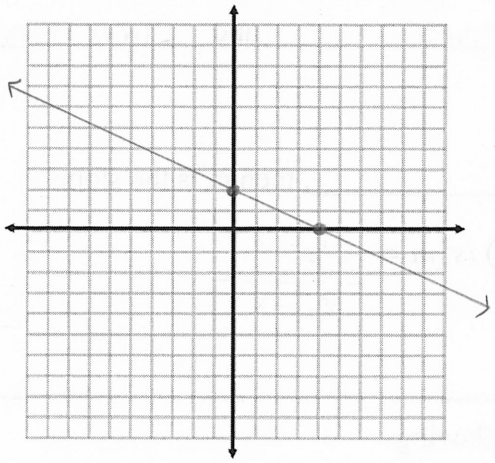
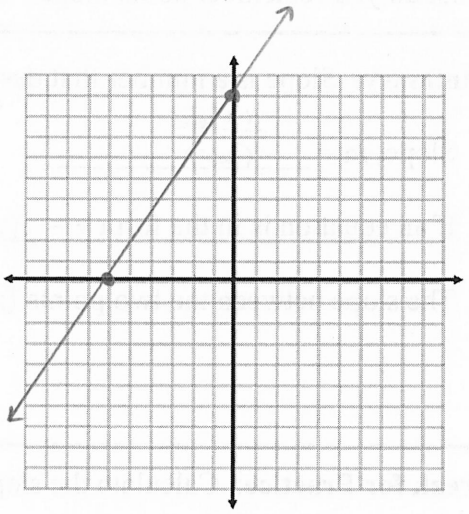
Extended Practice: Identify as linear or not linear. Graph the linear equations.

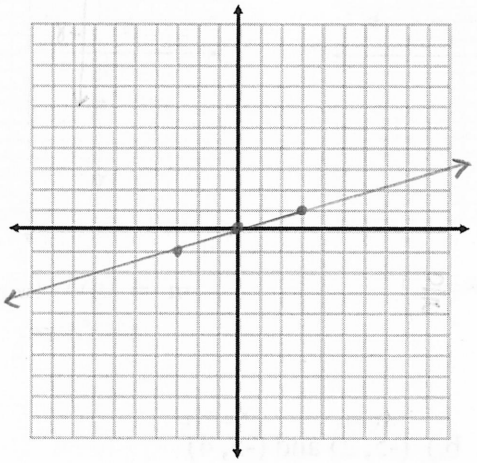
1. $x + 2y = 4$ (Linear)



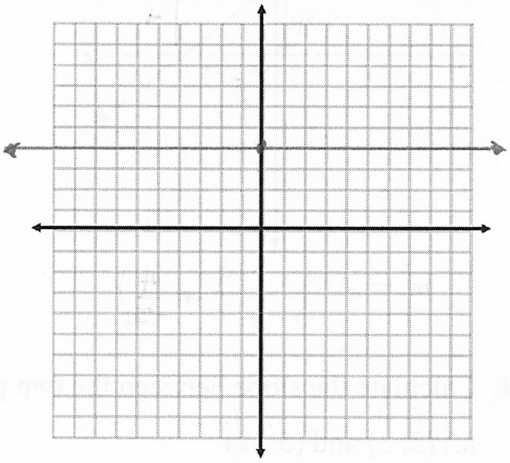
2. $3x - 2y + 18 = 0$ (Linear)



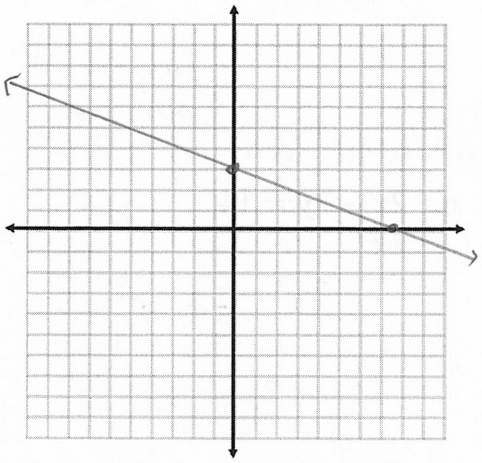
3. $x - 3y = 0$ (Linear)



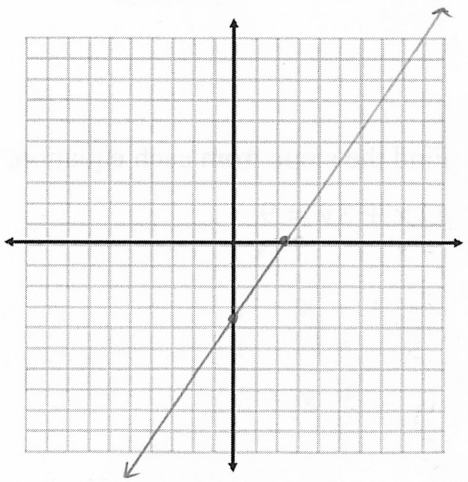
4. $2y - 8 = 0$ (Linear)



5. $2x + 5y = 15$ (Linear)



6. $3x - 2y = 7$ (Linear)



Slope

What do you remember about slope? Look at the previous section for ideas.

