Graphing Parabolas

It is possible to find the vertex of a parabola quickly when the equation is written in a form such as $y - 5 = 3(x - 4)^2$. This form is similar to the point-slope form of the linear function equation, except that the (x – 4) is squared.

Standard Quadratic Equation

a) Transform the above quadratic function to the form $y = ax^2 + bx + c$.

$y-5 = 3(x-4)^2 \Rightarrow y-5 = 3(x-4)(x-4)$

$y-5 = 3(x^2-4x-4x+16)$

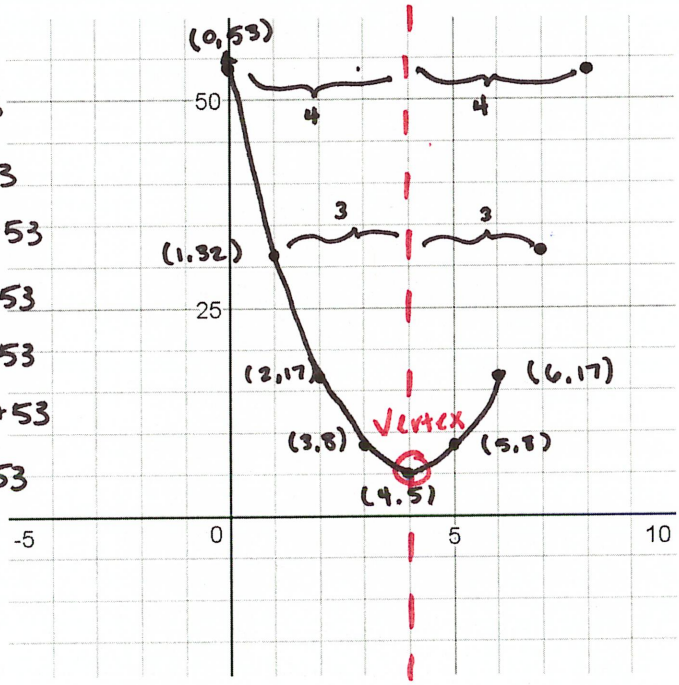
$y-5 = 3(x^2-8x+16)$

$y-5 = 3x^2-24x+48 \Rightarrow y = 3x^2-24x+53$

(+5 on both sides)

b) Complete the table for the above equation, and use these points to plot the parabola on the axes.

X	Y	
0	53	$= 3(0)^2-24(0)+53$
1	32	$= 3(1)^2-24(1)+53$
2	17	$= 3(2)^2-24(2)+53$
3	8	$= 3(3)^2-24(3)+53$
4	5	$= 3(4)^2-24(4)+53$
5	8	$= 3(5)^2-24(5)+53$
6	17	$= 3(6)^2-24(6)+53$



c) Mark the vertex on the graph, and write down the ordered pair.

(4, 5)

d) Where do the coordinates of the vertex show up in the equation? $\Rightarrow y-5 = 3(x-4)^2$

x-coordinate (opposite x=4)

y-coordinate (opposite y=5)

e) How could you find the values of y for x = 7, and x = 8 without substituting these numbers into the equation? ★ Use the Axis of Symmetry ★

x=7 ; y=32 x=8 ; y=53

From the previous activity, you saw that equations in the form $y - k = a(x - h)^2$ were easy to identify the coordinates of the vertex. This form is called **vertex form → $y - k = a(x - h)^2$.** Vertex point (K, h)

We will now learn how to transform an equation from $y = ax^2 + bx + c$ form into vertex form. We will also learn how to graph it by finding the y-intercept, the vertex, the axis of symmetry, and the x-intercept(s).

