Chapter 11.6: Areas of Regular Polygons

Center of a polygon: The center of a polygon is the center of its circumscribed circle.

Example: Point P

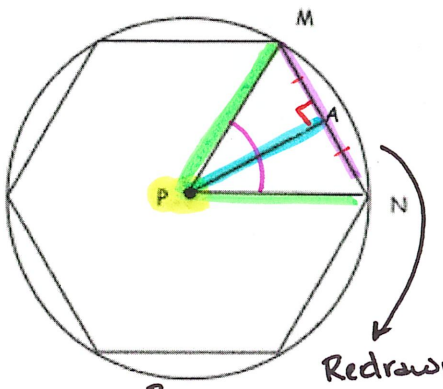
Radius of a polygon: The radius of a polygon is the radius of its circumscribed circle.

Example: $\overline{MP}$ and $\overline{NP}$

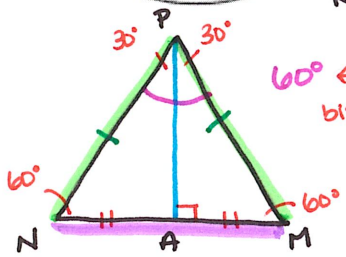
* Connects the center to the angle *

Apothem of a polygon: The distance from the center to any side of the polygon. → @ the side lengths midpoint @ a 90° angle

Example: $\overline{AP}$



Redrawn



60° ← Apothem bisects central angle

Central angle of a regular polygon: A central angle of a regular polygon is an angle formed by two radii drawn to consecutive vertices of the polygon.

Example: ∠MPN

Ex. $m\angle MPN = \frac{360°}{6} = 60°$

* can be found by $\frac{360°}{n}$ n = # of sides

Identify △ *MPN* by its sides.

Radii → $\overline{NP}$ and $\overline{MP}$

Side Length → $\overline{NM}$

How does the apothem relate to △ *MPN*?

It's the altitude

$\angle MPA \cong \angle NPA$ and $\overline{MA} \cong \overline{NA}$

Example #1: In the diagram, ABCDEF is a regular hexagon inscribed in ⊙*G*. If DE= 8cm, find each of the following.

↳ All ∠'s and sides ≅

a.) $m\angle EGF =$ $\frac{360°}{6} = 60°$

* Central ∠ *

b.) $m\angle EGH =$ $\frac{60°}{2} = 30°$

c.) $m\angle HEG =$ $180° - 90° - 30° = 60°$

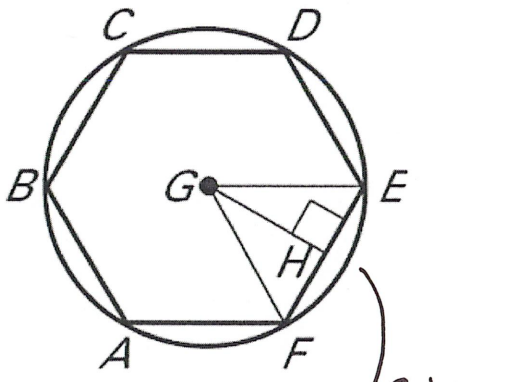
d.) FE = 8 cm

e.) HE = $\frac{8}{2} = 4$ cm

f.) GH = $4\sqrt{3}$ cm OR

$4 \cdot \tan 60° = \frac{a}{4} \cdot 4$

$a = 6.93$ cm



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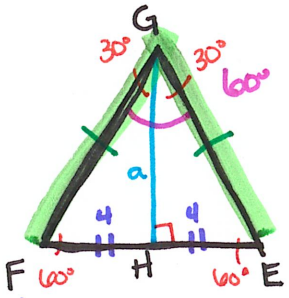


8 cm

g.) What is the perimeter of the hexagon?

← length of each side

$P = 8(6)$ ← # of sides

$P = 48$ cm

Example #2: Find the measure of a **central angle** of a regular polygon with the given number of sides.

