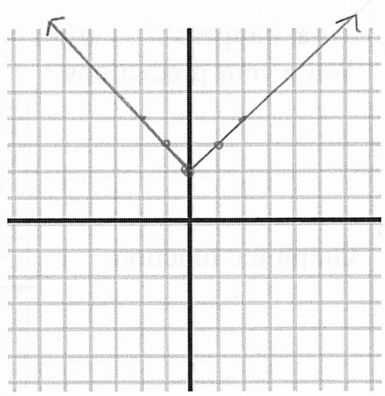
• V shape "Absolute Value"

Domain: $\mathbb{R}$

Range: $y \geq 0$

What happens if … $h(x) = |x| + 2$ ?
Write your predictions.

Graph the function.



Describe what changed.

Moved up when add 2 (outside)

→ Move down when subtract a #

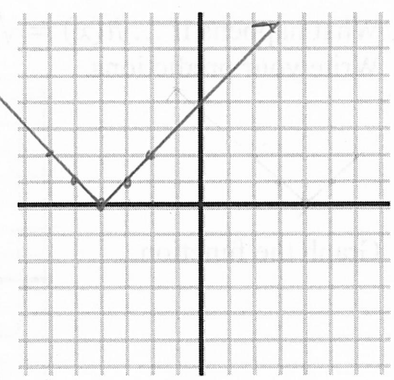
Domain: $\mathbb{R}$ (same)

Range: $y \geq 2$ (diff #)

change

What happens if … $g(x) = |x + 4|$ ?
Write your predictions.

Graph the function.



Describe what changed.

Shifts left when adding 4 (inside)

→ Shifts right when subtracting #

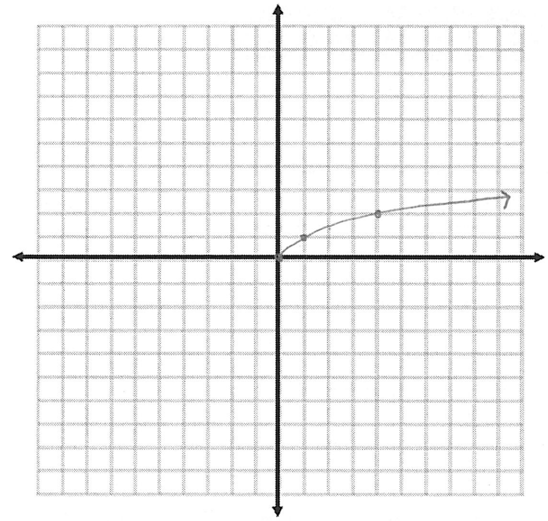
Domain: $\mathbb{R}$ (same)

Range: $y \geq 0$ (same)

## Square Root: $y = \sqrt{x}$

Graph the function by completing the table of values.

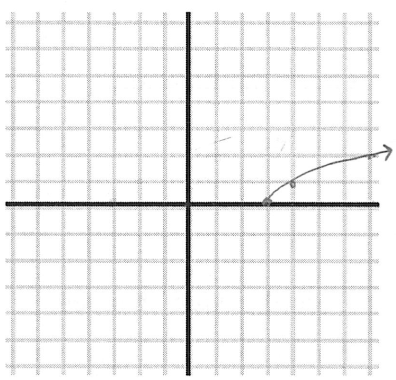
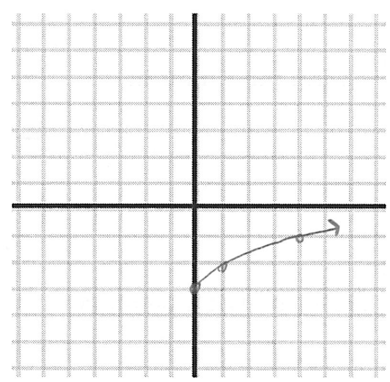
| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | ≈ 1.4 |
| 3 | ≈ 1.7 |
| 4 | 2 |



Describe the graph. What does it look like?

