Name: ______________________________

Algebra II

Unit 2: Linear Functions

Priority Standards: A.CED.2: Create equations in two or more variables to represent relationships between quantities.

Unit "I can" statements:

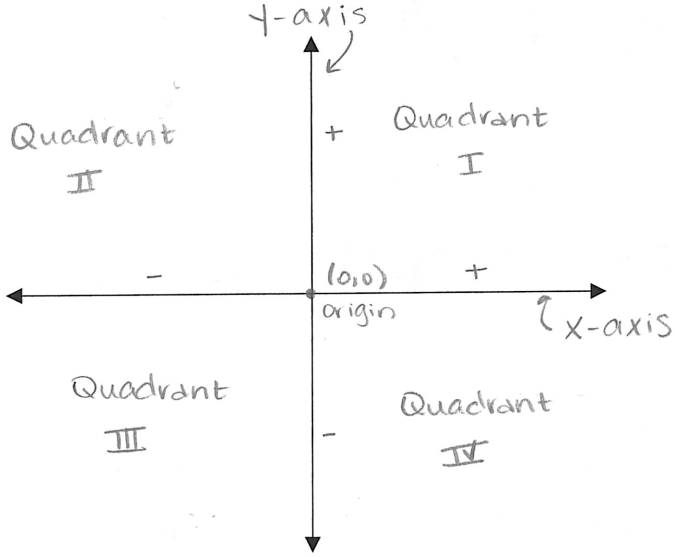
1. I can identify the domain and range, and the dependent and independent variables in a relation.
2. I can identify functions and correctly interpret function notation.
3. I can find the slope of a line and graph a line when given the slope and a point.
4. I can rewrite equations into slope/intercept form and graph them by using the slope and y-intercept.
5. I can find an equation of a line given its slope and y-intercept, its slope and a point, or two points.
6. I can graph a linear inequality in two variables.
7. I can describe how a simple quadratic, absolute value, or square root function is transformed by minor changes to the parent equation.

Common Core State Standards that are addressed in this unit include: F.IF.1a, F.IF.2a, F.IF.4, F.IF.5, F.IF.7b, F.IF.7c, A.CED.2a, A.CED.2, A.CED.3a, A.SSE.1a
For more information see www.corestandards.org/Math/

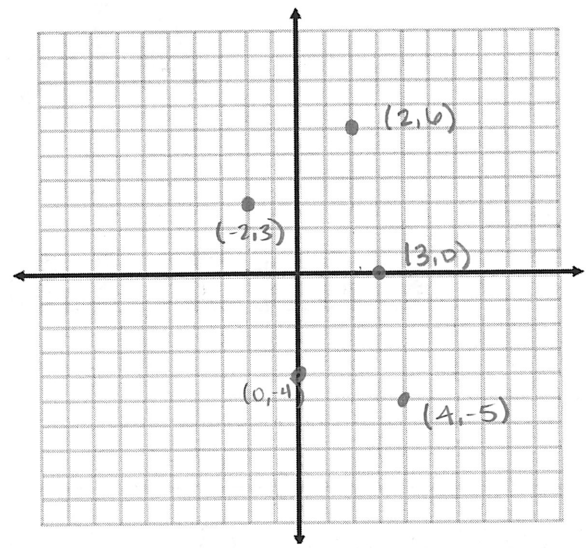
Relations

In this unit, our main focus is going to be on linear functions, and since this will involve graphing, we will have a quick review of the <u>Cartesian Coordinate System</u>.

Parts of the Cartesian Coordinate System



Points are graphed as ordered pairs. (x, y)



Graph the following points on the same coordinate plane:
(2, 6) (3, 0) (-2, 3) (4, -5) (0, -4)

There are several important definitions that you need to know as we begin this unit.

Relation: any set of ordered pairs

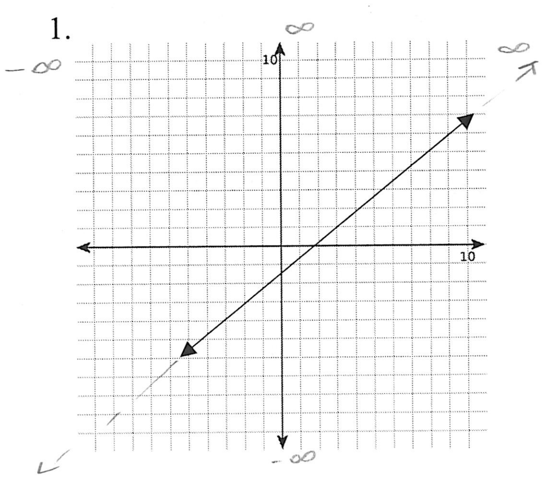
Domain: the set of values for the independent variable (x-value)

Range: the set of values for the dependent variable (y-value)

Note: Solid line or point (use ≤, ≥)
Dashed line or open point (use <, >)

Break for Practice: Try to determine the domain and range for the following relations.

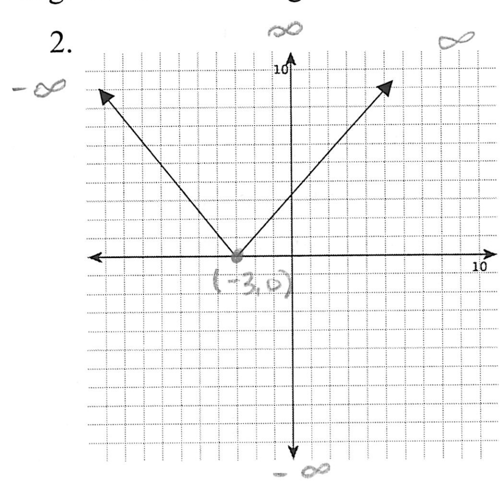
1.



