4. A 5 inch by 7inch photograph is surrounded by a frame of uniform width. The area of the frame equals the area of the photograph. Find the width of the frame.

$X = 1.21$ in

The Discriminant

In certain situations, it is not necessary to actually solve a quadratic equation, but it is useful to know what kind of solutions (roots) it has. To do this, you need to be able to use the discriminant test.

Consider the following four problems. Each one gives a different type of solution. Solve each with the quadratic formula, and then try to figure out what part of the quadratic formula determines the nature of the solutions.

1. $2x^2 - 8x + 8 = 0$ $a=2, b=-8, c=8$

$$X = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(8)}}{2(2)}$$

$$X = \frac{8 \pm \sqrt{0}}{4}$$ Discriminant $=0$

Result: 1 real

double root (same answer twice)

$$X = \frac{8+0}{4} \qquad X = \frac{8-0}{4}$$

$X = 2$ $X = 2$

2. $x^2 - x - 12 = 0$ $a=1, b=-1, c=-12$

$$X = \frac{1 \pm \sqrt{(1)^2 - 4(1)(-12)}}{2(1)}$$

$$X = \frac{1 \pm \sqrt{49}}{2}$$ Discriminant is a positive, perfect square

Result: 2 real, rational roots

$$X = \frac{1+7}{2} \qquad X = \frac{1-7}{2}$$

$X = 4$ $X = -3$

3. $x^2 - 2x + 2 = 0$ $a=1, b=-2, c=2$

$$X = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$X = \frac{2 \pm \sqrt{-4}}{2}$$ Discriminant is negative

Result: 2 imaginary roots

$$X = \frac{2+2i}{2} \qquad X = \frac{2-2i}{2}$$

$X = 1+i$ $X = 1-i$

4. $x^2 - 4x - 3 = 0$ $a=1, b=-4, c=-3$

$$X = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$X = \frac{4 \pm \sqrt{28}}{2}$$ $\sqrt{28} = \sqrt{4}\,\sqrt{7}$ Discriminant is a positive, NOT perfect square

Result: 2 real, irrational roots

$$X = \frac{4+2\sqrt{7}}{2} \qquad X = \frac{4-2\sqrt{7}}{2}$$

$X = 2+\sqrt{7}$ $X = 2-\sqrt{7}$

Result: The Discriminant, D = b^2-4ac

D is negative

a) If $D < 0$, then there are 2 imaginary roots

b) If $D = 0$, then there is 1 real double root

D is positive

c) If $D > 0$ and a perfect square, then there are 2 real, rational root

D is positive

d) If $D > 0$ but not a perfect square, then there are 2 real, irrational root

Beak for Practice: Use the discriminant to identify the type of solutions for each equation. $D=b^2-4ac$

1. $x^2 + 10x + 25 = 0$ $a=1, b=10, c=25$

$D=(10)^2-4(1)(25)$

$D=0$; 1 real double root

2. $5x^2 - 6x + 2 = 0$ $a=5, b=-6, c=2$

$D=(-6)^2-4(5)(2)$

$D=-4$; 2 imaginary roots

3. $3x^2 - 4x - 7 = 0$ $a=3, b=-4, c=-7$

$D=(-4)^2-4(3)(-7)$

$D=100$; 2 real, rational roots

4. $\frac{1x^2}{6} + 1 = x$ ← MUST be set = 0 first

$-x \quad -x$

$a=\frac{1}{6}, b=-1, c=1$

$\frac{1}{6}x^2 - x + 1 = 0$

$D=(-1)^2-4(\frac{1}{6})(1)$

$D=\frac{1}{3}$ OR $0.\overline{3}$; 2 real, irrational roots

5. Find the value(s) of k for which the equation $2x^2 + 4x + k = 0$ has the following.

$a=2, b=4, c=k$

a) One real double root.

happens when discriminant = 0

$b^2-4ac=0$

$(4)^2-4(2)(k)=0$

$16-8k=0$

$-16 \quad -16$

$\frac{-8k}{-8}=\frac{-16}{-8}$

$k=2$

b) Two different real roots

happens when discriminant is positive

$b^2-4ac>0$

$(4)^2-4(2)(k)>0$

$16-8k>0$

$-16 \quad -16$

$\frac{-8k}{-8}>\frac{-16}{-8}$

Flip

$k<2$

c) Two imaginary roots

happens when discriminant is negative

$b^2-4ac<0$

$(4)^2-4(2)(k)<0$

$16-8k<0$

$-16 \quad -16$

$\frac{-8k}{-8}<\frac{-16}{-8} \rightarrow k>2$

Flip

Extended Practice:

Use the discriminant to identify the type of solutions for each equation.

1. $x^2 + 3x - 9 = 0$ D= 45 2 real, irrational roots	2. $x^2 - 4x - 5 = 0$ D=36 2 real, rational roots	3. $t^2 + 8t + 20 = 0$ D=-16 2 imaginary roots
4. $3m^2 - 8m - 5 = 0$ D= 124 2 real, irrational roots	5. $2y^2 - 9y + 3 = 0$ D= 57 2 real, irrational roots	6. $5t^2 - 4t + 3 = 0$ D= -44 2 imaginary roots
7. $z^2 + \frac{5}{4} = z$ D= -4 2 imaginary roots.	8. $\frac{r^2}{4} + 1 = r$ D=0 1 real, double root	

Solve each equation using whichever method seems easiest to you.(Factoring, completing the Square or Quadratic Formula)

9. $x^2 - 6x + 5 = 0$ x=5, x=1	10. $y^2 + 2y - 24 = 0$ y=-6, y=4

11. $5(x+7)^2 = 0$

$X = -7 \quad X = -7$

12. $5(x+7)^2 = 25$

$X = -7 \pm \sqrt{5}$

13. $(2x+5)(X-3) = 0$

$X = -5/2 \quad X = 3$

14. Find the value(s) of k for which the equation $3x^2 - 6x - k = 0$ has the following.

a) One real double root.

$K = -3$

b) Two different real roots

$K > -3$

c) Two imaginary roots

$K < -3$