Summary: Slope is a number that describes the Steepness and direction of a line.

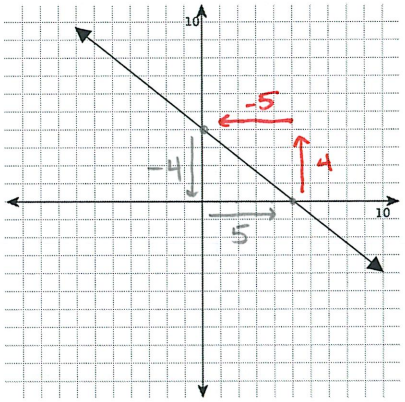
Slope, m = $\frac{\text{rise}}{\text{run}}$

If an equation is in the form y = $mx+b$, then m is the slope.

The slope between the two points (x_1, y_1) *and* (x_2, y_2) *is* $m = \frac{y_2 - y_1}{x_2 - x_1}$

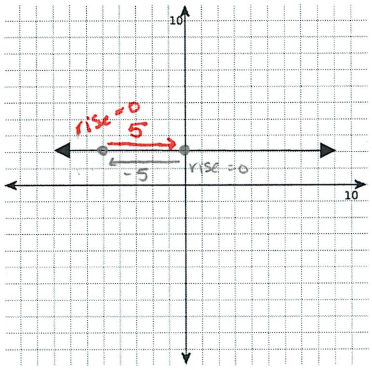
Break for Practice: Calculate the slope in each of the following

1.



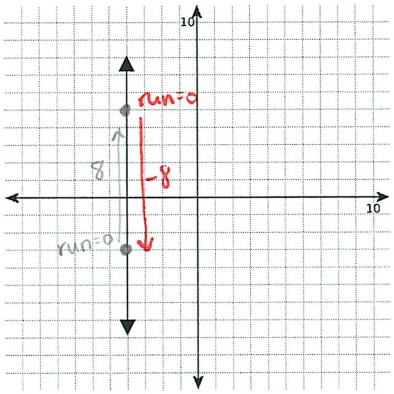
$m = -\frac{4}{5}$ $\left(\frac{-4}{5} \text{ or } \frac{4}{-5}\right)$

2.



$m = 0$ $\left(\frac{0}{-5} \text{ OR } \frac{0}{5}\right)$

3.



$m =$ undefined
No Slope $\left(\frac{8}{0} \text{ OR } \frac{-8}{0}\right)$

4. Calculate the slope between the two points given.

a.) (2, 3) and (5, -2) (x_1, y_1) (x_2, y_2)

$m = \frac{-2-3}{5-2} \Rightarrow m = -\frac{5}{3}$

b.) (-5, 2) and (-5, 4) (x_1, y_1) (x_2, y_2)

$m = \frac{4-2}{-5-(-5)} \Rightarrow m = \frac{2}{0}$

undefined
No Slope

5. Find the slope from each equation. Put into slope-intercept form $y = mx+b$

a.) $x + 3y = 9$

$-x \quad -x$

$\frac{3y}{3} = -\frac{x}{3} + \frac{9}{3}$

$y = -\frac{1}{3}x + 3$

$m = -\frac{1}{3}$

b.) $2x - 7y = 14$

$-2x \quad -2x$

$\frac{-7y}{-7} = \frac{-2x}{-7} + \frac{14}{-7}$

$y = \frac{2}{7}x - 2$

$m = \frac{2}{7}$

Result: The graph of any equation of the form $Ax + By = C$ is a line. These equations are called linear equations. Note: Only two points are needed to graph these equations, and often these points are the x-intercept and the y-intercept.

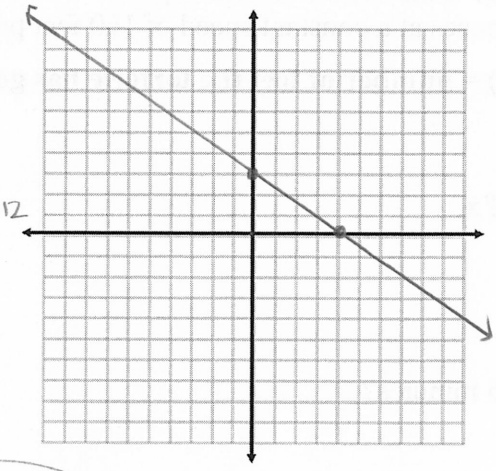
point when y=0 ↑ point when x=0

Break for Practice: Identify each equation as linear or not linear. Graph the linear equations.