Domain: $\mathbb{R}$ or $-\infty \le x \le \infty$

Range: $\mathbb{R}$ or $-\infty \le y \le$

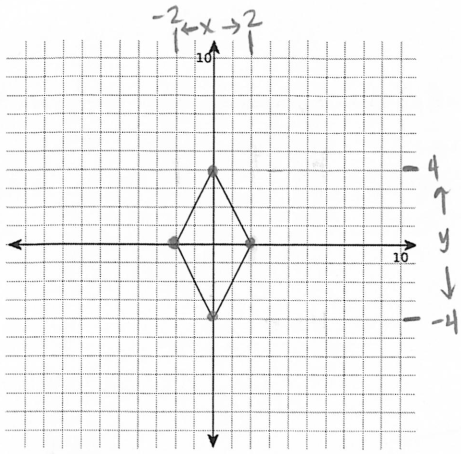
2.



Domain: $\mathbb{R}$ or $-\infty \le x \le \infty$

Range: $y \ge 0$

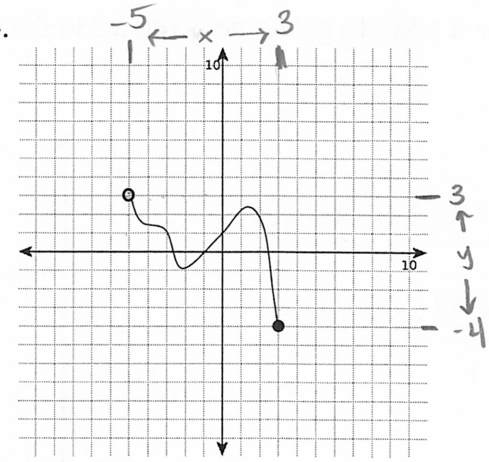
3.



Domain: $-2 \le x \le 2$

Range: $-4 \le y \le 4$

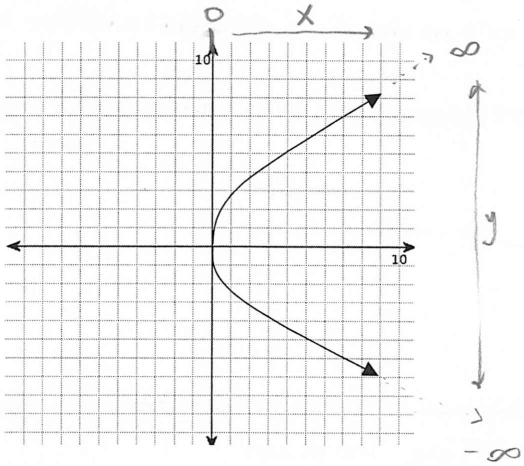
4.



Domain: $-5 < x \le 3$

Range: $-3 < y \le -4$

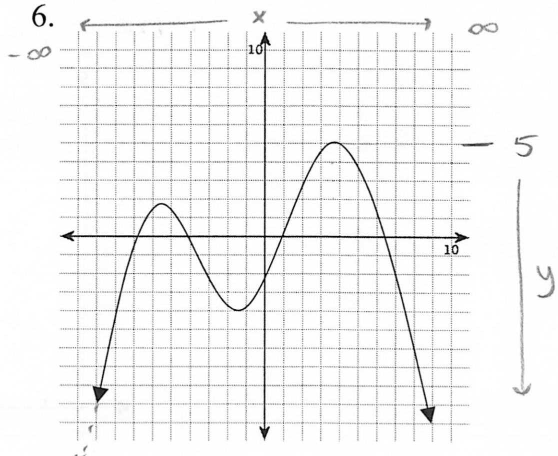
5.



Domain: $x \ge 0$

Range: $\mathbb{R}$ or $-\infty \le y \le \infty$

6.



Domain: $\mathbb{R}$ or $-\infty \le x \le \infty$

Range: $y \le 5$

Another skill we need to begin developing is that of Identifying the independent and dependent variables.

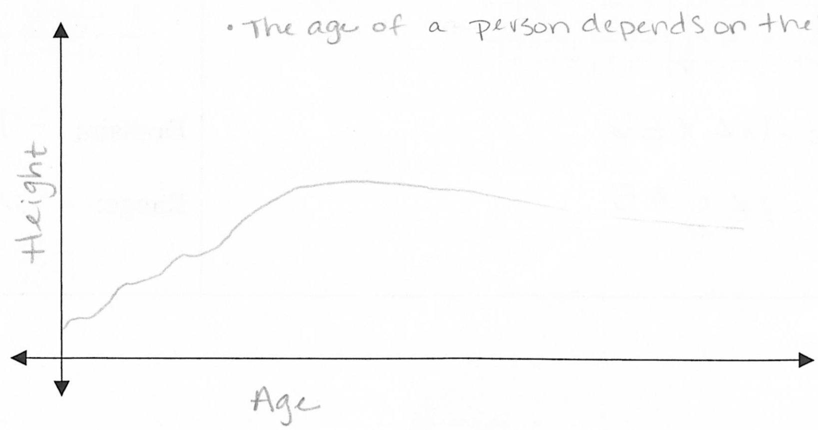
Break for Practice: For each problem: 1. Identify the independent and dependent variables.
2. Label the axes.
3. Sketch a reasonable graph.

1. The height of a person is related to their age.

What Sounds better?
- ✩ The height of a person depends on their age?
- The age of a person depends on their height?

Independent (x): Age

Dependent (y): Height



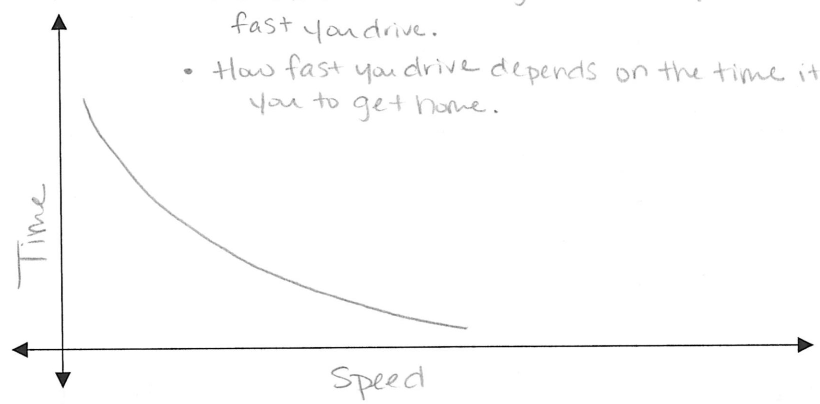
2. The time it takes to get home is related to the speed you drive.

