Applications with Quadratic Equations

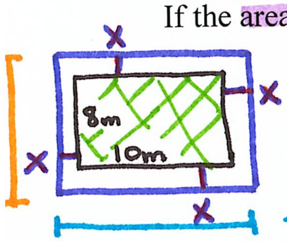
You can use... factoring
completing the square
quadratic formula

$* \; x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} \; *$

Now it's time to see where quadratic equations might be used.

Break for Practice: Solve

1. A flower garden with dimensions of 8 meters by 10 meters is enclosed by a walkway of uniform width. If the area of the walkway is 40 square meters, then what is the width of the path?



Total Area $= lw \Rightarrow 120 = (2x+10)(2x+8)$

$l = x + 10 + x$ $\quad w = x + 8 + x$

$l = 2x + 10$ $\quad w = 2x + 8$

-120

$0 = 4x^2 + 16x + 20x + 80 - 120$

$0 = 4x^2 + 36x - 40 \quad a=4,\ b=36,\ c=-40$

$A = 40\ m^2$ (walkway)
$A = 80\ m^2$ (garden)
Total Area

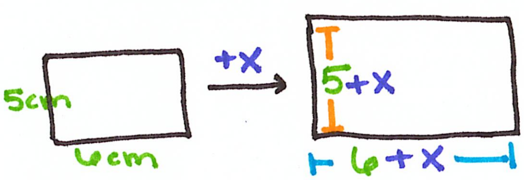
$x = \dfrac{-36 \pm \sqrt{(36)^2 - 4(4)(-40)}}{2(4)}$

$x = \dfrac{-36 \pm \sqrt{1936}}{8} \rightarrow x = \dfrac{-36+44}{8}$ $\quad x = 1m$ ← width of walkway

$\searrow x = \dfrac{-36-44}{8}$ $\quad x = \cancel{-10}m$

2. A rectangle is 6 cm long and 5 cm wide. When each dimension is increased by x cm, the area is tripled. Find the value of x.

Addition $\quad$ multiplication



New Area $= l \cdot w \Rightarrow 90 = (6+x)(5+x)$

-90

$0 = 30 + 6x + 5x + x^2 - 90$

$0 = x^2 + 11x - 60 \quad a=1,\ b=11,\ c=-60$

$A = 30\ cm^2$ (original)
$A = 3(30)$
$A = 90\ cm^2$ (new)

$l = 6 + x$

$w = 5 + x$

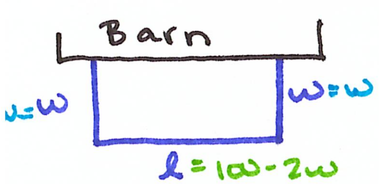
$x = \dfrac{-11 \pm \sqrt{(11)^2 - 4(1)(-60)}}{2(1)}$

$x = \dfrac{-11 \pm \sqrt{361}}{2} \rightarrow x = \dfrac{-11+19}{2}$ $\quad x = 4cm$ ← Amount the original rectangle was increased by.

$\searrow x = \dfrac{-11-19}{2}$ $\quad x = \cancel{-15}\ cm$

3. A rectangular animal pen with an area of 1200 square meters has one side along a barn. The other three sides are enclosed by 100 meters of fencing. Find the dimensions of the pen.

Area $\quad$ perimeter $\quad$ both w and l



$A = l \cdot w \Rightarrow 1200 = (100-2w)w$

-1200

$0 = 100w - 2w^2 - 1200$

$0 = -2w^2 + 100w - 1200$

$a=-2 \quad b=100 \quad c=-1200$

$A = 1200\ m^2$

$P = w + l + w$

$100 = 2w + l$ (Solve for 1 variable)
$-2w \quad -2w$

$l = 100 - 2w$

$w = \dfrac{-100 \pm \sqrt{(100)^2 - 4(-2)(-1200)}}{2(-2)}$

$w = \dfrac{-100 \pm \sqrt{400}}{-4} \rightarrow w = \dfrac{-100+20}{-4}$ $\quad w: 20m \times 60m$

$\searrow w = \dfrac{-100-20}{-4}$ $\quad w = 30m \times 40m$

Found w, but we also need to find l.

$\star\ A = l \cdot w\ \star$

$\dfrac{1200}{20} = \dfrac{l \cdot 20}{20}$

$l = 60$

$\dfrac{1200}{30} = \dfrac{l \cdot 30}{30}$

$l = 40$

Extended Practice: Solve

1. Each side of a square is 4 meters long. When each side is increased by x meters, the area is doubled. Find the value of x.

$x = 1.66m$

2. A rectangular field with area 5000 square meters is enclosed by 300 meters of fencing. Find the dimensions of the field.

50m x 100m OR 100m x 50m

3. A walkway of uniform width has area 72 meters squared and surrounds a swimming pool that is 8 meters wide and 10 meters long. Find the width of the walkway.

$x = 1.68m$

4. A 5 inch by 7inch photograph is surrounded by a frame of uniform width. The area of the frame equals the area of the photograph. Find the width of the frame.

$X = 1.21$ in

The Discriminant

In certain situations, it is not necessary to actually solve a quadratic equation, but it is useful to know what kind of solutions (roots) it has. To do this, you need to be able to use the discriminant test.

Consider the following four problems. Each one gives a different type of solution. Solve each with the quadratic formula, and then try to figure out what part of the quadratic formula determines the nature of the solutions.

1. $2x^2 - 8x + 8 = 0$ $a=2, b=-8, c=8$

$$X = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(8)}}{2(2)}$$

$$X = \frac{8 \pm \sqrt{0}}{4}$$ Discriminant $=0$

$X = \frac{8+0}{4}$ $X = \frac{8-0}{4}$

$X = 2$ $X = 2$

Result: 1 real double root (same answer twice)

2. $x^2 - x - 12 = 0$ $a=1, b=-1, c=-12$

$$X = \frac{1 \pm \sqrt{(1)^2 - 4(1)(-12)}}{2(1)}$$

$$X = \frac{1 \pm \sqrt{49}}{2}$$ Discriminant is a positive, perfect square

$X = \frac{1+7}{2}$ $X = \frac{1-7}{2}$

$X = 4$ $X = -3$

Result: 2 real, rational roots

3. $x^2 - 2x + 2 = 0$ $a=1, b=-2, c=2$

$$X = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$X = \frac{2 \pm \sqrt{-4}}{2}$$ Discriminant is negative

$X = \frac{2+2i}{2}$ $X = \frac{2-2i}{2}$

$X = 1+i$ $X = 1-i$

Result: 2 imaginary roots

4. $x^2 - 4x - 3 = 0$ $a=1, b=-4, c=-3$

$$X = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$X = \frac{4 \pm \sqrt{28}}{2}$$ $\sqrt{4}$ $\sqrt{7}$ Discriminant is a positive, NOT perfect square

$X = \frac{4+2\sqrt{7}}{2}$ $X = \frac{4-2\sqrt{7}}{2}$

$X = 2+\sqrt{7}$ $X = 2-\sqrt{7}$

Result: 2 real, irrational roots