1. $3x + 4y = 12$ (Linear)

y=0 ; 3x+4(0)=12
3x/3 = 12/3
x = 4

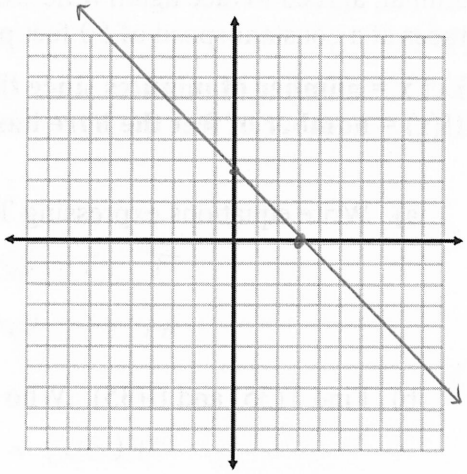
x=0 ; 3(0)+4y=12
4y/4 = 12/4
y = 3



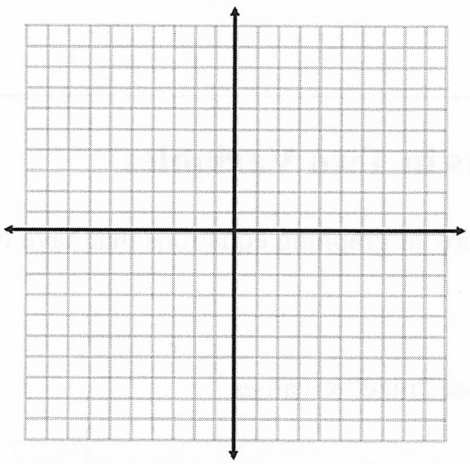
2. $3x + 2y - 9 = 0$ (Linear)

y=0 ; 3x+2(0)=9
3x/3 = 9/3
x = 3

x=0 ; 3(0)+2y=9
2y/2 = 9/2
y = 4.5



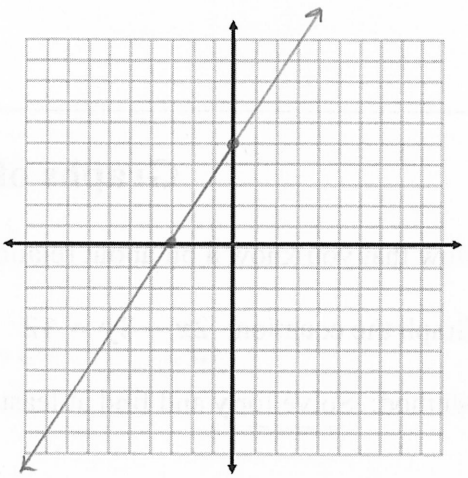
3. $2x^2 + y = 3$ (Not Linear)



4. $-2x + y = 6$ (Linear)

y=0 ; -2x+0=6
-2x/-2 = 6/-2
x = -3

x=0 ; -2(0)+y=6
y = 6

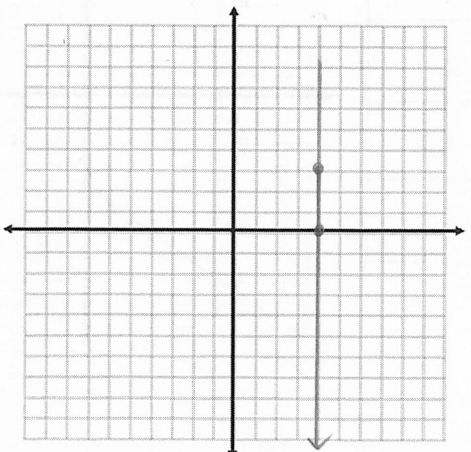


5. $x = 4 \Rightarrow x + 0y = 4$

y=0 ; x+0(0)=4
x = 4

y=0 ; 0+0y=4
0≠4
(not possible)

y=3 ; x+0(3)=4
x = 4



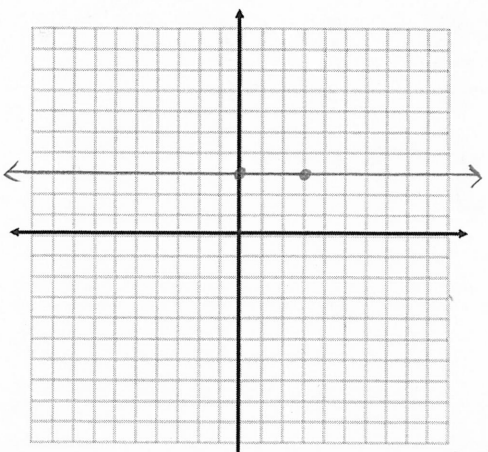
x=# are vertical lines (not a fn)

6. $y = -3 \Rightarrow 0x + y = -3$

y=0 ; 0x+0=3
0≠3
(not possible)

x=0 ; 0(0)+y=-3
y = -3

x=3 ; 0(3)+y=-3
y = -3



y=# are horizontal lines (fn)

10. $h(r)$ $(r, 4-3r+r^2)$	11. $g(m)$ $(m, 7-3m)$	12. $f(a+b)$ $(a+b, 2a+2b-5)$

13. Coming off an embarrassing loss to the tortoise, the hare wanted a rematch. The tortoise, being a wise old animal, agreed to race again if he was given a 30-minute lead. Both start at the same point. The Tortoise races at a constant speed of 50 feet per minute, and the hare races at a constant speed of 160 feet per minute.