What Sounds better?

- The time it takes to get home depends on how fast you drive.
- How fast you drive depends on the time it takes you to get home.

Independent (x): Speed

Dependent (y): Time



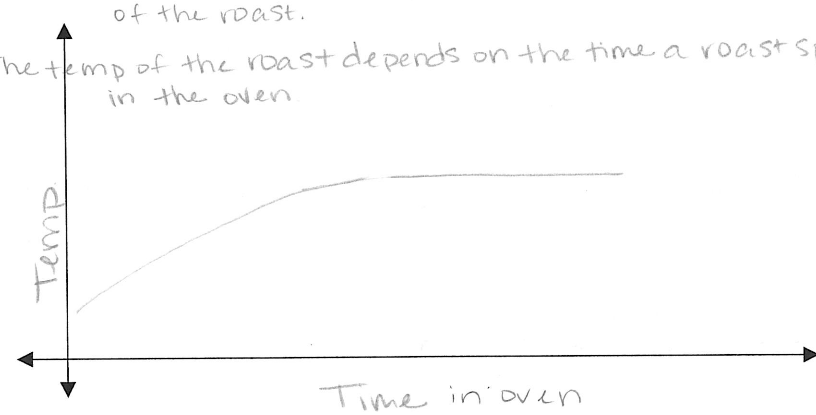
3. A roast is taken from a refrigerator and put in an oven. The time spent in the oven is related to the temperature of the roast.

What Sounds better?

- The time a roast spends in the oven depends on the temp. of the roast.
- The temp of the roast depends on the time a roast spends in the oven

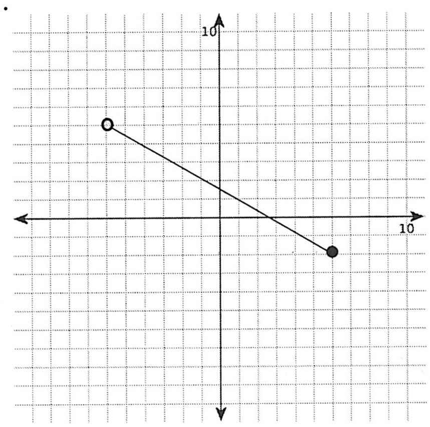
Independent (x): Time in oven

Dependent (y): Temp of Roast



Extended Practice: Try to determine the domain and range for the following relations.

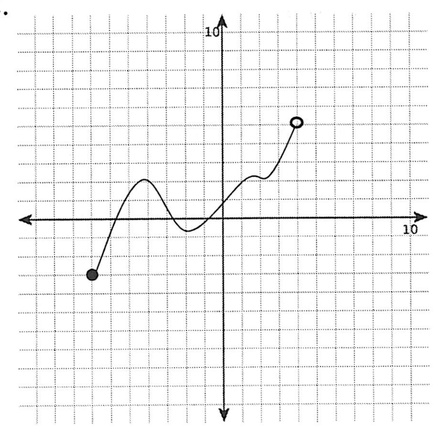
1.



Domain: $-6 < x \leq 6$

Range: $-2 \leq y < 5$

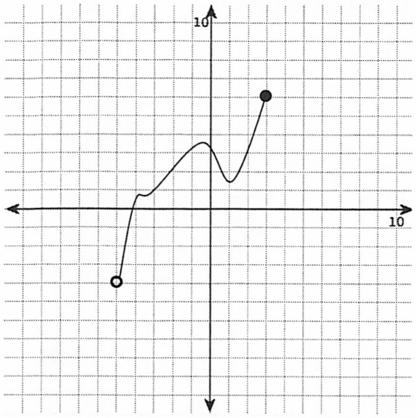
2.



Domain: $-7 \leq x < 4$

Range: $-3 \leq y < 5$

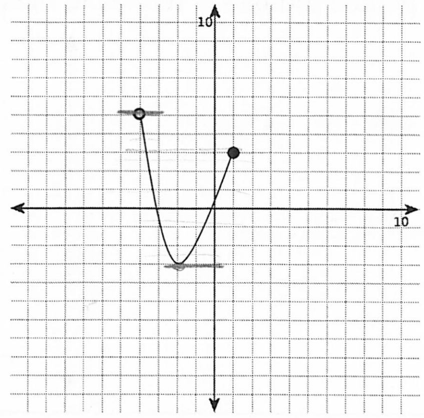
3.



Domain: $-5 < x \leq 3$

Range: $-4 < y \leq 6$

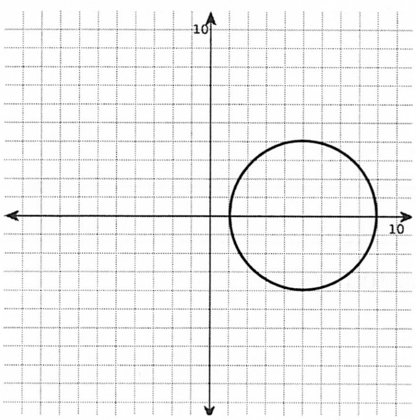
4.



Domain: $-4 < x \leq 1$

Range: $-3 \leq y < 5$

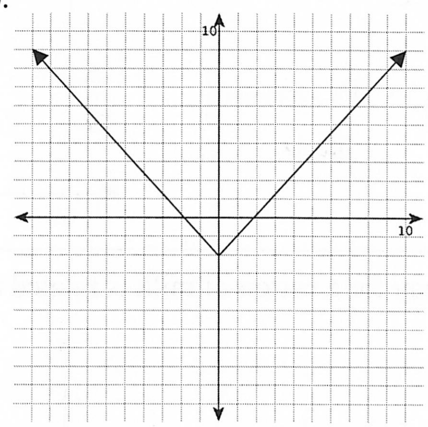
5.



