Extended Practice: Solve each equation.

1. $(x-1)(x-4)=0$

$\{1, 4\}$

2. $t(t+1)(t-2)=0$

$\{-1, 2, 0\}$

3. $z^2+3=4z$

$\{3, 1\}$

4. $x^2-12=4x$

$\{6, -2\}$

5. $3r^2=4r-1$

$\{1/3, 1\}$

6. $6x^2=1-x$

$\{1/3, -1/2\}$

7. $6(x+12)=x^2$

$\{12, -6\}$

8. $(u+3)(u-3)=8u$

$\{9, -1\}$

9. $3t(t+1) = 4(t+1)$

$\{4/3, -1\}$

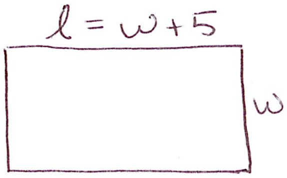
10. $2(r^2+1) = 5r$

$\{1/2, 2\}$

Problem Solving Using Polynomial Equations

Now it's time to see where polynomial equations can be used.

Example: A rectangle is 5 m longer than it is wide, and its area is 176 m^2. Find its dimensions.



Area = lw $\Rightarrow$

$176 = (w+5)w$

$176 = w^2 + 5w$

$-176 \qquad -176$

$0 = w^2 + 5w - 176$

$0 = (w+16)(w-11)$

$w+16=0 \qquad w-11=0$

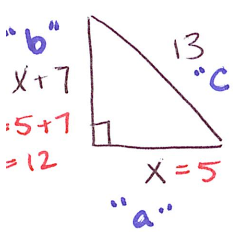
$-16 \quad -16 \qquad +11 \quad +11$

$w = \cancel{-16}$ doesn't make sense $\qquad w = 11$

-1, 176
-2, 88
-4, 44
-8, 22
-11, 16 ✓

$\boxed{\text{width} = 11\text{cm} \quad \text{length} = 11 + 5 = 16\text{cm}}$

Example: The hypotenuse of a right triangle is 13 in long. One leg is 7 in longer than the other leg. Find the length of each leg.



Pythagorean Thrm $\Rightarrow a^2+b^2=c$

$x^2 + (x+7)^2 = 13^2$

$x^2 + (x+7)(x+7) = 169$

$x^2 + x^2 + 7x + 7x + 49 = 169$

$-169 \quad -169$

$2x^2 + 14x - 120 = 0$

$2(x^2 + 7x - 60) = 0$

$2(x+12)(x-5) = 0$

$x+12=0 \qquad x-5=0$

$-12 \quad -12 \qquad +5 \quad +5$

$x = \cancel{-12} \qquad x = 5$

$\boxed{\text{The legs are 5 inches and 12 inches}}$

Vertical motion (ex: a thrown ball, a rocket, etc.) affected only by gravity leads to a formula that is a polynomial.

h = height v = initial velocity t = time in seconds

If h is in meters, then $h = vt - 4.9t^2$. If h is in feet, then $h = vt - 16t^2$.

Example: A batter hits a baseball directly upward with a speed of 96 ft/sec.

a) How long is the ball in the air before being caught by the catcher?

★ Assume that the height when caught is the same as the height when hit and designate that height as zero. ★

$h = 0$

$v = 96$ ft/sec

$h = vt - 16t^2 \Rightarrow 0 = 96t - 16t^2$

$0 = 16t(6 - t)$

$\frac{16t}{16} = \frac{0}{16}$ $\qquad$ $6 - t = 0$, $+t \quad +t$

$t = 0$ sec $\qquad$ $t = 6$ sec

time when hit $\qquad$ time when caught

The ball was in the air for 6 seconds

b) How high did the ball go?

★ The ball would be at its highest point halfway between being hit and being caught. This would be at time $t = 3$ seconds ★

$v = 96$

$t = 3$

$h = vt - 16t^2 \Rightarrow h = 96(3) - 16(3)^2$

$h = 288 - 144$

$h = 144$ ft

Extended Practice: Solve

1. A rectangle is 4 cm longer than it is wide, and its area is 117 cm^2. Find its dimensions.

9cm by 13cm

2. The top of a 15-foot ladder is 3 ft. farther up a wall than the foot of the ladder is from the bottom of the wall. How far is the foot of the ladder from the bottom of the wall?