Let x = number of minutes since the tortoise started, T(x) = number of feet the tortoise has gone, and H(x) = number of feet the hare has gone.

a) Write equations expressing T(x) and H(x) in terms of x.

$T(x) = 50x$

$H(x) = 160(x-30)$

b) Find T(35) and H(35). Who is ahead at the end of 35 minutes?

$T(35) = 1750$ ft

$H(35) = 800$ ft

Graphs of Linear Equations in Two Variables

Now that you know a bit about relations and functions, let's look at graphing equations that have two variables.

Graph the equation: $2x - 3y = 12$ (subtract $2x$ from both sides)

Method: Solve for y and find at least 4 points to graph. Choose "nice" x-values.

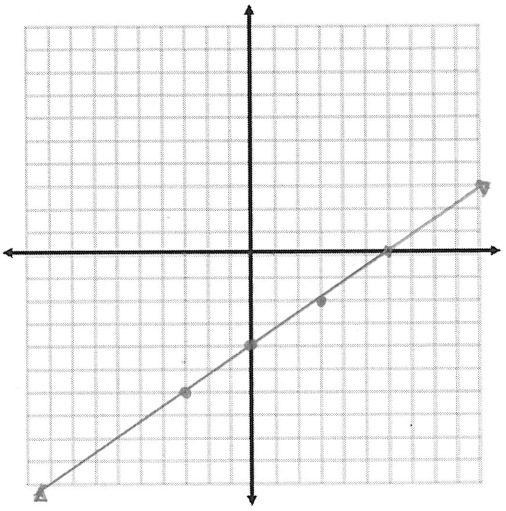
$2x - 3y = 12$

$-2x \quad -2x$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{12}{-3}$

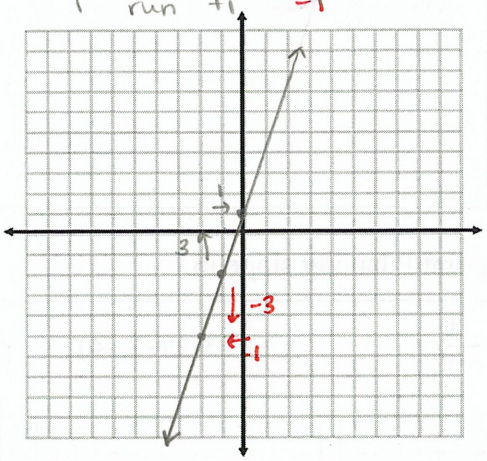
$y = \frac{2}{3}x - 4$

X	Y
0	-4
3	-2
-3	-6
6	0

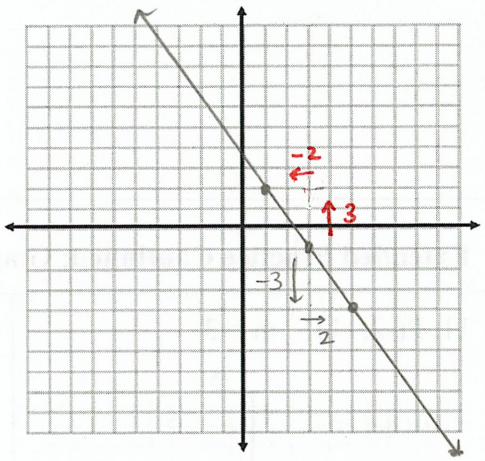


6. Graph the line through P with a slope of m.

a.) P(-1, -2) $m = \frac{3}{1}$ $\frac{rise}{run}$ $\frac{+3}{+1}$ OR $\frac{-3}{-1}$



b.) P(3, -1) $m = \frac{-3}{2}$ $\frac{rise}{run}$ $\frac{-3}{2}$ OR $\frac{3}{-2}$



Extended Practice: Find the slope of the line containing the given points.

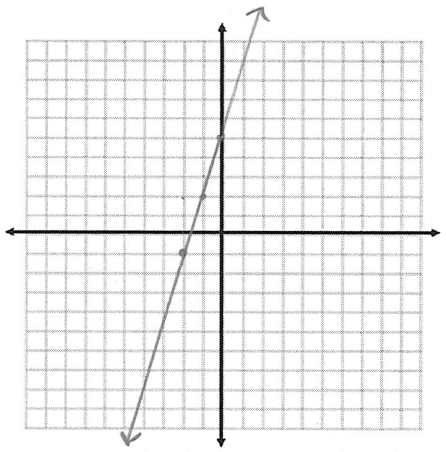
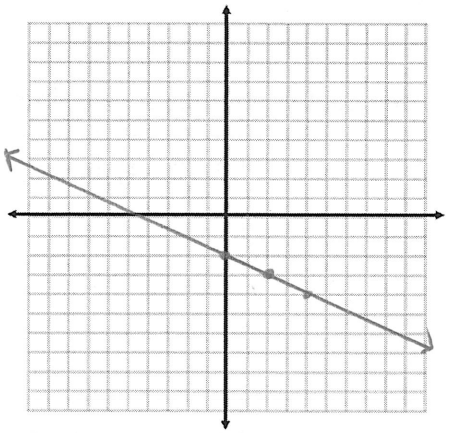
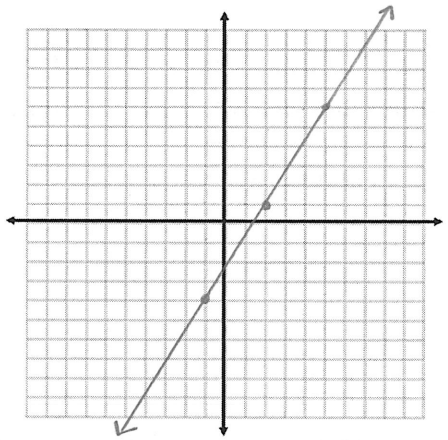
1. (3, 1) and (5, 5) $m = 2$	2. (3, -4) and (3, -2) Undefined; No Slope
3. (4, -1) and (-2, 3) $m = -\frac{2}{3}$	4. (-5, -2) and (5, -2) $m = 0$

