Name: ________________________________

Geometry

Unit 5: Relationships within Triangles

<u>Priority Standard:</u>

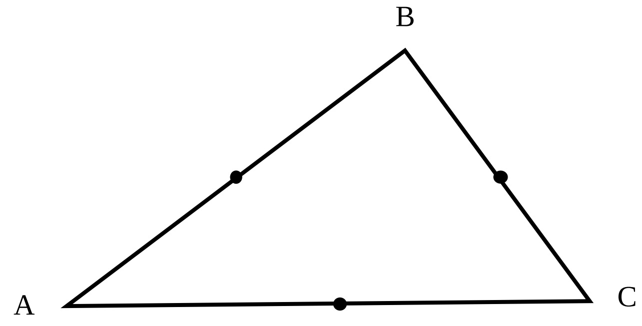
Unit "I can" statements:

1. I can identify and use the properties of midsegments in triangles to find unknown measures.
2. I can identify and use the properties of perpendicular bisectors (circumcenter) in triangles to find unknown measures.
3. I can identify and use the properties angles bisectors (incenter) in triangles to find unknown measures.
4. I can identify and use the properties medians (centroid) in triangles to find unknown measures.
5. I can identify the altitudes (orthocenter) in triangles.
6. I can use the relationships between sides and angles in triangles to find unknown measures.
7. I can use inequalities to make comparisons in two triangles

Common Core State Standards that are addressed in this unit include:
For more information see www.corestandards.org/Math/

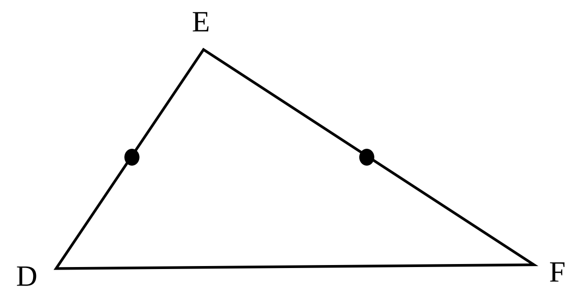
Chapter 5.1: Midsegment Theorem

______________________ **of a triangle** is a segment that connects the ______________ of two sides of a triangle.



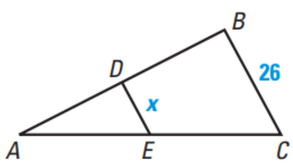
__________________ **Theorem (Theorem 5.1):**

The segment connecting ______________________ of two sides of a triangles is _________________ to the third side and is ________ as long as that side.

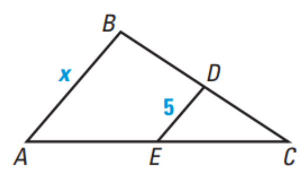


1. $\overline{DE}$ is a midsegment of $\triangle ABC$. Find the value of *x*.

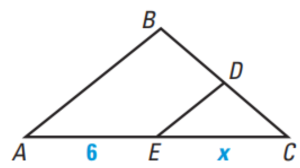
a.)



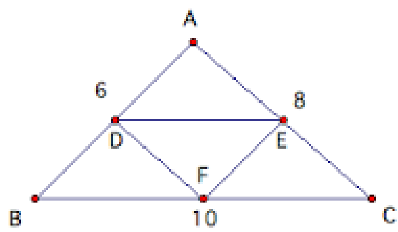
b.)



c.)



