Not parallelograms
↳ they do not have 2 sets of parallel sides.

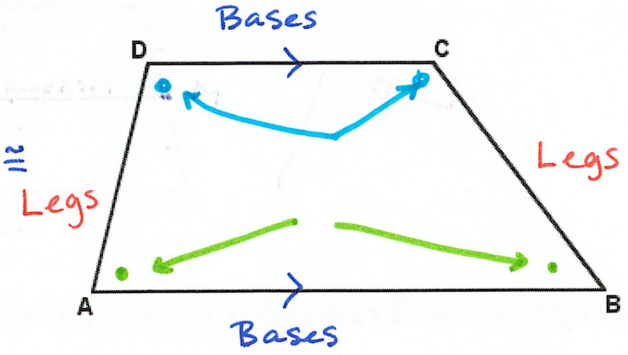
Chapter 8.5: Use Properties of Trapezoids and Kites

Trapezoid: a quadrilateral with exactly one pair of parallel sides.

Bases: Parallel sides

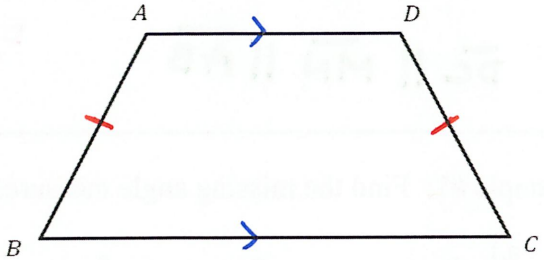
Base Angles: Two pair ⇒ not necessarily ≅
(∠C and ∠D, ∠A and ∠B)

Legs: Non-parallel sides



Isosceles Trapezoid:

A trapezoid with congruent legs.



Isosceles Trapezoid Theorem (Theorem 8.14):

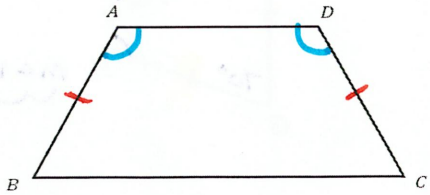
If a trapezoid is isosceles, then each pair of base angels are congruent

∠A ≅ ∠D ∠B ≅ ∠C

Trapezoid with Congruent Base Angles Theorem (Theorem 8.15):

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid

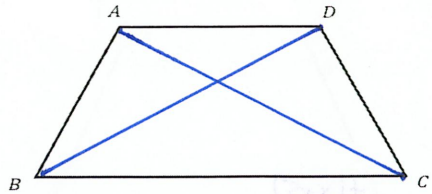
If ∠A ≅ ∠D, then ABCD is an isosceles trapezoid



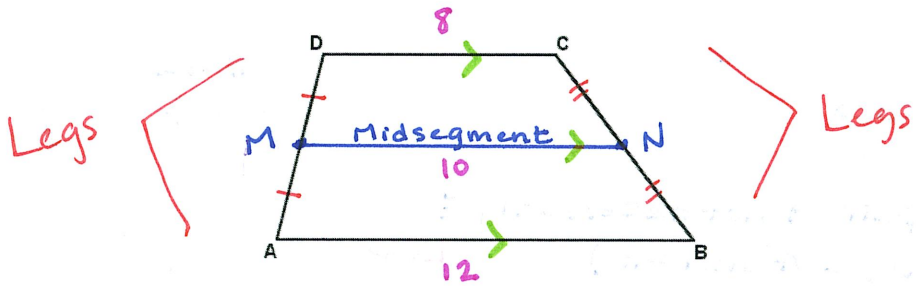
Trapezoid with Congruent Diagonals Theorem (Theorem 8.16):

A trapezoid is isosceles iff its diagonals are congruent.

If $\overline{AC} \cong \overline{BD}$, then ABCD is an isosceles trapezoid.



Midsegment of a Trapezoid: The segment that connects the midpoints of its legs.