Extended Practice Continued: Find the slope of each line.

5. $x - y + 1 = 0$ $m = 1$	6. $2x + 4y = 5$ $m = -\frac{1}{2}$
7. $4x - 3y = 3$ $m = \frac{4}{3}$	8. $3x - 3y = 5$ $m = 1$

9. $4y - 5 = 6x$	10. $x = 3y + 2$
$m = \frac{3}{2}$	$m = \frac{1}{3}$

Extended Practice Continued: Graph the line through point P having slope m.

11. P(-2, -1) m = 3	12. P(2, -3) $m = -\frac{1}{2}$	13. P(-1, -4) $m = \frac{5}{3}$
		

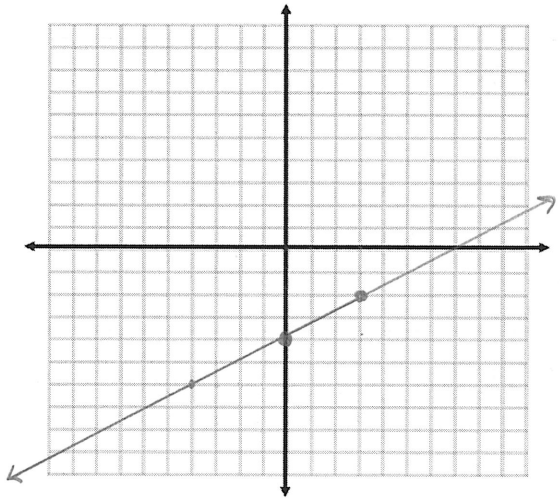
Graphing Equations using Slope/Intercept Form

Consider $y = \frac{2}{3}x - 4$

$x = 0$; $y = \frac{2}{3}(0) - 4$ $\quad y = -4$

$x = 3$; $y = \frac{2}{3}(3) - 4$ $\quad y = -2$

Graph it.



What is the value of the slope? $\frac{2}{3}$

Where do you see the slope in the given equation?

It is the coefficient (# infront of) x

Calculate the y-intercept. Remember that this is the value of y when x = 0. (0, -4)

Where do you see the y-intercept in the given equation?

It is the constant (# w/out a variable)

Summary:

When an equation is in the form $y = mx + b$

m: slope

b: y-intercept (where the graph crosses the y-axis)

Break for Practice: Graph each of the following by first identifying m and b.

