Completing the Square

In this unit, we will be investigating quadratic equations and functions. Just like all linear functions could be written in the form $= mx + b$, all quadratic functions can be written in the form:

$$y = ax^2 + bx + c, \;\; \boldsymbol{where\ a, b, and\ c \in reals.}$$

In this unit, we will learn how to solve, graph, and apply quadratic functions. The first technique that we will learn in this unit for solving quadratics is called completing the square. First we should consider quadratics that you already know how to solve.

Break for Practice: Solve

1. $\sqrt{x^2} = \sqrt{25}$

   $x = \pm 5$

2. $\sqrt{x^2} = \sqrt{12}$ $\sqrt{4}$ $\sqrt{3}$

   $x = \pm 2\sqrt{3}$

3. $\sqrt{(x-2)^2} = \sqrt{5}$

   $x - 2 = \pm\sqrt{5}$

   $+2 \quad +2$

   $x = 2 \pm \sqrt{5}$

4. $\sqrt{(5x+4)^2} = \sqrt{-36}$

   $5x + 4 = \pm 6i$

   $-4 \quad -4$

   $\frac{5x}{5} = \frac{-4 \pm 6i}{5}$

   $x = \frac{-4}{5} \pm \frac{6i}{5}$

If we can transform a quadratic into the above form, then we can solve it. This needed technique is called **completing the square.**

Example: Solve by completing the square.

$x^2 + 12x - 45 = 0$

$+45 \quad +45$

① $\frac{1x^2}{1} + \frac{12x}{1} = \frac{45}{1}$

② $x^2 + 12x + \underline{36} = 45 + \underline{36}$

③ $\frac{12}{2} = (6)^2 = 36$

④ $\sqrt{(x+6)^2} = \sqrt{81}$

⑤ $x + 6 = \pm 9$

$-6 \quad -6$

$x = 9 - 6 \qquad x = -9 - 6$

$x = 3 \qquad x = -15$

Steps:

1. Isolate the constant on one side.
2. Divide through by the coefficient of x^2 and add a blank to both sides.
3. Add to both sides (fill in the blanks) the square of half of the coefficient of x.
4. Write in the form $(x \pm q)^2 = r$.
5. Solve.

Note: The Ancient Babylonians knew how to do all of this with pictures!

Break for Practice: Solve by completing the square.

1. $2x^2 - 8x - 12 = 0$

$+12 \quad +12$

① $\frac{2x^2}{2} - \frac{8x}{2} = \frac{12}{2}$

② $x^2 - \underbrace{4}x + \underline{4} = 6 + \underline{4}$

③ $\frac{-4}{2} = (-2)^2 = 4$

④ $\sqrt{(x-2)^2} = \sqrt{10}$

⑤ $x - 2 = \pm\sqrt{10}$

$+2 \quad +2$

$x = 2 \pm \sqrt{10}$

2. $3x^2 - 10 = 12x$

$-12x \quad +10 \quad -12x \quad +10$

① $\frac{3x^2}{3} - \frac{12x}{3} = \frac{10}{3}$

② $x^2 - \underbrace{4}x + \underline{4} = \frac{10}{3} + \underline{4} \rightarrow \frac{12}{3}$

③ $\frac{-4}{2} = (-2)^2 = 4$

④ $\sqrt{(x-2)^2} = \frac{\sqrt{22} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}}$

⑤ $x - 2 = \pm\frac{\sqrt{66}}{3}$

$+2 \quad +2$

$x = 2 \pm \frac{\sqrt{66}}{3}$

Extended Practice:

1. Solve each equation.

a.) $x^2 = 3$ $x = \pm\sqrt{3}$	b.) $(x-1)^2 = 3$ $x = 1 \pm \sqrt{3}$	c.) $(2x-1)^2 = 3$ $x = \frac{1 \pm \sqrt{3}}{2}$ or $x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$
d.) $x^2 = -4$ $x = \pm 2i$	e.) $(x+7)^2 = -4$ $x = -7 \pm 2i$	f.) $(2x+7)^2 = -4$ $x = -\frac{7}{2} \pm i$ OR $x = \frac{-7 \pm 2i}{2}$

2. Solve by completing the square.

a.) $x^2 - 2x - 5 = 0$ $x = 1 \pm \sqrt{6}$	b.) $y^2 + 6y - 2 = 0$ $y = -3 \pm \sqrt{11}$
c.) $t^2 + 8 = 4t$ $t = 2 \pm 2i$	d.) $3n^2 + 12n = -1$ $n = -2 \pm \dfrac{\sqrt{33}}{3}$

The Quadratic Formula

In this section we will learn a more efficient method for solving quadratic equations. It is a formula derived from the method of completing the square.

Quadratic Formula: If $ax^2 + bx + c = 0$, then

opposite
NOT negative

$$x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$

AKA: equation MUST be set equal to 0.

Note – In order to use this formula, the problem needs to be in the form $ax^2 + bx + c = 0$.