Domain: $1 \leq x \leq 9$

Range: $-4 \leq y \leq 4$

6.



Domain: $\mathbb{R}$ OR $-\infty \leq x \leq \infty$

Range: $y \geq -2$

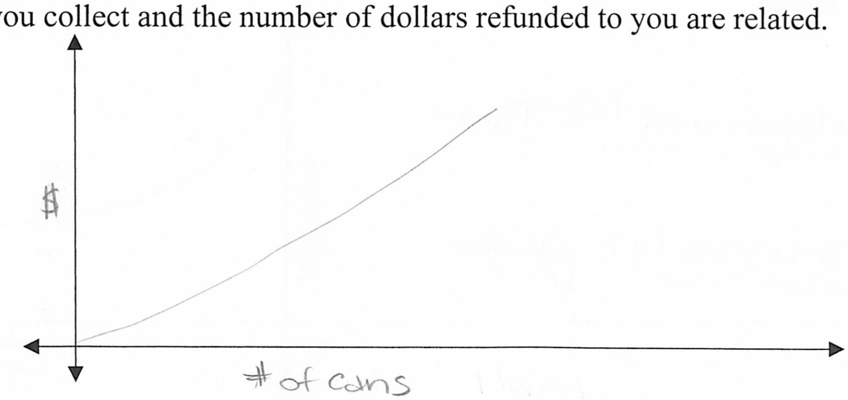
Extended Practice Continued:

For each problem: a.) Identify the independent and dependent variables.
b.) Label the axes.
c.) Sketch a reasonable graph.

7. The number of used aluminum cans you collect and the number of dollars refunded to you are related.

Independent (x): # of cans

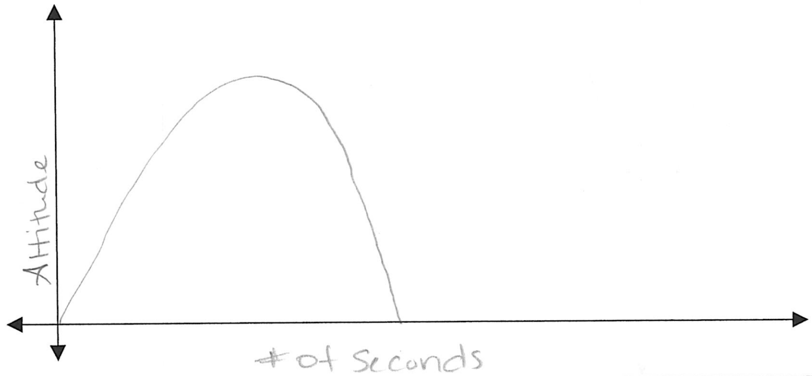
Dependent (y): Money



8. The altitude of a punted football is related to the number of seconds since it was kicked.

Independent (x): # of sec

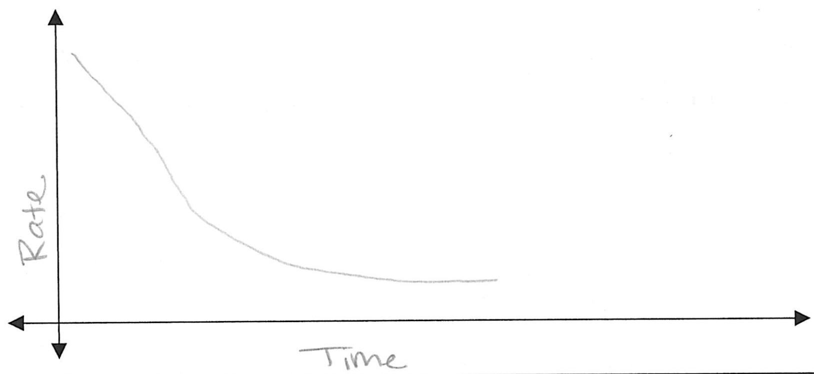
Dependent (y): Altitude



9. The rate at which you are breathing is related to how long it has been since you finished running a race.

Independent (x): Time

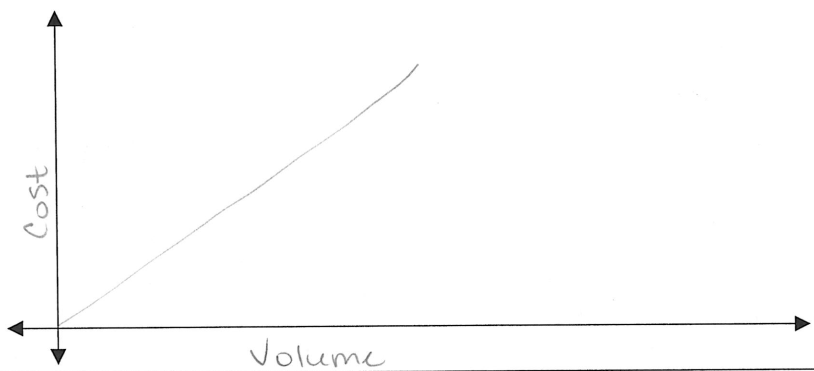
Dependent (y): Breathing Rate



10. The price you pay for a carton of milk is related to how much milk the carton holds.

Independent (x): Volume

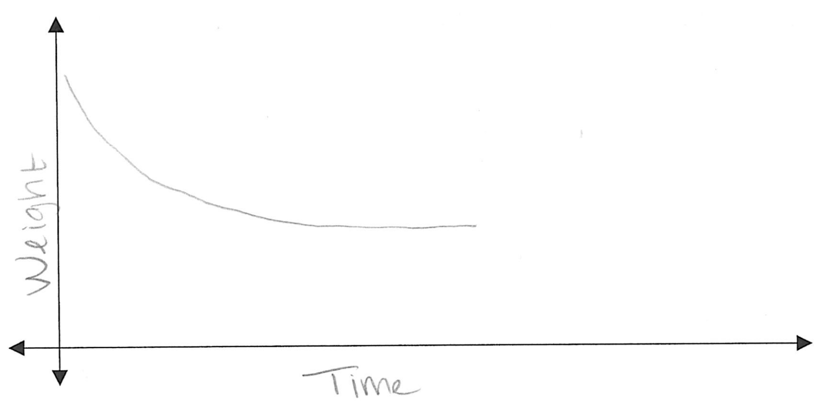
Dependent (y): Cost



11. Calvin Butterball desires to lose some weight, so he reduces his food intake from 8000 calories per day to 2000 calories per day. His weight is related to the number of days that have elapsed since he reduced his food intake.