→ $\frac{360°}{n}$

a.) 9 sides

$\frac{360°}{9} = 40°$

b.) 15 sides

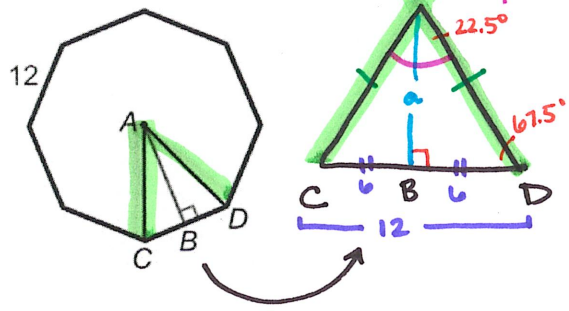
$\frac{360°}{15} = 24°$

c.) 30 sides

$\frac{360°}{30} = 12°$

Example #3: Find the length of the apothem in the regular octagon. Round your answer to the nearest hundredth.

→ 8 sides



Redrawn

$m\angle CAD = \frac{360°}{8}$

$m\angle CAD = 45°$

$6 \cdot \tan 67.5° = \frac{a}{6} \cdot 6$

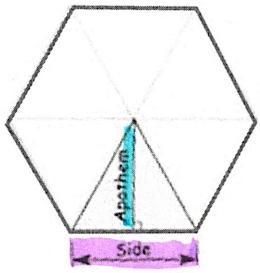
$a = 14.49$ units

Area of a Regular Polygon (Theorem 11.11):

The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P

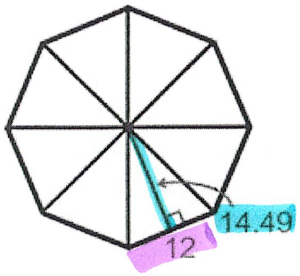
↳ side length • # of sides ($P = s \cdot n$)

$A = \frac{1}{2} a \cdot P$ OR $A = \frac{aP}{2}$



Example # 4: Find the area of the regular octagon. Round your answer to the nearest hundredth.

↳ 8 sides



Regular Polygon: $A = \frac{1}{2} \cdot a \cdot P \Rightarrow A = \frac{1}{2}(14.49)(96)$

$a = 14.49$ units

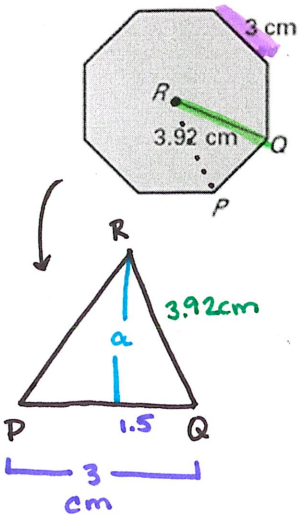
$P = 12 \cdot 8$

$P = 96$ units

$A = 695.52 \text{ units}^2$

→ 8 sides

Example #5: A wooden coaster is a regular octagon with 3 cm sides and a radius of about 3.92 cm. What is the area of the coaster? Round your answer to the nearest hundredth.



Regular Polygon: $A = \frac{1}{2} \cdot a \cdot P \Rightarrow A = \frac{1}{2}(3.62)(24)$

$a^2 + 1.5^2 = 3.92^2$ $\quad P = 3(8)$

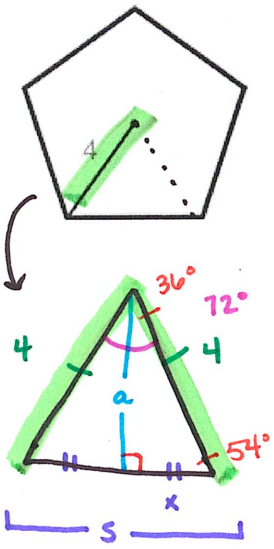
$\sqrt{a^2} = \sqrt{13.1164}$ $\quad P = 24$ cm

$a = 3.62$ cm

$A = 43.44$ cm^2

→ 5 sides

Example #6: Find the area of the regular pentagon with radius 4. Round your answer to the nearest hundredth.