2. Find the perimeter of $\triangle ABC$, then the perimeter of $\triangle DEF$. What do you notice about these values?

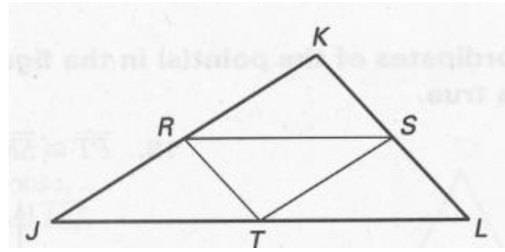


3. In $\triangle JKL, \overline{JR} \cong \overline{RK}, \overline{KS} \cong \overline{SL}$ and $\overline{JT} \cong \overline{TL}$.

a.) $\overline{RS} \parallel$ _______

b.) $\overline{ST} \parallel$ _______

c.) $\overline{KL} \parallel$ _______



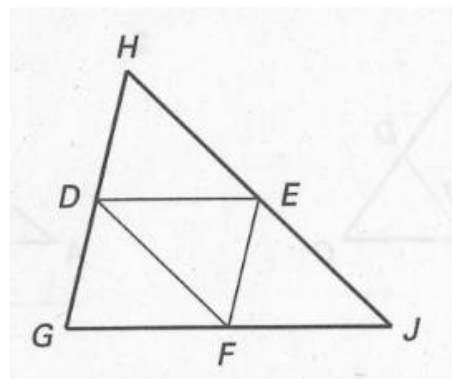
d.) $\overline{SL} \cong$_______$\cong$_______

e.) $\overline{JR} \cong$_______$\cong$_______

f.) $\overline{JT} \cong$_______$\cong$_______

4. Use $\triangle GHJ$, where D, E and F are midpoints of the sides.

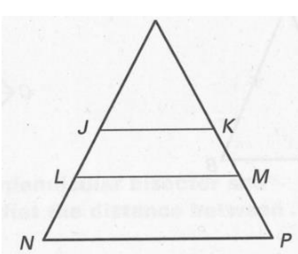
a.) If $DE = 4x + 5$ and $GJ = 3x + 25$, what is DE?



b.) If $GD = 2x + 7$ and $GH = 5x - 1$, what is EF?

c.) If $HE = 8x - 7$ and $DF = 2x + 11$, what is HJ?

5. In an A-frame house, the floor of the second level, labeled $\overline{LM}$, is closer to the first floor, $\overline{NP}$, then midsegment $\overline{JK}$. If $\overline{JK}$ is 14 feet long, can $\overline{LM}$ be 12 feet long? 14 feet long? 20 feet long? 24 feet long? 30 feet long?

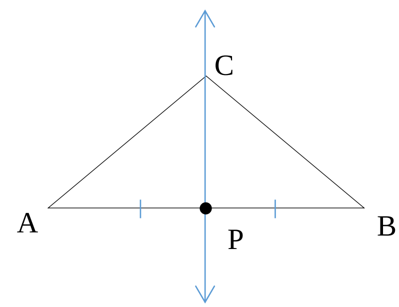


Perpendicular Bisector: A segment, ray, line, or plane that is perpendicular to a segment at its_____________________ .

Equidistant: A point is equidistant from two figures if the point is the ____________________________ from each figure.

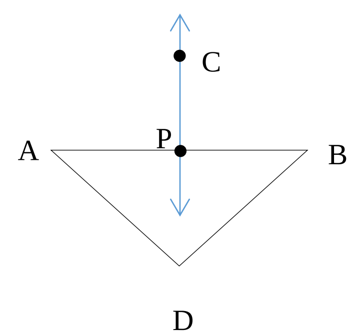
Perpendicular Bisector Theorem (Theorem 5.2):

In a plane, if a point is on the perpendicular bisector of a segment, then it is ____________________________ from the endpoints of the segment.



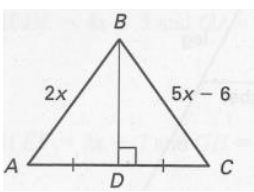
Converse of the Perpendicular Bisector Theorem (Theorem 5.3):

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the ____ of the segment.

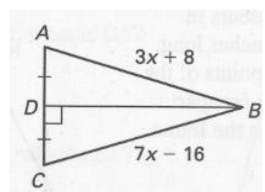


1. Find the length of $\overline{AB}$.

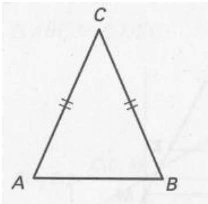
a.)