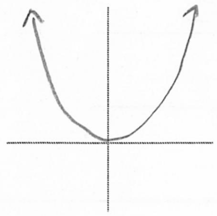
• Half of a sideways "U"; √ ish

Domain: $x \geq 0$ Range: $y \geq 0$

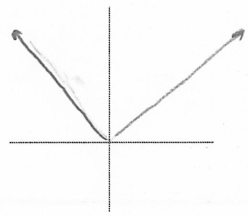
| | |
|---|---|
| What happens if … $h(x) = \sqrt{x-3}$ ?<br>Write your predictions.<br><br>Graph the function.<br><br><br>Describe what changed.<br>Shifts right when subtracting 3 (inside)<br>→ shifts left when adding a #<br><br>Domain: $x \geq 3$ ← (changed #)<br><br>Range: $y \geq 0$ (same) | What happens if … $g(x) = \sqrt{x} - 3$ ?<br>Write your predictions.<br><br>Graph the function.<br><br><br>Describe what changed.<br>Moved down when subtracting 3 (outside)<br>→ Moves up when adding a #.<br><br>Domain: $x \geq 0$ (same)<br><br>Range: $y \geq -3$ ← (changed #) |

## Summary:

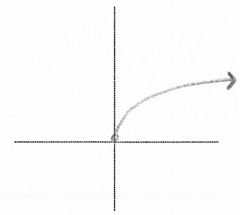
**Original Graphs:**



$f(x) = x^2$



$h(x) = |x|$



$g(x) = \sqrt{x}$

What happens when…

1. A number is added or subtracted **OUTSIDE** of the main function operation, the graph will move up or down

   Examples: $y = x^2 + 4$ $\quad y = |x| + 4$ $\quad y = \sqrt{x} + 4$ ← up

   $y = x^2 - 2$ $\quad y = |x| - 2$ $\quad y = \sqrt{x} - 2$ ← down

   What will happen to the Domain of $f(x) = x^2$ $\mathbb{R}$ & Range $f(x) = x^2$ $y \geq$ # add/sub.

   $f(x) = |x|$ $\mathbb{R}$ & Range $f(x) = |x|$ $y \geq$ # add/sub

   $f(x) = \sqrt{x}$ $x \geq 0$ & Range $f(x) = \sqrt{x}$ $y \geq$ # add/sub

   Same as original domain — Changes from original domain

2. A number is added or subtracted **INSIDE** of the main function operation, the graph will shift right or left

   Examples: $y = (x+2)^2$ $\quad y = |x+2|$ $\quad y = \sqrt{x+2}$ ← left

   $y = (x-3)^2$ $\quad y = |x-3|$ $\quad y = \sqrt{x-3}$ ← right

   What will happen to the Domain of $f(x) = x^2$ $\mathbb{R}$ & Range $f(x) = x^2$ $y \geq 0$

   $f(x) = |x|$ $\mathbb{R}$ & Range $f(x) = |x|$ $y \geq 0$

   only one that changes → $f(x) = \sqrt{x}$ $x \geq$ opposite of what is added & Range $f(x) = \sqrt{x}$ $y \geq 0$

   same as original domain

3. A number multiplies the main function operation, the graph will become more narrow

   Examples: $\left.\begin{matrix} y = 2x^2 \\ y = 2|x| \\ y = 2\sqrt{2} \end{matrix}\right\}$ narrow $\quad \left.\begin{matrix} y = \frac{1}{2}x^2 \\ y = \frac{1}{4}|x| \\ y = \frac{1}{2}\sqrt{x} \end{matrix}\right\}$ wider $\quad \left.\begin{matrix} y = -4x^2 \\ y = -4|x| \\ y = -4\sqrt{3} \end{matrix}\right\}$ Flipped

   wider, or/& flipped

   What will happen to the Domain of $f(x) = x^2$ $\mathbb{R}$ & Range $f(x) = x^2$ $y \geq 0$ or $y \leq 0$

   $f(x) = |x|$ $\mathbb{R}$ & Range $f(x) = |x|$ $y \geq 0$ or $y \leq 0$

   $f(x) = \sqrt{x}$ $x \geq 0$ & Range $f(x) = \sqrt{x}$ $y \geq 0$ or $y \leq 0$

**Extended Practice:** Find the domain and range for each of the following functions.

| | |
|---|---|
| 1. $f(x) = \sqrt{x+2}$ | Domain: $x \geq -2$ <br> Range: $y \geq 0$ |
| 2. $g(x) = \lvert x \rvert - 5$ | Domain: $\mathbb{R}$ <br> Range: $y \geq -5$ |
| 3. $h(x) = x^2 + 4$ | Domain: $\mathbb{R}$ <br> Range: $y \geq 4$ |
| 4. $f(x) = \lvert x+3 \rvert - 6$ | Domain: $\mathbb{R}$ <br> Range: $y \geq -6$ |
| 5. $h(x) = -\sqrt{x}$ | Domain: $x \geq 0$ <br> Range: $y \leq 0$ |
| 6. $g(x) = -2 \cdot \lvert x \rvert + 3$ | Domain: $\mathbb{R}$ <br> Range: $y \leq 3$ |
| 7. $f(x) = \frac{1}{2}(x-7)^2$ | Domain: $\mathbb{R}$ <br> Range: $y \geq 0$ |