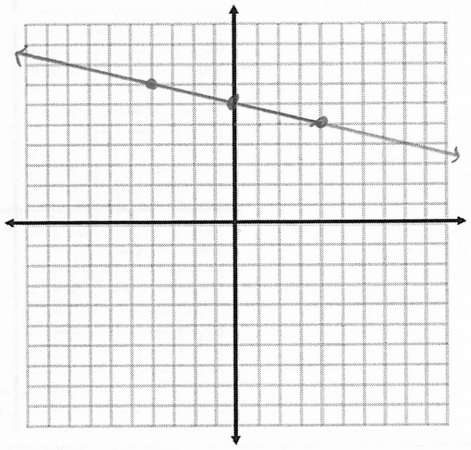
1. $y = -\frac{1}{4}x + 6$

$m = -\frac{1}{4}$ $\left(m = \frac{-1}{4} \text{ OR } m = \frac{1}{-4}\right)$

$b = 6$

↑

Graph 1st



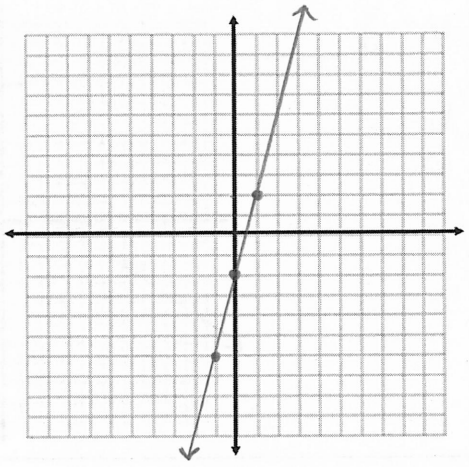
2. $y = 4x - 2$

$m = 4$ $\left(m = \frac{4}{1} \text{ OR } m = \frac{-4}{-1}\right)$

$b = -2$

↑

Graph 1st



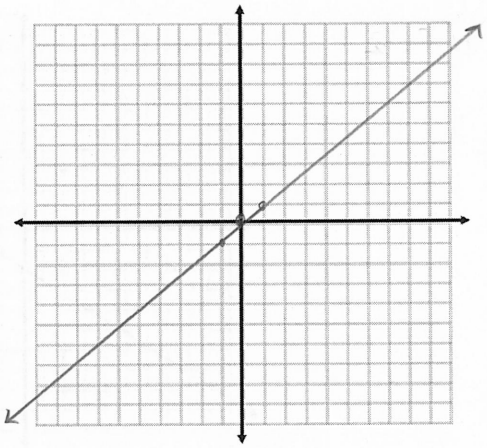
3. $y = x$

$m = 1$ $\left(m = \frac{1}{1} \text{ OR } m = \frac{-1}{-1}\right)$

$b = 0$

↑

Graph 1st



4. $3x + 4y = 12$

$-3x \qquad -3x$

$\frac{4y}{4} = \frac{-3x}{4} + \frac{12}{4} \Rightarrow y = -\frac{3}{4}x + 3$

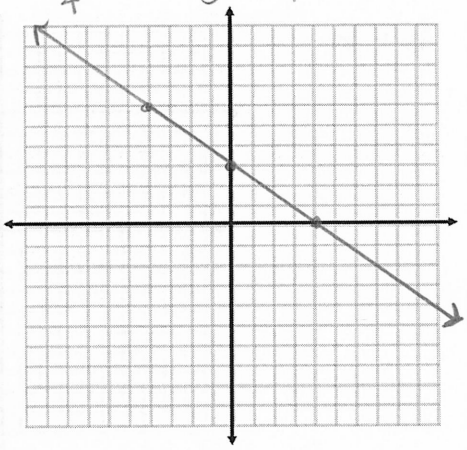
$m = -\frac{3}{4}$

$\left(m = \frac{-3}{4} \text{ OR } m = \frac{3}{-4}\right)$

$b = 3$

↑

Graph 1st



5. $-2x + 4y = 8$

$+2x \qquad +2x$

$\frac{4y}{4} = \frac{2x}{4} + \frac{8}{4} \Rightarrow y = \frac{1}{2}x + 2$

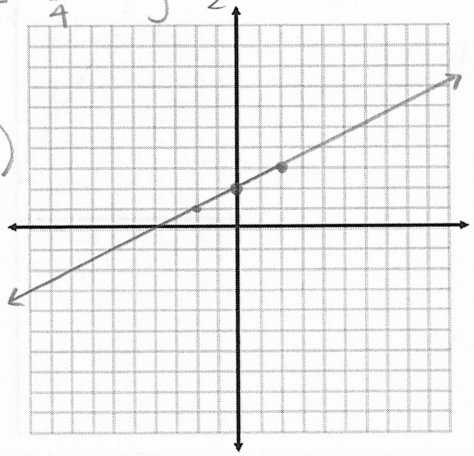
$m = \frac{1}{2}$

$\left(m = \frac{1}{2} \text{ OR } m = \frac{-1}{-2}\right)$

$b = 2$

↑

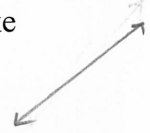
Graph 1st



Summary:

A positive slope looks like

$m = \frac{\#}{\#}$ OR $m = \frac{-\#}{-\#}$



A negative slope looks like

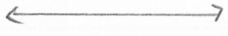
$m = \frac{-\#}{\#}$ OR $m = \frac{\#}{-\#}$

No slope looks like

$m = \frac{\#}{0}$

A slope of zero looks like

$m = \frac{0}{\#}$

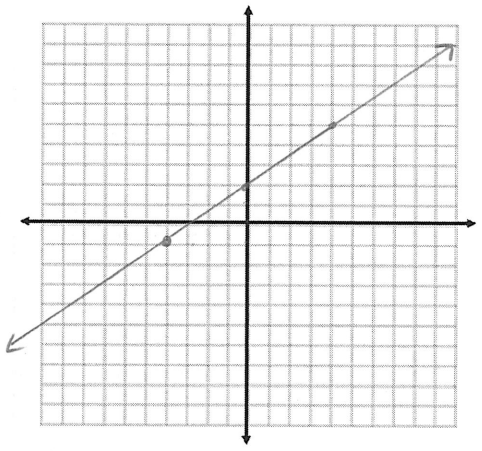


Extended Practice: Identify the slope, m and y-intercept, b of each equation, and graph.