Regular Polygon: $A = \frac{1}{2} \cdot a \cdot P \Rightarrow A = \frac{1}{2}(3.24)(23.5)$

Central Angle: $\frac{360°}{5} = 72°$

$A = 38.07$ units^2

apothem: $4 \cdot \sin 54° = \frac{a}{4} \cdot 4$

$a = 3.24$ units

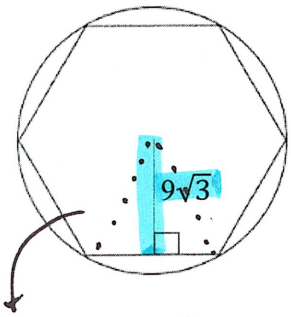
Perimeter: $4 \cdot \cos 54° = \frac{x}{4} \cdot 4$ $\quad P = 4 \cdot 1.5$

$x = 2.35$ $\quad P = 23.5$ units

$s = 2(2.35) = 4.7$

Example #7: Find the area of the inscribed hexagon. Round your answer to the nearest hundredth.

↳ 6 sides

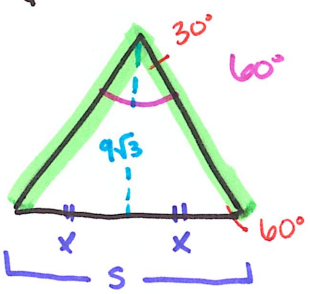


Regular Polygon: $A = \frac{1}{2} \cdot a \cdot P \Rightarrow A = \frac{1}{2} \cdot (9\sqrt{3})(108)$

$a = 9\sqrt{3}$ units

$A = 841.78$ units^2

Central Angle: $\frac{360°}{6} = 60°$



$x = \frac{9\sqrt{3}}{\sqrt{3}}$ OR $x \cdot \tan 60° = \frac{9\sqrt{3}}{x} \cdot x$

$x = 9$ $\quad x \cdot \frac{\tan 60°}{\tan 60°} = \frac{9\sqrt{3}}{\tan 60°}$ $\quad P = 18 \cdot 6$

$x = 9$ $\quad P = 108$ units

$s = 2(9) = 18$ units

Finding Lengths in a Regular N-gon

To find the area of a regular n-gon with radius r, you may need to first find the apothem a or the side length s.

You can use...	...when you know n and...	Example(s) to Reference
$a^2 + b^2 = c^2$	Need to Know 2 side lengths. (r and a OR r and s)	Example # 5
SoH-CAH-ToA	Need to only Know one side length * Need to find central angle	Example # 6
Special Right Δ's 30°-60°-90° or 45°-45°-90°	Need to Know only one side length * Need to find central angle	Example #7

Chapter 11.7: Use Geometric Probability

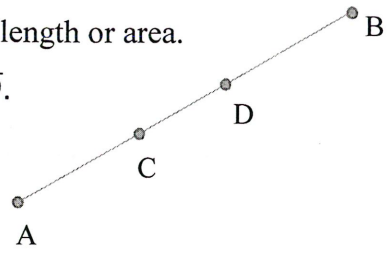
Probability: the likelihood that an event will occur.

$$\text{Probability} = \frac{\text{\# of favorable outcomes (what you want to happen)}}{\text{\# of possible outcomes}}$$

$P = 0$	$P = 0.25$	$P = 0.50$	$P = 0.75$	$P = 1$
0%	25%	50%	75%	100%
Impossible	Unlikely	Equally likely to occur or not occur	Likely	Certain

Geometric Probability: A ratio that involves a geometric measure such as length or area.

Probability and Length: Let $\overline{AB}$ be a segment that contains the segment $\overline{CD}$. If a point K on $\overline{AB}$ is chosen at random, then the probability that it is on $\overline{CD}$ is the ratio of the length of $\overline{CD}$ to the length of $\overline{AB}$.



$$P(K \text{ is on } \overline{CD}) = \frac{\text{length of } \overline{CD} \text{ (what you want to happen/land)}}{\text{length of } \overline{AB} \text{ (Total distance)}}$$

Probability and Area: Let J be a region that contains region M. If a point K in J is chosen at random, then the probability that it is in region M is that ratio of the area of M to the area of J.

$$P(K \text{ is in region } M) = \frac{\text{Area of M (what you want to happen/land)}}{\text{Area of J (Total Area)}}$$