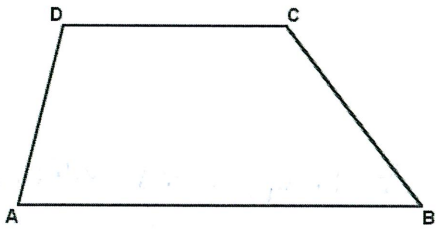


Midsegment for Trapezoids Theorem (Theorem 8.17):

Midsegment of a trapezoid is parallel to each base and its length is 1/2 the sum of the 2 bases.

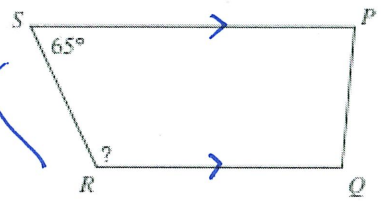
$\overline{DC} \parallel \overline{MN} \parallel \overline{AB}$

$\frac{DC + AB}{2} = MN$



Example #1: Find the missing angle measures in the trapezoids below.

a.)

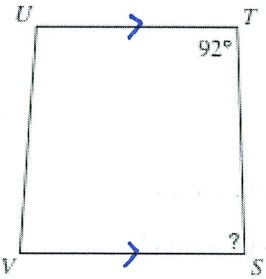


+UP to 180°

$m\angle R = 180° - 65°$

$m\angle R = 115°$

b.)



$m\angle S = 180° - 92°$

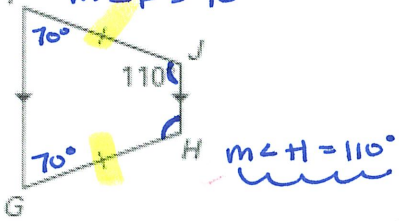
$m\angle S = 88°$

Example #2: ~~Find~~ $m\angle F$, $m\angle G$, and $m\angle H$

a.)

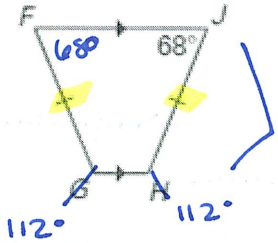
$\angle F = 180° - 110°$

$m\angle F = 70°$



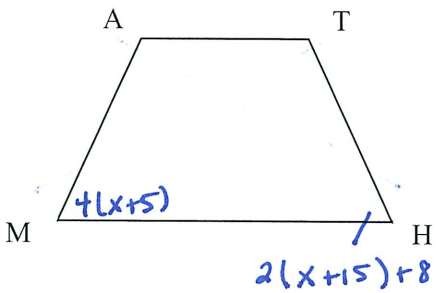
$m\angle H = 110°$

b.)



$m\angle H = 180° - 68°$

Example #3: In isosceles trapezoid MATH, the $m\angle M = 4(x + 5)°$ and $m\angle H = 2(x + 15) + 8°$. Find the measure of all 4 angles of the trapezoid.



$4(x+5) = 2(x+15) + 8$

$4x + 20 = 2x + 30 + 8$

$-2x \quad -2x$

$2x + 20 = 38$

$-20 \quad -20$

$\frac{2x}{2} = \frac{18}{2}$

$x = 9$

$\angle M = 4(9+5)$

$= 56°$

$\angle H = 56°$

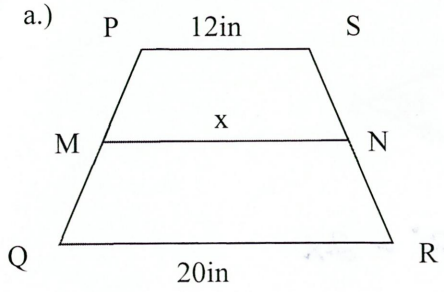
$\angle A = 180° - 56°$

$= 124°$

$\angle T = 124°$

Example #4: In the diagrams below, $\overline{MN}$ is the midsegment of trapezoid PQRS. Solve for x.