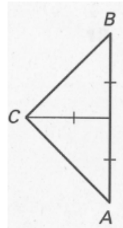


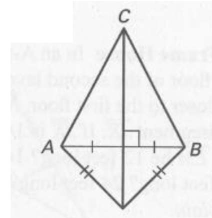
b.)



2. Tell whether the information in the diagram allows you to conclude that C is on the perpendicular bisector of $\overline{AB}$.

a.) 

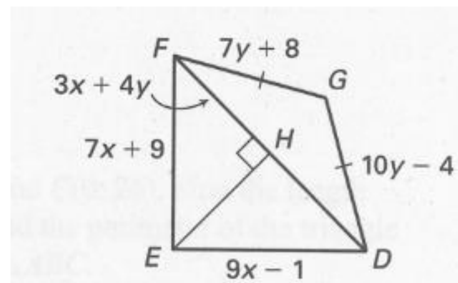
b.) 

c.) 

3. Use the diagram. $\overline{EH}$ is the perpendicular bisector of $\overline{DF}$. Find the indicated measure.

a.) Find EF

b.) Find FG



c.) Find FH

d.) Find DF

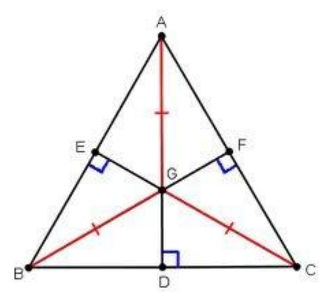
Concurrent: When three or more lines, rays, or line segments intersect in the same point, they are called concurrent lines, rays or segments.

Point of Concurrency: The point of intersection of concurrent lines, rays, or segments.

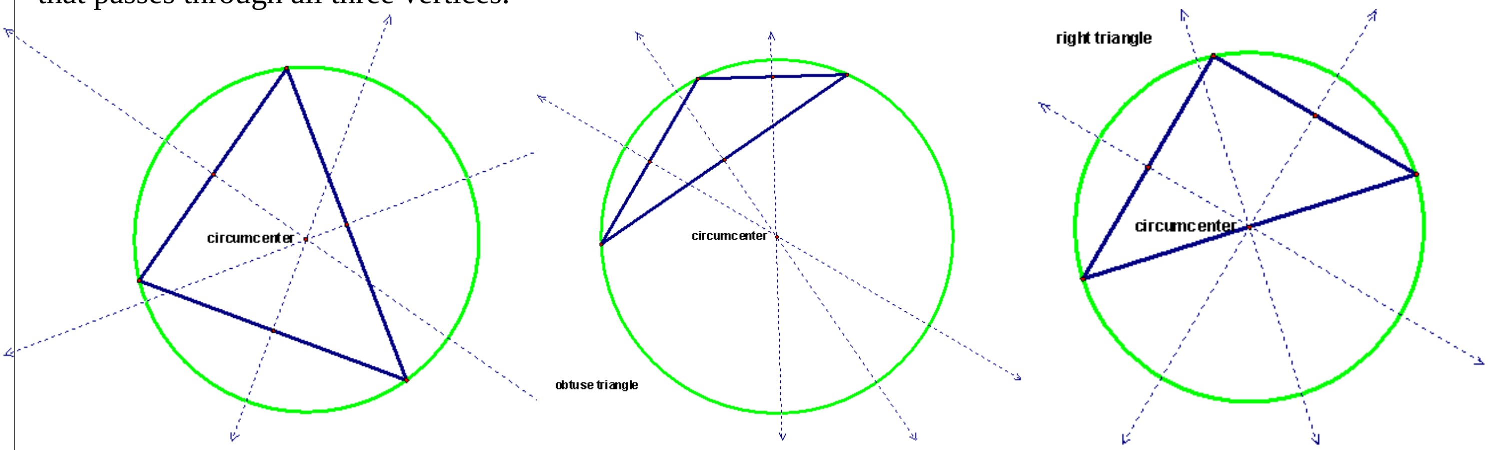
Concurrency of Perpendicular Bisectors of a Triangle (Theorem 5.4):

The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of a triangle called the ___________________________.