9 ft from the bottom of the wall

3. A rectangle is 15 cm wide and 18 cm long. If both dimensions are decreased by the same amount, the area of the new rectangle formed is 116 cm^2 less than the area of the original. Find the dimensions of the new rectangle.

11 cm x 14cm

4. A projectile is launched upward from ground level with an initial speed of 98 m/sec.

a) When will it return to the ground?

t= 0 sec (starting point)

t= 20sec (return to the ground)

b.) How high will it go?

$h = 490m$

5. A ball is thrown directly upward from ground level with an initial speed of 80 ft/sec.

a.) When will it return to the ground?

$t = 0$ (starting point)

$t = 50 sec$ (return to the ground)

b.) How high will it go?

$h = 100ft$

Solving Polynomial Inequalities

The final section in this unit is solving polynomial inequalities. The use of factoring and graphing are the keys to solving these problems.

What does $ab > 0$ and $ab < 0$ mean?

the product of a·b is greater than 0

$ab > 0$ → means… that the product of a and b is a positive number

** So, a and b are either both positive numbers OR both negative numbers. (Example: 20 → (4)(5) OR (-4)(-5))

the product of a·b is less than 0

$ab < 0$ → means… that the product of a and b is a negative number

** So, a and b are either (-a)(b) OR (a)(-b)
(Example: -20 → (-4)(5) OR (4)(-5))

Example: Find and graph the solution set of $x^2 + 3x < -2$.
$+2 \quad +2$

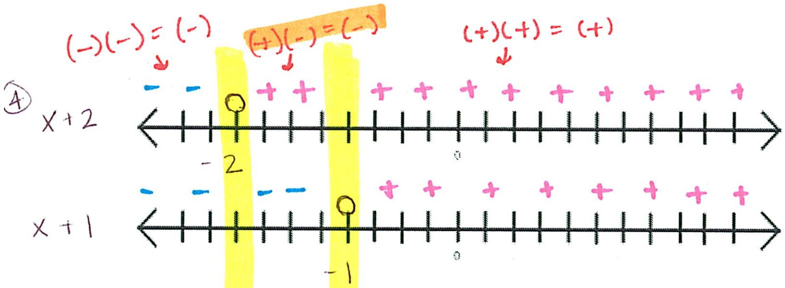
① $x^2 + 3x + 2 < 0$ open points
Need to find the region with the negative product.

② Factor: $(x+2)(x+1) < 0$

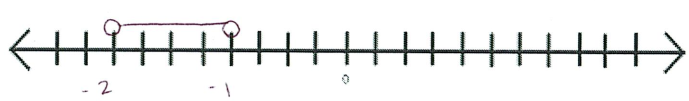
③ $x + 2 = 0$ $\quad$ $x + 1 = 0$
$-2 \quad -2$ $\quad$ $-1 \quad -1$
$x = -2$ $\quad$ $x = -1$

Steps:

1. Set one side of the inequality to zero.
2. Factor
3. Identify the "breaking points"
4. Graph separate number lines and combine
5. Identify the solution



⑤ Solution set: $-2 < x < -1$



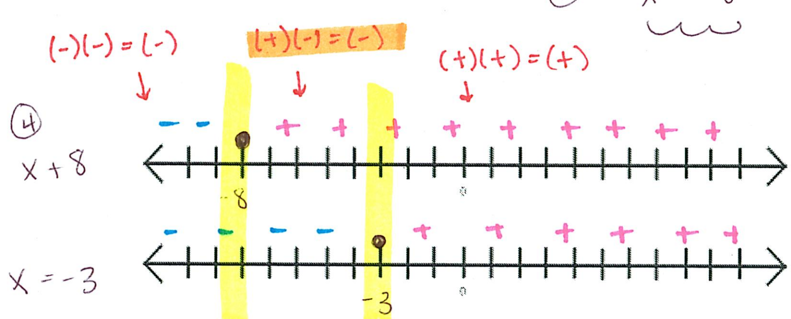
Break for Practice: Find and graph the solution set to each inequality.

1. $x^2 + 15x \le 4x - 24$ ⟹ ① $x^2 + 11x + 24 \le 0$ Closed points
$+24 \quad -4x \quad -4x \quad +24$
Need to find the region with a negative product.

② $(x+8)(x+3) \le 0$

$x + 8 = 0$ $\quad$ $x + 3 = 0$
$-8 \quad -8$ $\quad$ $-3 \quad -3$

③ $x = -8$ $\quad$ $x = -3$



⑤ Solution set: $-8 \le x \le -3$