

Finding Quadratic Equations from Points

Earlier in the year we learned how to find an equation for a line in the form $y = mx + b$ if we knew the coordinates for two points on the line. In this section we will learn how to find the equation for a parabola in the form $y = ax^2 + bx + c$ if we know the coordinates for three points on the line.

Example: Find the equation for the parabola that passes through the points $(-1, 6)$, $(-2, 17)$, and $(2, 9)$.

$$ax^2 + bx + c = y$$

$$(-1, 6) \rightarrow a(-1)^2 + b(-1) + c = 6$$

$$(-2, 17) \rightarrow a(-2)^2 + b(-2) + c = 17$$

$$(2, 9) \rightarrow a(2)^2 + b(2) + c = 9$$

Matrix A (x)	Matrix B (y)
$\begin{bmatrix} 1 & -1 & 1 \\ 4 & -2 & 1 \\ 4 & 2 & 1 \\ x^2 & x & c \end{bmatrix}$	$\begin{bmatrix} 6 \\ 17 \\ 9 \\ y \end{bmatrix}$

Calculator Instructions: We are calculating $[A]^{-1} \times [B]$

Press 2^{nd} x^{-1} \rightarrow \rightarrow (Edit is highlighted) 1 Make the matrix 3×3

Enter matrix information

Press 2^{nd} x^{-1} \rightarrow \rightarrow (Edit is highlighted) 2 Make the matrix 3×1

Enter matrix information

(To get to home screen 2^{nd} Mode)

Press 2^{nd} x^{-1} (Names is highlighted) 1 (will be back to home screen) x^{-1} \times

Then 2^{nd} x^{-1} (Names is highlighted) 2 (will be back to home screen) Enter

$$[A]^{-1} \cdot [B] = \begin{bmatrix} 3a \\ -2b \\ 1c \end{bmatrix} \leftarrow \text{Answer Matrix}$$

The equation that contains $(-1, 6)$, $(-2, 17)$, and $(2, 9)$ is $y = 3x^2 - 2x + 1$

Break for Practice:

- Find the equation for the parabola that contains the points $(-2, -11)$, $(4, 13)$, and $(6, 29)$.

Matrix A Matrix B

$$\begin{bmatrix} 4 & -2 & 1 \\ 16 & 4 & 1 \\ 36 & 6 & 1 \\ x^2 & x & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -11 \\ 13 \\ 29 \\ y \end{bmatrix} = \begin{bmatrix} 0.5a \\ 3b \\ -7c \end{bmatrix} \Rightarrow y = 0.5x^2 + 3x - 7$$

Extended Practice: Find the equations ($y = ax^2 + bx + c$) that contain the following points.

1. (1,6), (3,26), (-2,21)

$$y = 3x^2 - 2x + 5$$

2. (-2,-41), (-3,-72), (5,-48)

$$y = -4x^2 + 11x - 3$$

3. (10,40), (-20,160), (-5,10)

$$y = 0.4x^2$$

4. (-4,-37), (2,11), (0,-1)

$$y = -0.5x^2 + 7x - 1$$

5. (4,7.3), (6,12.7), (-3,1.0)

$$y = 0.2x^2 + 0.7x + 1.3$$

6. (0,0), (-1,7), (6,42)

$$y = 2x^2 - 5x$$