Independent (x): Time

Dependent (y): Weight



Beginning Functions

Throughout the year you will be investigating many special families of functions, but first you need to understand what a function is.

Review: **Relation**: any set of ordered pairs

New: **Function**: A relation where the independent (x) variable NEVER repeats (or a relation where no two points have the same first coordinate.)

Break for Practice: Which of the following are functions:

1. {(1, 5), (2, 6), (3, 7)} Yes

2. {(1, 2), (2, 2), (3, 2)} Yes

3. {(1, 1), (1, 2), (1, 3)} No
(Repeating x-values)

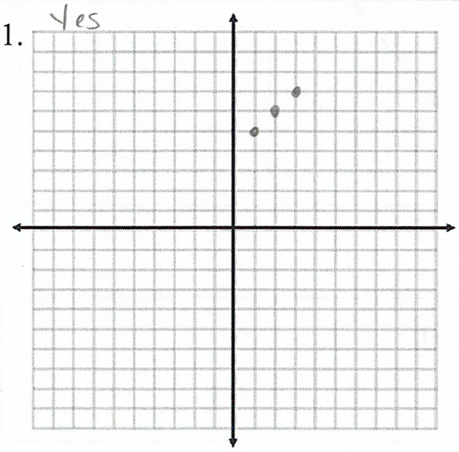
4. {(2, -2), (2, 2), (3, -3), (3, 3)} No
(Repeating x-values)

2. {(-2, 2), (2, 2), (-3, 3), (3, 3)} Yes

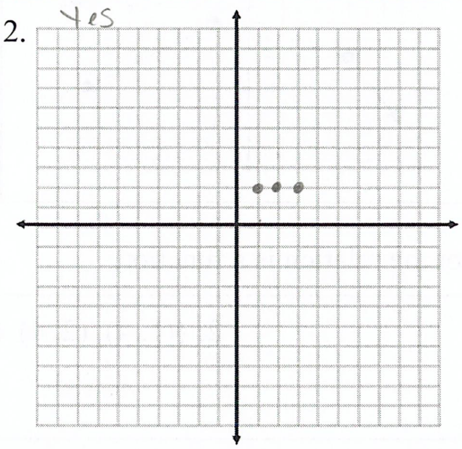
It is also possible to identify a function from a graph. Sketch a graph for each of the above relations.

If a vertical line passes through the graphs, what do you notice?

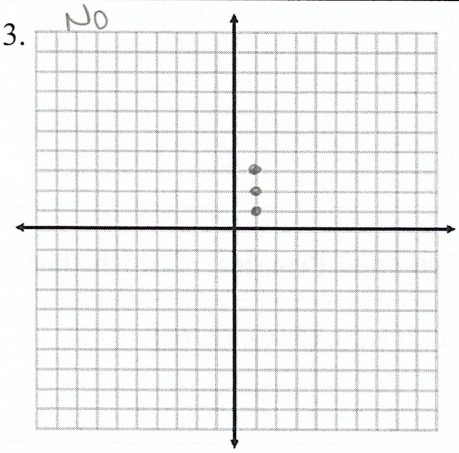
1. Yes



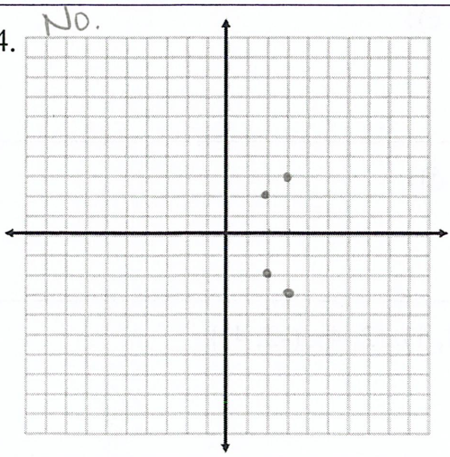
2. Yes



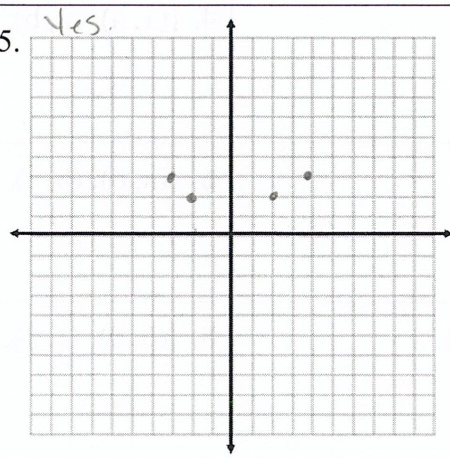
3. No



4. No.



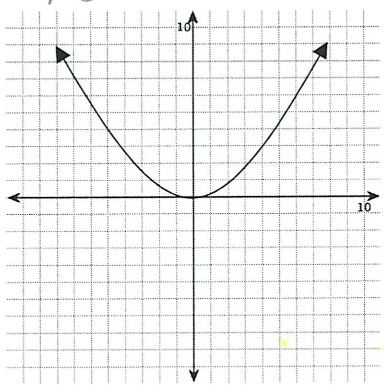
5. Yes.



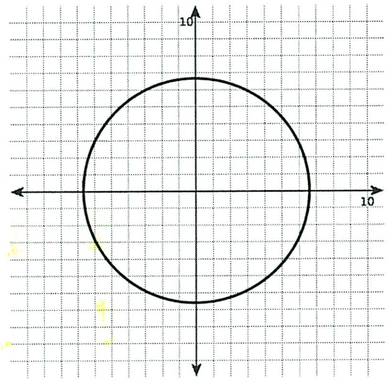
Vertical Line Test: If a vertical line can never pass through more than one point at a given time, then the relation is a function

Break for Practice: Decide whether or not each of the following graphs represent functions.