Example: Consider the function $y = x^2 + 10x + 21$.

a) Identify the y-intercept. ($x = 0$)

$y = (0)^2 + 10(0) + 21$

$y = 21$ (0, 21)

b) Transform the equation to vertex form by completing the square. (pg. 2 for steps)

① $y = x^2 + 10x + 21$ (−21, −21)

ALWAYS the same #

② does not need to be done because $a = 1$

$y - 21 = x^2 + 10x$

③ $\frac{10}{2} = (5)^2 = 25$

$y - 21 + 25 = x^2 + 10x + 25$

$(x+5)(x+5)$

④ $y + 4 = (x+5)^2$

y-coordinate; x-coordinate: $x = -5$

$y = -4$ Vertex form

c) Identify the coordinates of the vertex. → (x, y)

(−5, −4)

d) Identify the equation for the axis of symmetry → x = ___

$x = -5$

e) Identify the x-intercept(s). ($y = 0$)

$y + 4 = (x+5)^2$

⑤ $0 + 4 = (x+5)^2$

$\sqrt{4} = \sqrt{(x+5)^2}$

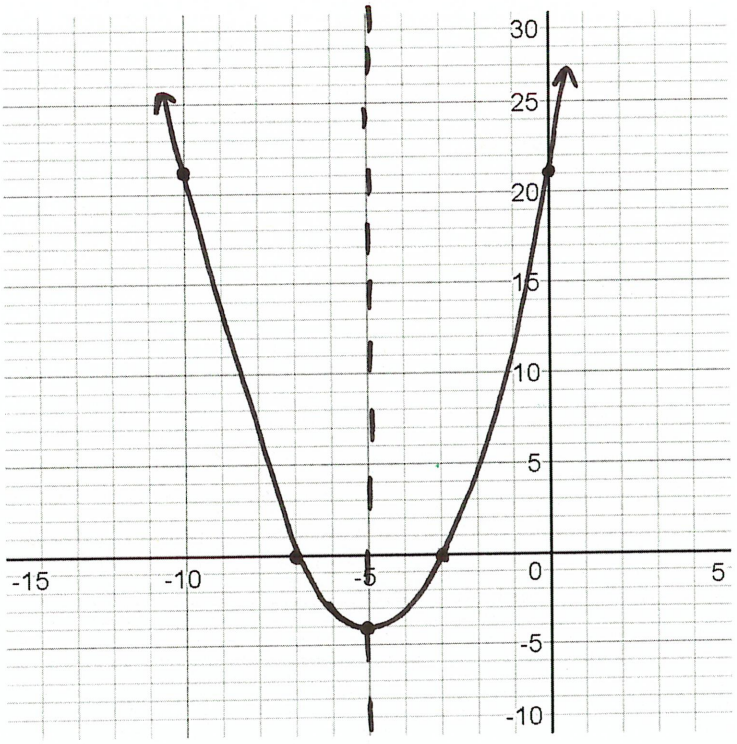
$\pm 2 = x + 5$

$-5 \quad -5$

$x = -5 + 2$ $x = -5 - 2$

$x = -3$ (−3, 0) $x = -7$ (−7, 0)

f) Graph



Break for Practice: Use the method of completing the square to find each of the requested pieces of information, and graph the parabola.

