Always look for GCF 1st!

Break for Practice: Factor Completely

1. $x^3 + 64$

$a = \sqrt[3]{x^3}$ $\quad b = \sqrt[3]{64}$

$a = x$ $\quad b = 4$

$(x+4)(x^2 - x \cdot 4 + 4^2)$

$= (x+4)(x^2 - 4x + 16)$

2. $2n^3 - 54 \rightarrow 2(n^3 - 27)$

$a = \sqrt[3]{n^3}$ $\quad b = \sqrt[3]{27}$

$a = n$ $\quad b = 3$

$(n-3)(n^2 + n \cdot 3 + 3^2)$

$= 2(n-3)(n^2 + 3n + 9)$

3. $\cancel{16}x^3 + 250 \rightarrow 2(8x^3 + 125)$

$a = \sqrt[3]{8x^3}$ $\quad b = \sqrt[3]{125}$

$a = 2x$ $\quad b = 5$

$(2x+5)((2x)^2 - 2x \cdot 5 + 5^2)$

$= 2(2x+5)(4x^2 - 10x + 25)$

Extended Practice: Factor Completely. (These are a mixture of the different types we have studied.)

1. $x^3 - 8$ $(x-2)(x^2+2x+4)$	2. $x^2 - 16$ $(x+4)(x-4)$
3. $x^2 + 5x - 14$ $(x+7)(x-2)$	4. $64x^3 + 125$ $(4x+5)(16x^2-20x+25)$
5. $3x^2 + 9x - 30$ $3(x+5)(x-2)$	6. $x^2 - 9x + 20$ $(x-5)(x-4)$
7. $25x^2 - 9$ $(5x-3)(5x+3)$	8. $2x^2 - 4x - 70$ $2(x-7)(x+5)$
9. $8x^3 - 27$ $(2x-3)(4x^2+6x+9)$	10. $4x^2 - 12x + 9$ $(2x-3)(2x-3)$

Solving Polynomial Equations

In this section we will use the factoring techniques you have learned to solve polynomial equations. A key property to remember when solving these is if $ab = 0$, then $a = 0$, or $b = 0$, or both = 0.

Steps for Solving Polynomial Equations:

1. Set the equation equal to 0.
   $(x + \#)$ or $(x - \#)$
2. Factor the equation.
3. Set each factor equal to 0 and solve. to find the root, Solution or zero $(x = \#)$

Break for Practice: Solve each equation. The answers are called "roots", "zeros", or "solutions".

1. $x^2 + 7x = 18 \Rightarrow x^2 + 7x - 18 = 0$

   -18

   Factors: $(x+9)(x-2) = 0$

   $x + 9 = 0$ $\quad$ $x - 2 = 0$

   $-9 \quad -9$ $\quad$ $+2 \quad +2$

   Roots or Solutions or Zeros: $x = -9$ $\quad$ $x = 2$

2. $3r^2 = 10r + 8 \Rightarrow 3r^2 - 10r - 8 = 0$

   keep highest degree positive

   $-10r \quad -8 \quad -10r \quad -8$

   $= -24$ $\quad$ $-12, 2$

   $3r^2 - 12r + 2r - 8 = 0$

   $3r(r-4) + 2(r-4) = 0$

   $(3r+2)(r-4) = 0$

   $3r + 2 = 0$ $\quad$ $r - 4 = 0$

   $-2 \quad -2$ $\quad$ $+4 \quad +4$

   $\frac{3r}{3} = \frac{-2}{3}$ $\quad$ $r = 4$

   $r = -2/3$

3. $x^2 + 25 = 10x$ The solution(s) to this is called a double root. Why?

   $-10x \quad -10x$

   The factor happens twices

   $x^2 - 10x + 25 = 0$

   $(x-5)(x-5) = 0$

   $x - 5 = 0$ $\quad$ $x - 5 = 0$

   $+5 \quad +5$ $\quad$ $+5 \quad +5$

   $x = 5$ $\quad$ $x = 5$ OR $x = 5$ double root

4. $(a+3)(a-3) = 40$

   $a^2 - 3a + 3a - 9 = 40$

   $-40 \quad -40$

   $a^2 - 49 = 0$

   $(a-7)(a+7) = 0$

   $a - 7 = 0$ $\quad$ $a + 7 = 0$

   $+7 \quad +7$ $\quad$ $-7 \quad -7$

   $a = 7$ $\quad$ $a = -7$

5. $(c-6)^2 = c \rightarrow (c-6)(c-6) = c$

   $c^2 - 6c - 6c + 36 = c$

   $-c \quad -c$

   $c^2 - 13c + 36 = 0$

   $(c-9)(c-4) = 0$

   $c - 9 = 0$ $\quad$ $c - 4 = 0$

   $+9 \quad +9$ $\quad$ $+4 \quad +4$

   $c = 9$ $\quad$ $c = 4$

Extended Practice: Solve each equation.

1. $(x-1)(x-4)=0$

$\{1, 4\}$

2. $t(t+1)(t-2)=0$

$\{-1, 2, 0\}$

3. $z^2+3=4z$

$\{3, 1\}$

4. $x^2-12=4x$

$\{6, -2\}$

5. $3r^2=4r-1$

$\{1/3, 1\}$

6. $6x^2=1-x$

$\{1/3, -1/2\}$

7. $6(x+12)=x^2$

$\{12, -6\}$

8. $(u+3)(u-3)=8u$

$\{9, -1\}$

9. $3t(t+1) = 4(t+1)$

$\{4/3, -1\}$

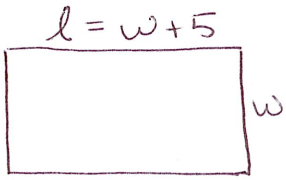
10. $2(r^2+1) = 5r$

$\{1/2, 2\}$

Problem Solving Using Polynomial Equations

Now it's time to see where polynomial equations can be used.

Example: A rectangle is 5 m longer than it is wide, and its area is 176 m^2. Find its dimensions.



Area = lw $\Rightarrow$ $176 = (w+5)w$

$176 = w^2 + 5w$

$-176 \qquad -176$

$0 = w^2 + 5w - 176$

$0 = (w+16)(w-11)$

$w+16=0 \qquad w-11=0$

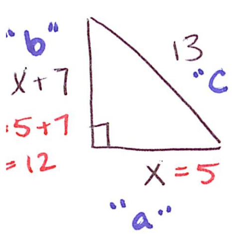
$-16 \quad -16 \qquad +11 \quad +11$

$w = \cancel{-16}$ doesn't make sense $\qquad w = 11$

-1, 176
-2, 88
-4, 44
-8, 22
-11, 16 ✓

width = 11cm
length = 11 + 5 = 16 cm

Example: The hypotenuse of a right triangle is 13 in long. One leg is 7 in longer than the other leg. Find the length of each leg.



Pythagorean Thrm $\Rightarrow a^2+b^2=c$

$x^2 + (x+7)^2 = 13^2$

$x^2 + (x+7)(x+7) = 169$

$x^2 + x^2 + 7x + 7x + 49 = 169$

$-169 \quad -169$

$2x^2 + 14x - 120 = 0$

$2(x^2 + 7x - 60) = 0$

$2(x+12)(x-5) = 0$

$x+12=0 \qquad x-5=0$

$-12 \quad -12 \qquad +5 \quad +5$

$x = \cancel{-12} \qquad x = 5$

The legs are 5inches and 12 inches