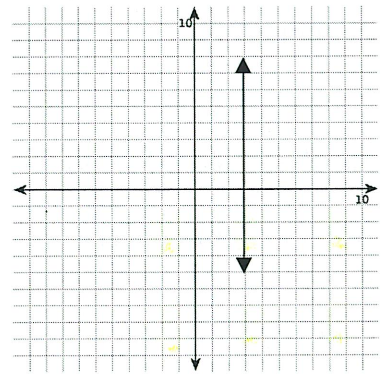
1. Yes



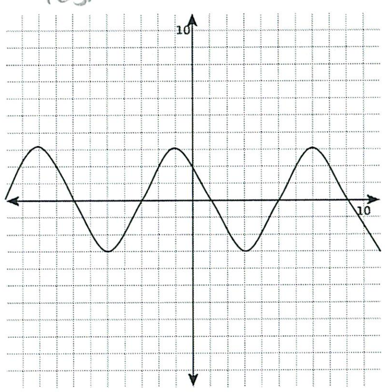
2. No.



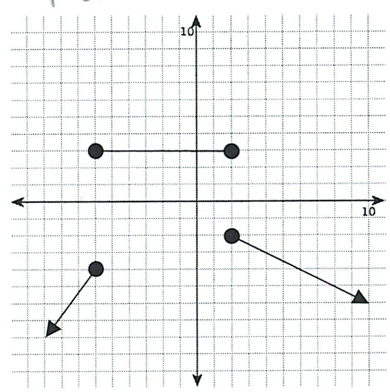
3. No.



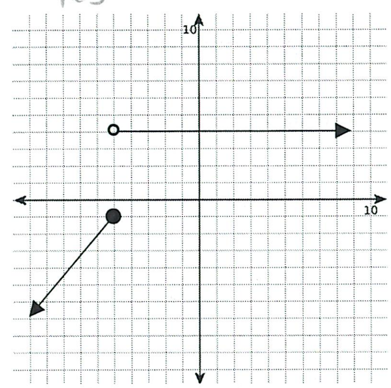
4. Yes.



5. No.



6. Yes

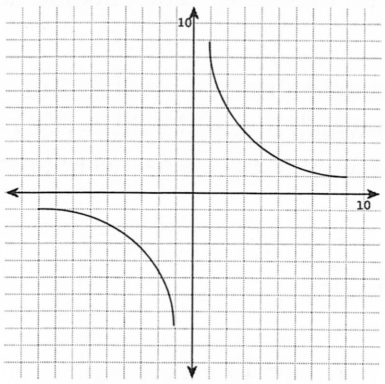


Extended Practice: Tell whether or not the relation is a function.

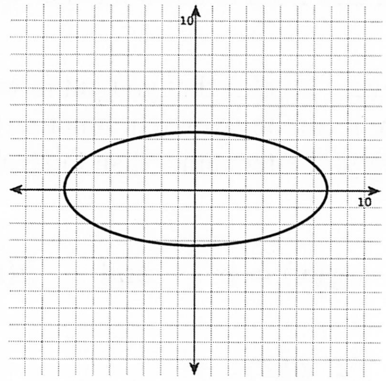
1. {(1, 3), (5, 6), (6, 6)} Yes.	2. {(1, 4), (2, 4), (3, 4)} Yes.
3. {(-1, -2), (0, 0), (1, 2)} Yes	4. {(1, a), (2, b), (3, c)} Yes
5. {(1, a), (2, a), (3, a)} Yes	6. {(a, b), (b, c), (c, d)} Yes
7. {(0, 0), (1, 1), (1, -1), (2, 4), (2, -4)} No.	8. {(0, 0), (1, 1), (-1, 1), (2, 4), (-2, 4)} Yes.

Extended Practice continued: Decide whether or not each of the following graphs represents a function.

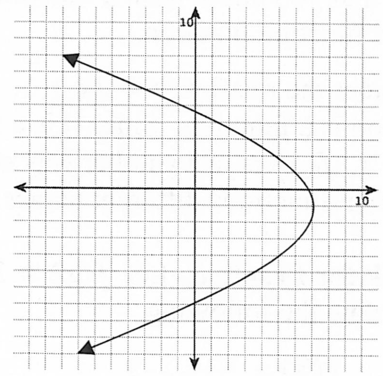
9. Yes



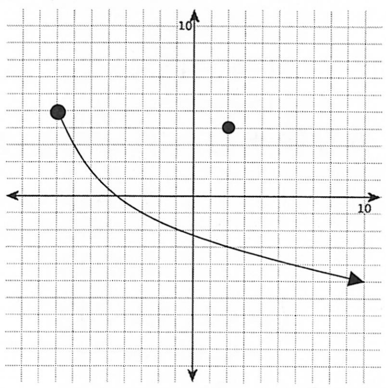
10. No



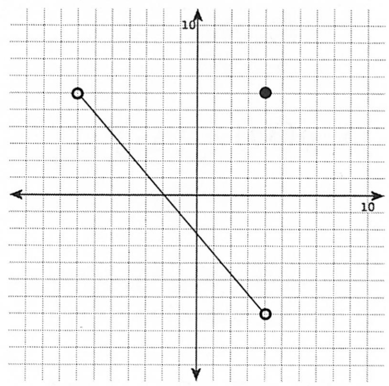
11. No



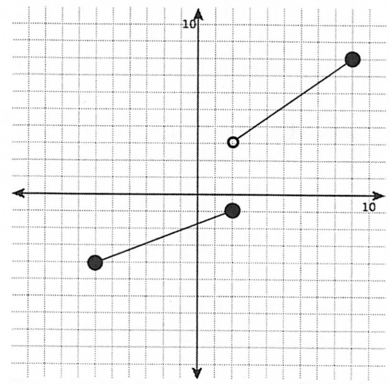
12. No.