$y = x^2 + 6x + 13$

$-13 \quad -13$

$y - 13 + 9 = x^2 + 6x + 9$

$\frac{6}{2} = (3)^2 = 9$

$y - 4 = (x+3)^2$ Vertex Form

$*y = 0$

$0 - 4 = (x+3)^2$

$\sqrt{-4} = \sqrt{(x+3)^2}$

$\pm 2i = x + 3$

Imaginary Root means NO X-INTERCEPTS

$(x=0)$

y-intercept

$(0, 13)$

Vertex

$(-3, 4)$

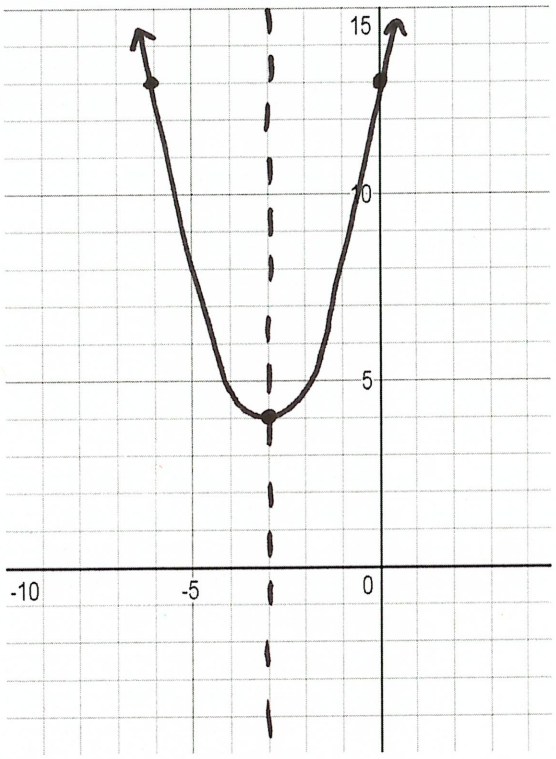
Axis of Sym.

$x = -3$

$(y=0)$

x-intercept(s)

None



Extended Practice: Use the method of completing the square to find each of the requested pieces of information, and graph the parabola.

1. $y = x^2 - 6x + 8$

y-intercept

$(0, 8)$

Vertex

$(3, -1)$

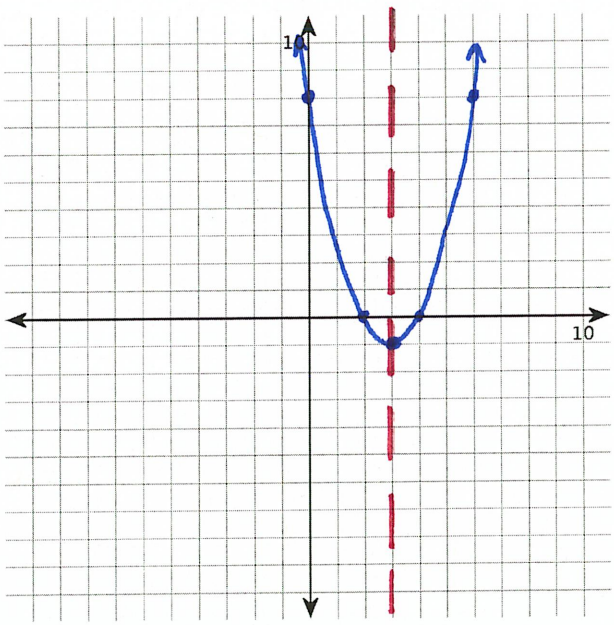
Axis of Sym.

$x = 3$

x-intercept(s)

$(4, 0)$

$(2, 0)$



2. $y = x^2 - 8x + 7$

y-intercept

(0, 7)

Vertex

(4, -9)

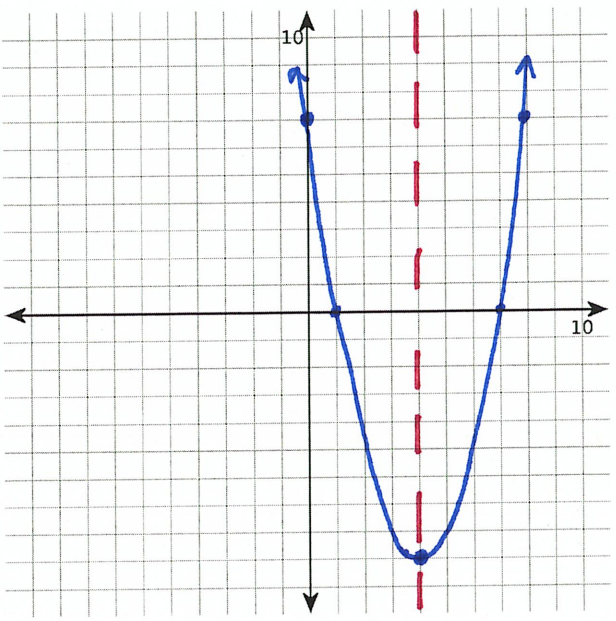
Axis of Sym.