If $\overline{GE}$, $\overline{GF}$ and $\overline{GD}$ are perpendicular bisectors, then _________________________



Circumcenter: The point of concurrency of the three perpendicular bisectors of a triangle is called the circumcenter of the triangle. The circumcenter is equidistant from the three vertices, so the center of a circle that passes through all three vertices.



4. In the diagram, the perpendicular bisectors of △ABC meet at point G and are shown dashed. Find the indicated measure.

a.) Find AG

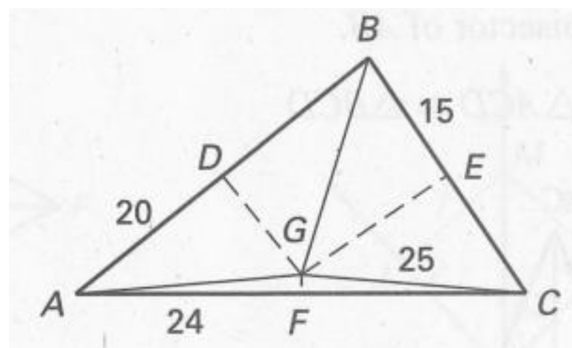
b.) Find BD

c.) Find CF

d.) Find BG

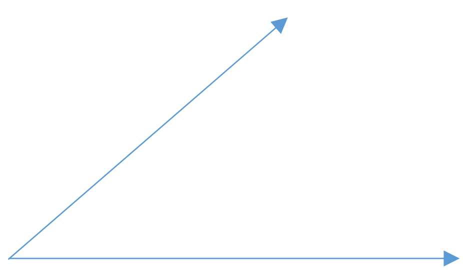
e.) Find CE

f.) Find AC



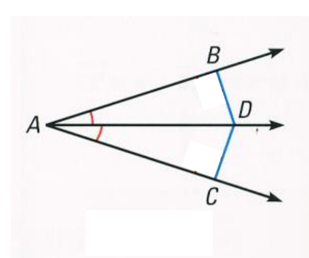
Chapter 5.3: Use Angles Bisectors of Triangles

________________________________: a ray that divides an angle into two congruent adjacent angles.



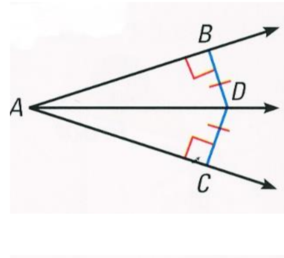
Angle Bisector Theorem **(Theorem 5.5) :**

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.



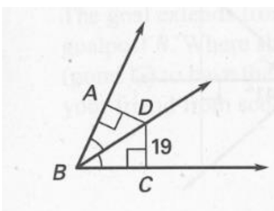
Converse of the Angle Bisector Theorem (Theorem 5.6) :

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

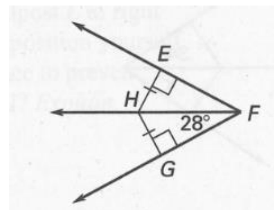


1. Use the information in the diagram to find the measure.

a.) Find AD

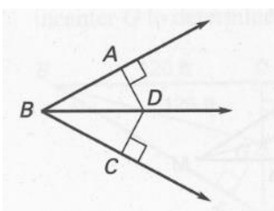


b.) Find $m\angle EFH$

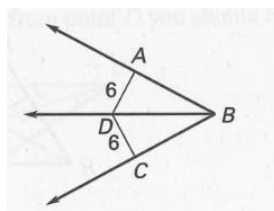


2. Can you conclude that $\overrightarrow{BD}$ bisects $\angle ABC$?

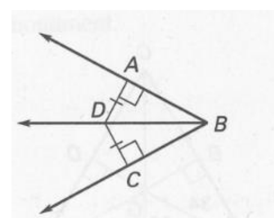
a.)



b.)