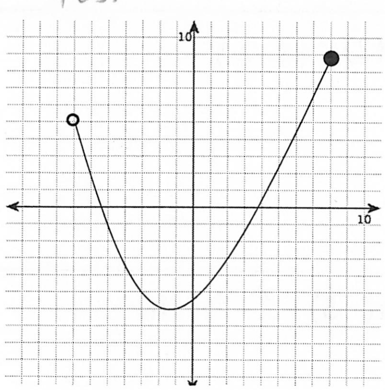
13. Yes



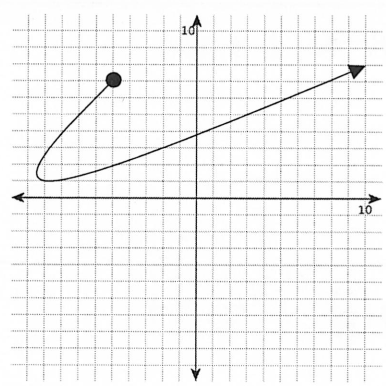
14. Yes.



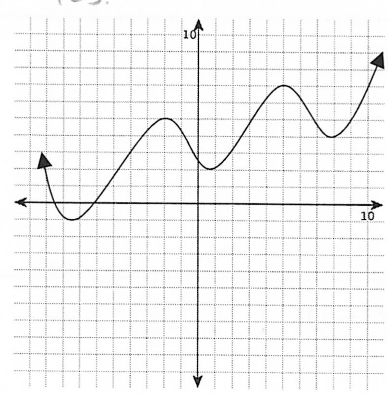
15. Yes.



16. No.



17. Yes.



Function Notation

Now that you know what a function is, you also need to know another way to write functions. This method is called function notation.

Consider the relation $y = 3x + 1$. Since every x-value that you put in would generate a unique y-value, this relation is a function. Since it is a function, it can be written in function notation which looks like this: $f(x) = 3x + 1$. $f(x)$ is just another way of writing y. Letters other than f can also be used, and this allows us to give different functions different names.

Break for Practice: Let $f(x) = 3x - 2$ $\quad g(x) = x^2$ $\quad$ and $\quad h(x) = |x| - 2$
Calculate the following.

1. $f(3) = 3(3)-2$
$= 9-2$
$= 7$
$(3,7)$

2. $g(2) = 2^2$
$= 4$
$(2,4)$

3. $h(-1) = |-1|-2$
$= 1-2$
$= -1$
$(-1,-1)$

4. $f(-4) = 3(-4)-2$
$= -12-2$
$= -14$
$(-4,-14)$

5. $g(3a) = (3a)^2$
$= 9a^2$
$(3a, 9a^2)$

6. $f(a+2) = 3(a+2)-2$
$= 3a+6-2$
$= 3a+4$
$(a+2, 3a+4)$

Extended Practice: Let $f(x) = 2x - 5$ $\quad g(x) = 7 - 3x$ $\quad$ and $\quad h(x) = 4 - 3x + x^2$.
Evaluate the following.

1. $f(4)$ $(4,3)$	2. $g(-3)$ $(-3,16)$	3. $h(2)$ $(2,2)$
4. $f(-5)$ $(-5,-15)$	5. $g\left(\frac{4}{3}\right)$ $(4/3, 3)$	6. $h(-5)$ $(-5,44)$
7. $h(0)$ $(0,4)$	8. $g(-5)$ $(-5,22)$	9. $f(a)$ $(a, 2a-5)$

10. $h(r)$ $(r, 4-3r+r^2)$	11. $g(m)$ $(m, 7-3m)$	12. $f(a+b)$ $(a+b, 2a+2b-5)$

13. Coming off an embarrassing loss to the tortoise, the hare wanted a rematch. The tortoise, being a wise old animal, agreed to race again if he was given a 30-minute lead. Both start at the same point. The Tortoise races at a constant speed of 50 feet per minute, and the hare races at a constant speed of 160 feet per minute.

Let x = number of minutes since the tortoise started, T(x) = number of feet the tortoise has gone, and H(x) = number of feet the hare has gone.

a) Write equations expressing T(x) and H(x) in terms of x.

$T(x) = 50x$

$H(x) = 160(x-30)$

b) Find T(35) and H(35). Who is ahead at the end of 35 minutes?

$T(35) = 1750$ ft

$H(35) = 800$ ft

Graphs of Linear Equations in Two Variables

Now that you know a bit about relations and functions, let's look at graphing equations that have two variables.

Graph the equation: $2x - 3y = 12$

$-2x \qquad -2x$

Method: Solve for y and find at least 4 points to graph. Choose "nice" x-values.

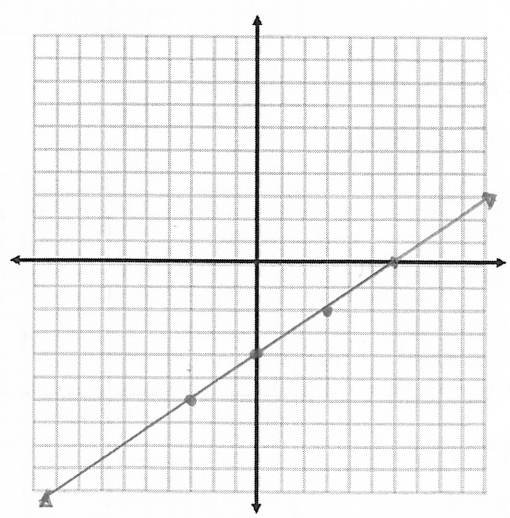
$2x - 3y = 12$

$-2x \qquad -2x$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{12}{-3}$

$y = \frac{2}{3}x - 4$

X	Y
0	-4
3	-2
-3	-6
6	0



Result: The graph of any equation of the form $Ax + By = C$ is a line. These equations are called linear equations. Note: Only two points are needed to graph these equations, and often these points are the x-intercept and the y-intercept.

point when $y=0$ point when $x=0$

Break for Practice: Identify each equation as linear or not linear. Graph the linear equations.