$x = 4$

x-intercept(s)

(7, 0)

(1, 0)



3. $y = x^2 + 4x + 8$

y-intercept

(0, 8)

Vertex

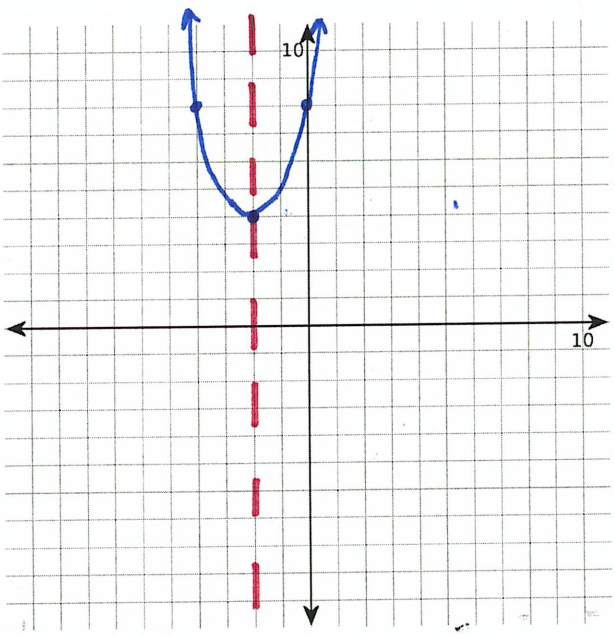
(-2, 4)

Axis of Sym.

$x = -2$

x-intercept(s)

none



Use Step #2 from Completing the Square

Next we should see what we need to adjust to work with equations that have a coefficient in front of the x^2

Break for Practice: Complete the following.

1. $y = 3x^2 + 30x + 72$

$-72 \quad -72$

$y - 72 = 3x^2 + 30x$

$\frac{}{3} \quad \frac{}{3}$

$y - 72 + 75 = 3(x^2 + 10x + 25)$

$\frac{10}{2} = (5)^2 = 25$

$y + 3 = 3(x+5)^2$

Vertex Form

$y = 0$

$\frac{3}{3} = \frac{3(x+5)^2}{3}$

$\sqrt{1} = \sqrt{(x+5)^2}$

$\pm 1 = x + 5$

$-5 \quad -5$

$x = 1 - 5$

$x = -4$

$x = -1 - 5$

$x = -6$

($x = 0$) y-intercept

$(0, +72)$

Vertex

$(-5, -3)$

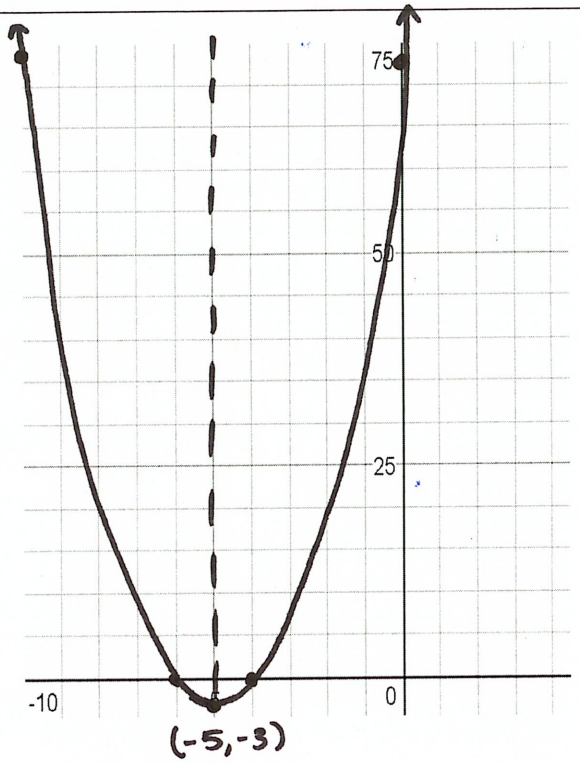
Axis of Sym.