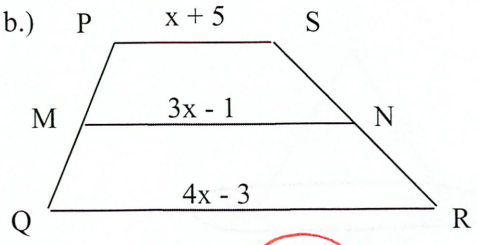
a.)



$x = \frac{12+20}{2}$

$x = \frac{32}{2} \rightarrow x = 16$ in

b.)



$2\left(\frac{x+5+4x-3}{2}\right) = (3x-1)2$

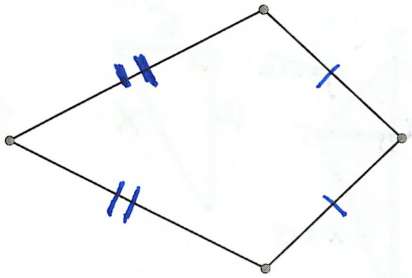
$5x + 2 = 6x - 2$

$-5x \quad +2 \quad -5x \quad +2$

$4 = x$

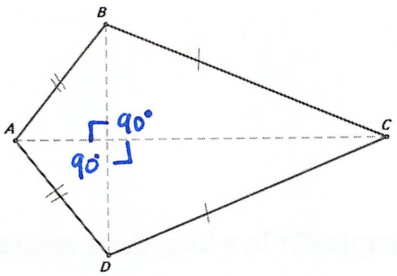
Kite: A quadrilateral that has…

- 2 pairs of Consecutive congruent sides
- But opposite sides are NOT CONGRUENT



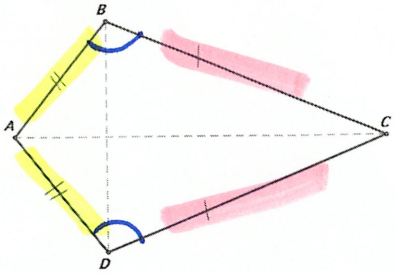
Quadrilateral with Perpendicular Diagonals Theorem (Theorem 8.18):

If a quadrilateral is a kite, then its diagonals are perpendicular.



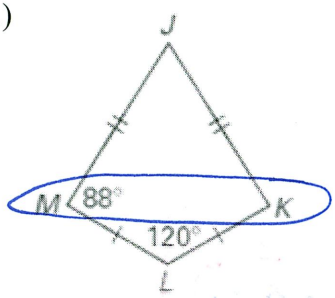
Quadrilateral with One Pair of Opposite Congruent Angles Theorem (Theorem 8.19):

If a quadrilateral is a kite, then exactly 1 pair of opposite angles are congruent.



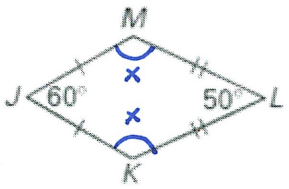
Example #5: JKLM is a kite. Find the $m\angle K$

a.)



$m\angle K = 88°$

b.)

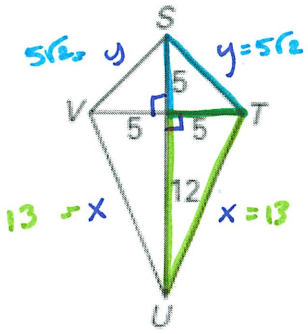


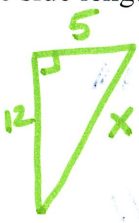
$360° - 60° - 50° = 2x$

$\frac{250°}{2} = \frac{2x}{2}$

$125° = x$

Example #6: Find the side lengths of Kite STUV



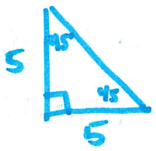


$5^2 + 12^2 = x^2$

$25 + 144 = x^2$

$\sqrt{169} = \sqrt{x^2}$

$13 = x$



$y = 5\sqrt{2}$

Example #7: In a kite, the measures of the angles are $3x°, 75°, 90°$ and $120°$. Find the value of x.