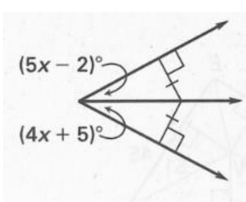


c.)

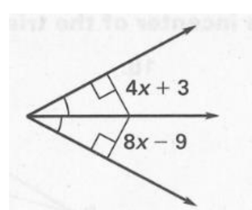


3. Find the value of *x*

a.)

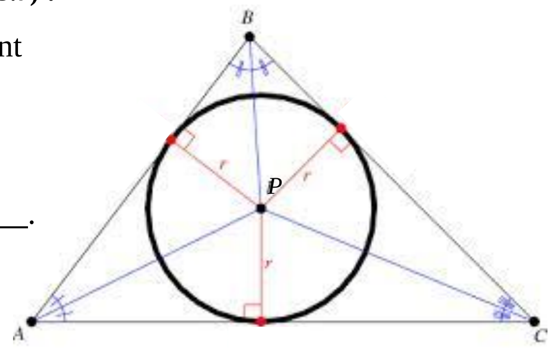


b.)



Concurrency of Angles Bisectors of a Triangle **(Theorem 5.7) :**

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle called the ________________.

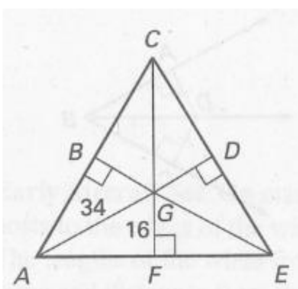


If $\overline{AP}, \overline{BP}$ and $\overline{CP}$ are angle bisectors then ______________________.

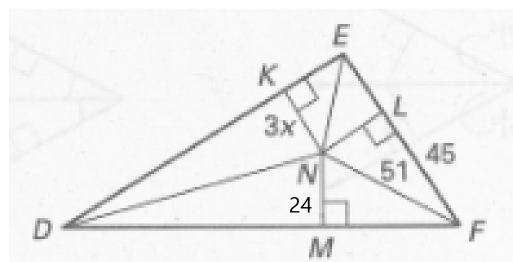
The incenter _______________ lies inside the triangle.

Because the incenter is equidistant from the three sides of the triangle, a circle drawn using the incenter as the center and the distance to one side as the radius will just touch the other two sides. The circle is said to be __________________ with the triangle.

4. Point G is the incenter of $\triangle ABC$.
   Find BG.



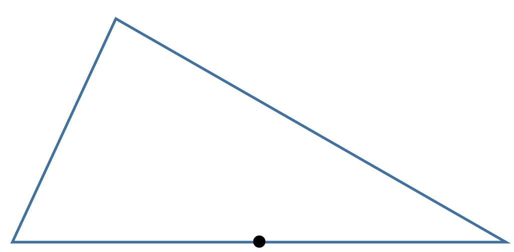
5. Find the value of *x* that makes N the incenter of the triangle.



Chapter 5.4: Use Medians and Altitudes

Median of a Triangle:

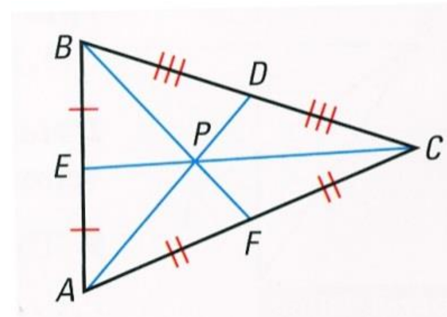
A segment from a vertex to the ________________________ of the opposite side.



_________________**:** The point of concurrency of the three **medians** of a triangle.

Concurrency of Medians of a Triangle **(Theorem 5.8):**

The medians of a triangle intersect at a point (_______________) that is ___________ of the distance from each vertex to the midpoint of the opposite side.