$x = -5$

($y = 0$) x-intercept(s)

$(-4, 0)$

$(-6, 0)$



2. $y = -3x^2 + 12x - 42$

$+42 \quad +42$

$y + 42 = -3x^2 + 12x$

$\frac{}{-3} \quad \frac{}{-3}$

$y + 42 + (-12) = -3(x^2 - 4x + 4)$

$\frac{-4}{2} = (-2)^2 = 4$

$y + 30 = -3(x-2)^2$

Vertex Form

$y = 0$

$\frac{30}{-3} = \frac{-3(x-2)^2}{-3}$

$\sqrt{-10} = \sqrt{(x-2)^2}$

$\pm i\sqrt{10} = x - 2$

↑ Imaginary Root means NO X-INTERCEPTS

($x = 0$) y-intercept

$(0, -42)$

Vertex

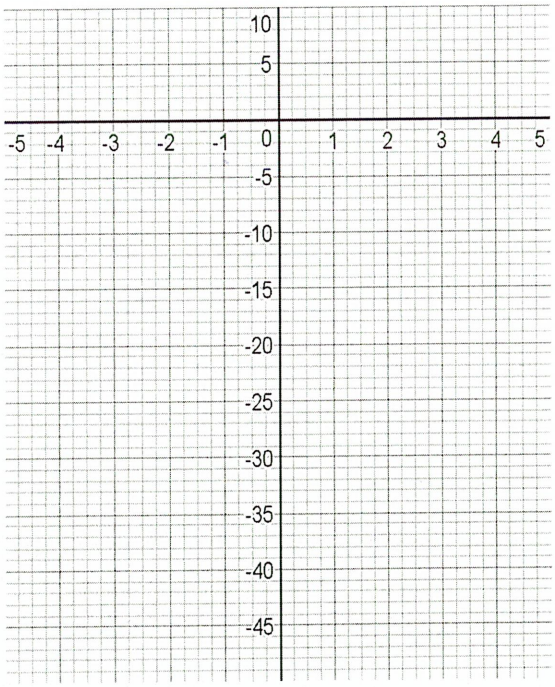
$(2, -30)$

Axis of Sym.

$x = 2$

($y = 0$) x-intercept(s)

None



Extended Practice: Complete the following.

1. $y = 2x^2 + 12x + 10$

y-intercept

(0, 10)

Vertex

(-3, -8)

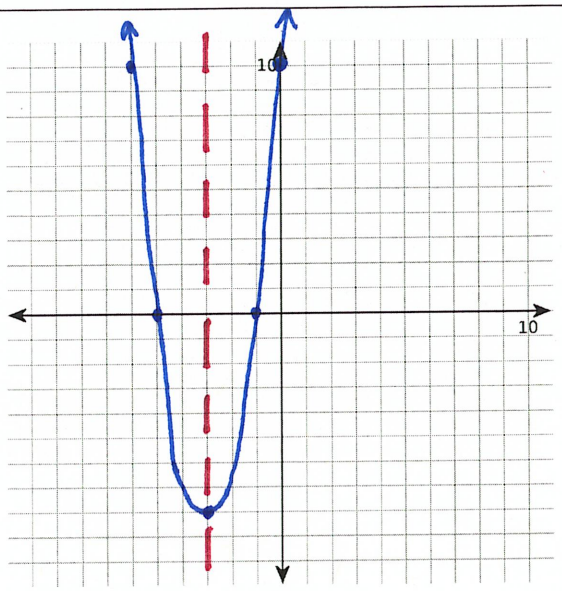
Axis of Sym.

$x = -3$

x-intercept(s)

(-1, 0)

(-5, 0)



2. $y = -x^2 + 4x - 3$

y-intercept

(0, -3)

Vertex

(2, 1)

Axis of Sym.

$x = 2$

x-intercept(s)

(3, 0)

(1, 0)