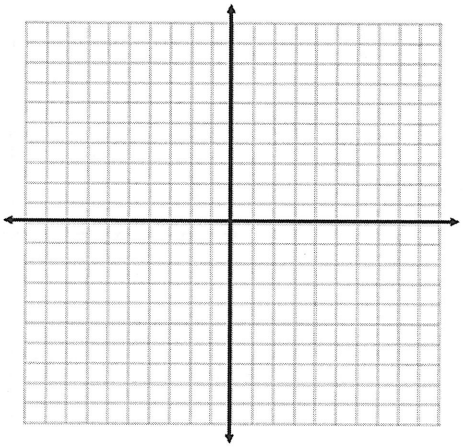
1. $y = \frac{3}{4}x + 2$

$m = \frac{3}{4}$

$b = 2$



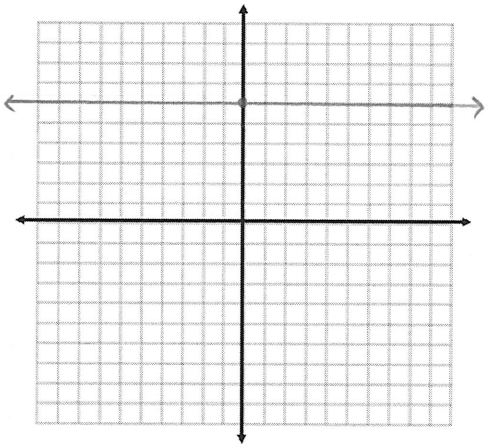
2. $y = -\frac{1}{3}x + 4$



3. $y = 6$

$m = 0$

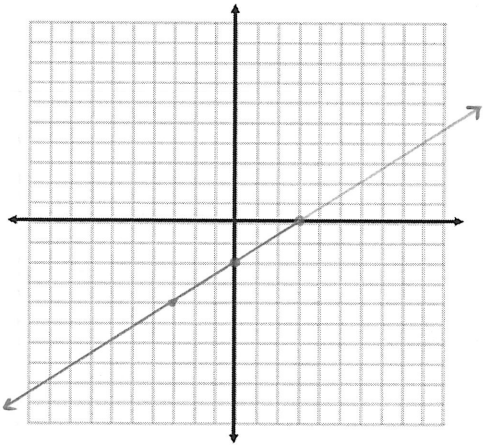
$b = 6$



4. $2x - 3y = 6$

$m = \frac{2}{3}$

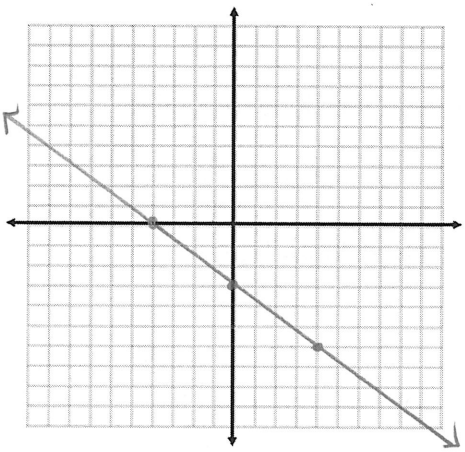
$b = -2$



5. $3x + 4y = -12$

$m = -\frac{3}{4}$

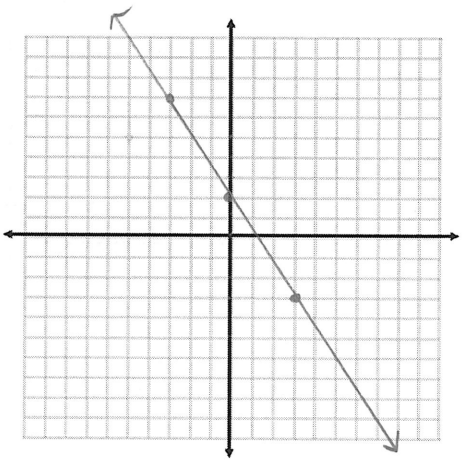
$b = -3$



6. $5x + 3y = 6$

$m = -\frac{5}{3}$

$b = 2$



Finding an Equation of a Line

Now that you know several ways to graph linear equations, the next thing to learn is how to find an equation for a line when given key pieces of information. In order to do this, it helps to know three different forms for linear equations.

I. Slope-Intercept Form:

Answer Format ⟶ $y = \#\,x + \#$ (values you will put in.)

- A line has y-intercept b if it intersects the y-axis at the point $(0, b)$
- A line has x-intercept a if it intersects the y-axis at the point $(a, 0)$

I b is the y-intercept of a line, the equation $y = mx + b$ is called the **slope-intercept form** of the equation of the line.

$x, y \rightarrow$ x, y coordinates (ordered pair) $\quad m \rightarrow$ slope $\quad b \rightarrow$ y-intercept

---

II. Standard Form:

$\#\,x + \#\,y = \#$ ← Answer Format (values you will put in.)

- $Ax + By = C$
- "A" and "B" need to be whole numbers (No Fractions, No Decimals) "A" should also be positive

$x, y \rightarrow$ x, y coordinates (ordered pairs) $\quad m \rightarrow -\frac{A}{B} \quad b \rightarrow \frac{C}{B}$

III. Point-Slope Form:

- The line containing the point (x_1, y_1) and having slope m has the equation.

$$y - y_1 = m(x - x_1)$$ ← notice it's just the slope formula $m = \frac{y - y_1}{x - x_1}$

** Notice that we are working with only ONE point

Break for Practice: Depending on the information that you are given will help you to decide which form you want to start with.