The medians of triangle ABC meet at P and

AP = BP= CP=

DP = FP= EP=

Example:

a.) If $BF = 12\ cm$, find BP and FP

b.) If $CP = 6\ cm$, find EP and CE

c.) If $DP = 2\ cm$, find AP and AD

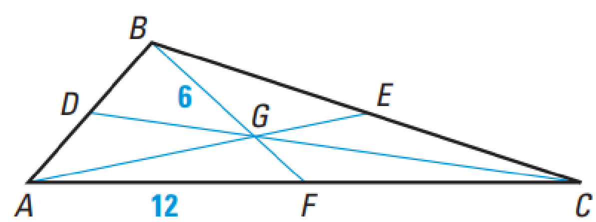
1. In $\triangle ABC$, G is the centroid. $BG\ 6\ ft, AF = 12\ ft$ and $AE = 15\ ft$. Find the following lengths.

a.) FC

b.) BF

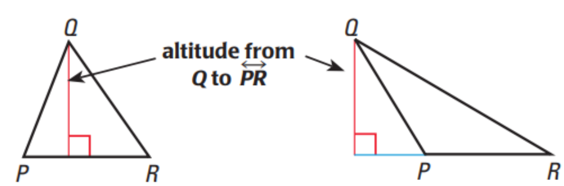
c.) AG

d.) GE

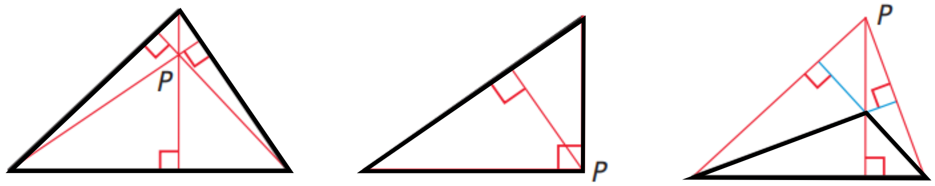


Altitude of a Triangle:

The _____________________________ segment from a vertex to the opposite side or to the line that contains the opposite side.



Orthocenter: The point (point of concurrency) at which the lines containing the three altitudes of a triangle intersect.



Acute triangle
P is inside triangle.

Right triangle
P is on triangle.

Obtuse triangle
P is outside triangle.

Point of Concurrency Review:

1. Fill in the blanks

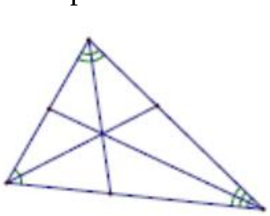
The three **perpendicular bisectors** of a triangles meet at the ___________________________

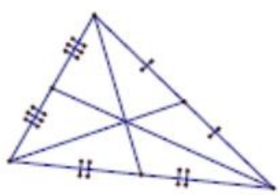
The three **altitudes** of a triangles meet at the ___________________________

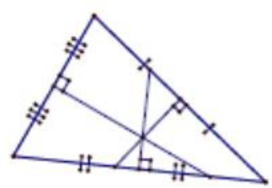
The three **medians** of a triangles meet at the ___________________________

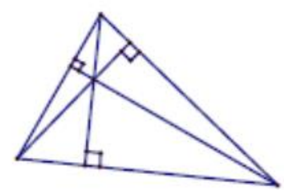
The three **angle bisectors** of a triangles meet at the ___________________________

2. In each figure below, tell what point of concurrency is illustrated and identify the line segments that forms that point







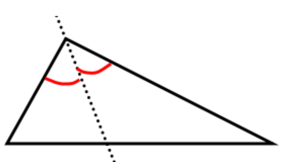


Point: ________________ Point: ________________ Point: ________________ Point: ________________

Line: _________________ Line: _________________ Line: _________________ Line: ________________

3. Given the following pictures and markings identify if the dotted line is a(n) Angle Bisector,Perpendicular Bisector, Altitude or Median **List All the Apply!**

a.)