1. $3x + 4y = 12$ (Linear)

$y=0$; $3x+4(0)=12$

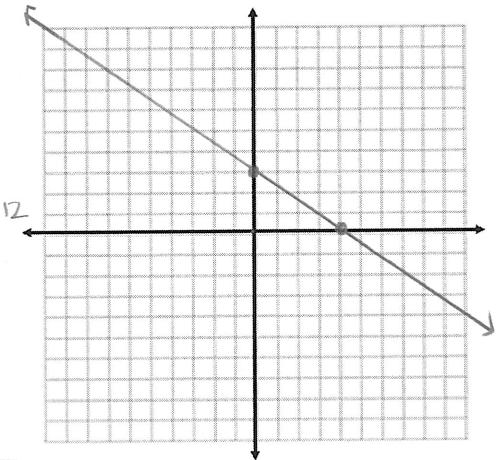
$\frac{3x}{3} = \frac{12}{3}$

$x=4$

$x=0$; $3(0)+4y=12$

$\frac{4y}{4}=\frac{12}{4}$

$y=3$



2. $3x + 2y - 9 = 0$ (Linear)

$y=0$; $3x+2(0)=9$

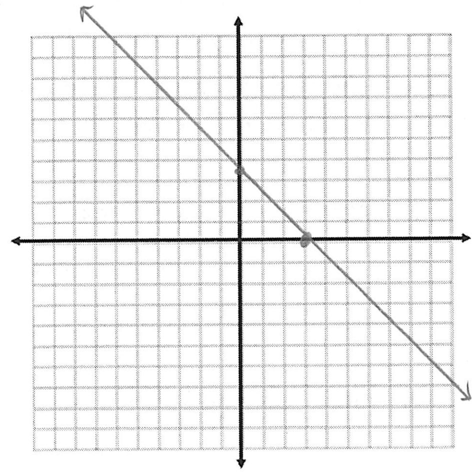
$\frac{3x}{3}=\frac{9}{3}$

$x=3$

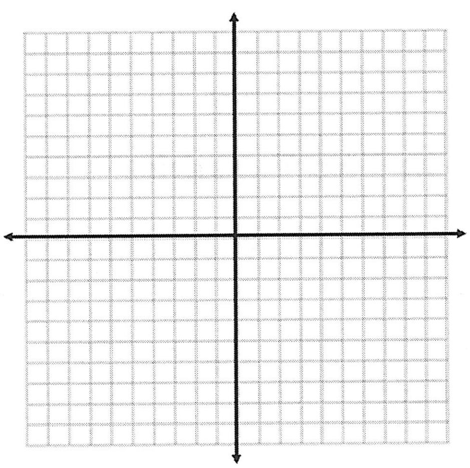
$x=0$; $3(0)+2y=9$

$\frac{2y}{2}=\frac{9}{2}$

$y=4.5$



3. $2x^2 + y = 3$ (Not Linear)



4. $-2x + y = 6$ (Linear)

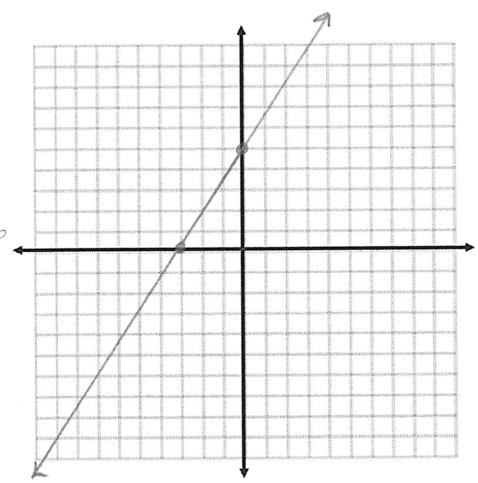
$y=0$; $-2x+0=6$

$\frac{-2x}{-2}=\frac{6}{-2}$

$x=-3$

$x=0$; $-2(0)+y=6$

$y=6$



5. $x = 4 \Rightarrow x + 0y = 4$

$y=0$; $x+0(0)=4$

$x=4$

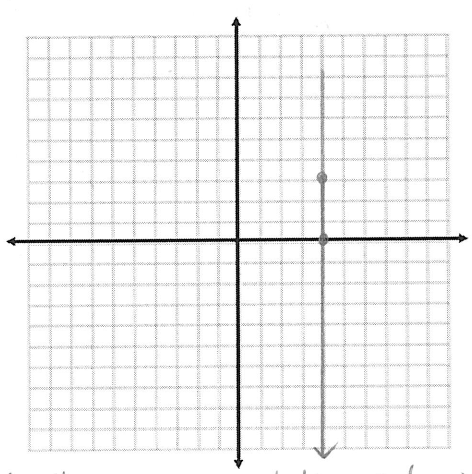
$x=0$; $0+0y=4$

$0 \neq 4$

(not possible)

$y=3$; $x+0(3)=4$

$x=4$



$x=\#$ are vertical lines (not a fn)

6. $y = -3 \Rightarrow 0x + y = -3$

$y=0$; $0x+0=3$

$0 \neq 3$

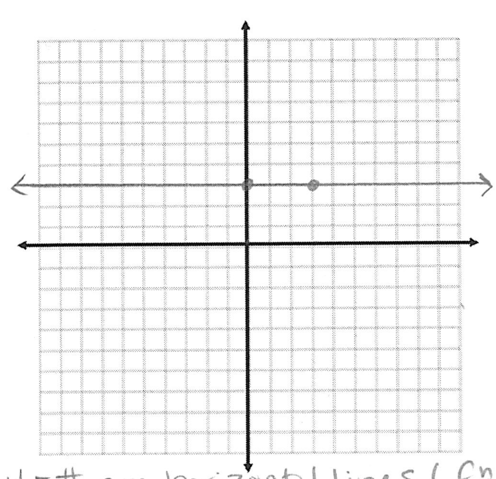
(not possible)

$x=0$; $0(0)+y=3$

$y=3$

$x=3$; $0(3)+y=3$

$y=3$



$y=\#$ are horizontal lines (fn)