Finding an Equation of a Line: $y = mx + b$

1. Given: slope and y-intercept

Example: Find an equation in slope-intercept form of the line having slope $-\frac{3}{4}$ and y-intercept 2.

$y = -\frac{3}{4}x + 2$

2. Given: standard form

Example: Put $3x + 4y = 1$ into slope-intercept form (solve for y)

$-3x \qquad -3x$

$\frac{4y}{4} = \frac{-3x}{4} + \frac{1}{4}$

$y = -\frac{3}{4}x + \frac{1}{4}$

3. Given: a point and the slope

Example: Find an equation in slope-intercept form of the line containing the point (4, -3) and having slope $-\frac{2}{5}$.

x_1, y_1

$-\frac{2}{5} = m$

using point-slope: $y - y_1 = m(x - x_1)$

$5\left(y - (-3)\right) = \left(-\frac{2}{5}(x - 4)\right)5$

$5(y + 3) = -2(x - 4)$

$5y + 15 = -2x + 8$

$-15 \quad -15$

$\frac{5y}{5} = -\frac{2}{5}x - \frac{7}{5} \Rightarrow y = -\frac{2}{5}x - \frac{7}{5}$

using slope intercept: $y = mx + b$

$-3 = -\frac{2}{5}(4) + b$

$-3 = -\frac{8}{5} + b$

$-\frac{15}{5} \quad +\frac{8}{5} \quad +\frac{8}{5}$

$-\frac{7}{5} = b \Rightarrow y = -\frac{2}{5}x - \frac{7}{5}$

4. Given: two points

Example: Find an equation in slope-intercept form (2, -3) and (-1, 2).

x_1, y_1 $\quad x_2, y_2$

$m = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{2 - (-3)}{-1 - 2}$

$m = \frac{5}{-3}$

using point-slope

$3\left((y - 2)\right) = \left(-\frac{5}{3}(x - (-1))\right)3$

$3y - 6 = -5(x + 1)$

$3y - 6 = -5x - 5$

$+6 \quad +6$

$\frac{3y}{3} = -\frac{5}{3}x + \frac{1}{3}$

$y = -\frac{5}{3}x + \frac{1}{3}$

using slope intercept

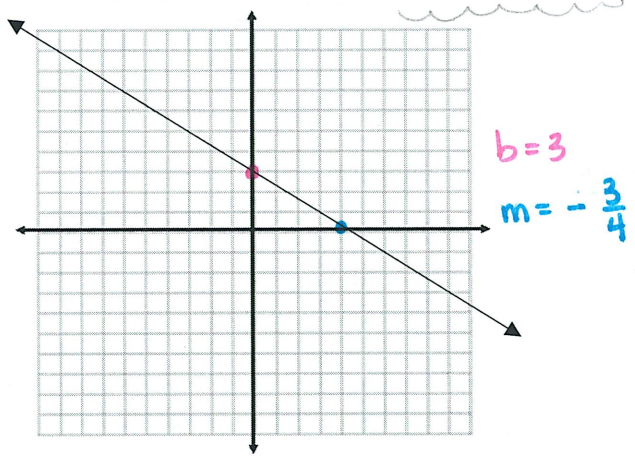
$2 = -\frac{5}{3}(-1) + b$

$2 = \frac{5}{3} + b$

$\frac{6}{3} \quad -\frac{5}{3} \quad -\frac{5}{3}$

$\frac{1}{3} = b \Rightarrow y = -\frac{5}{3}x + \frac{1}{3}$

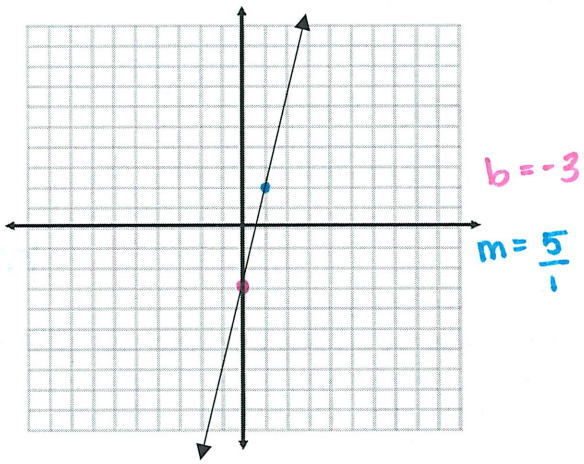
5. Given: a graphed line



$b = 3$

$m = -\frac{3}{4}$

$y = -\frac{3}{4}x + 3$



$b = -3$

$m = \frac{5}{1}$

$y = 5x - 3$

Finding an Equation of a Line: $Ax + By = C$

6. Given: slope and y-intercept

Example: Find an equation in ~~slope-intercept~~ standard form of the line having slope -1 and y-intercept 3. (0, 3)

$y - 3 = -1(x - 0)$

$y - 3 = -1(x)$

$y - 3 = -x$

$+x \quad +x$

$x + y - 3 = 0 \Rightarrow x + y = 3$

$+3 \quad +3$