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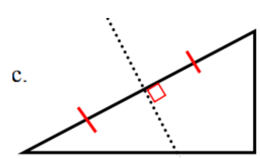

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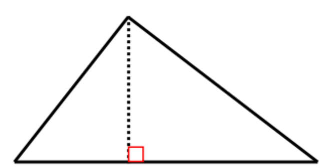

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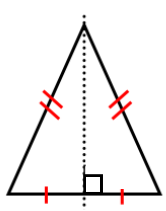
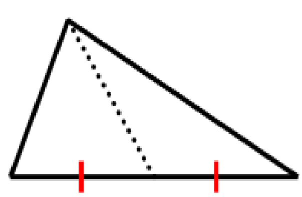
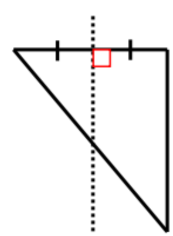


e.)

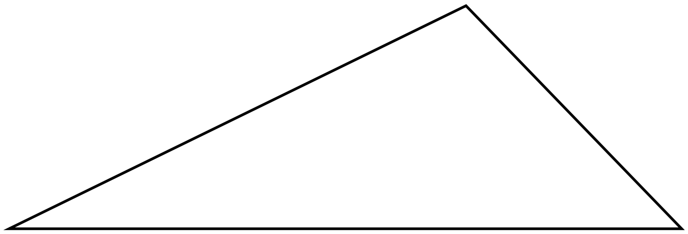


f.)



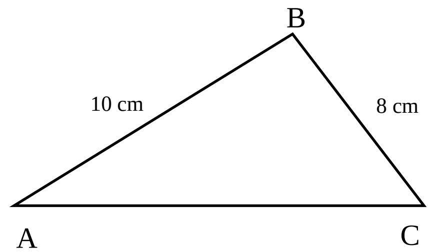
Chapter 5.5: Use Inequalities in a Triangle

Scalene Triangle:



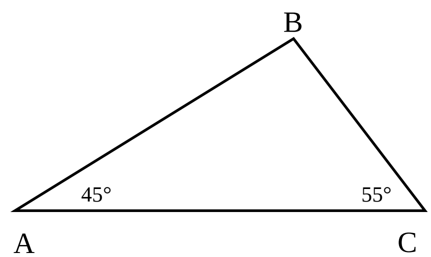
Theorem 5.10:

If one side of a triangle is longer than another side, then the angles opposite the longer side is _______________ than the angle opposite the shorter side.

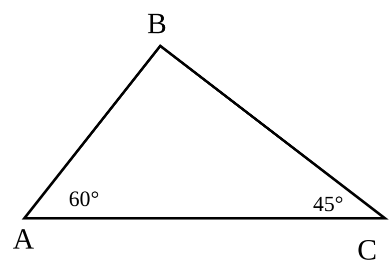


Theorem 5.11:

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is _______________ than the side opposite the smaller angle.

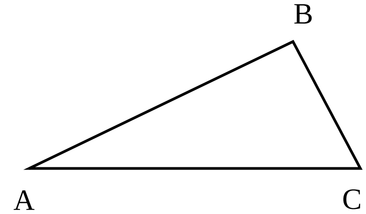


1. List the sides of $\triangle ABC$ in order from shortest to longest.



Triangle Inequality Theorem **(Theorem 5.12):**

The ____________ of the lengths of any two sides of a triangle is greater than the length of the third side.



2. Is it possible to construct a triangle with the given side lengths?

a.) 6, 7, 11 b.) 6, 3, 9 c.) 30, 10, 14

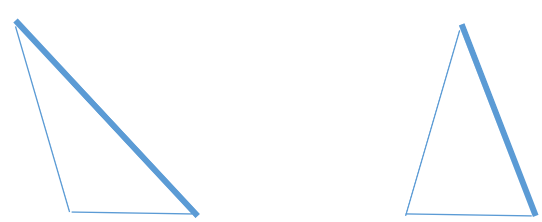
3. A triangle has one side length of 14in and another length of 10in. Describe the possible lengths of the third side.

4. A triangle has one side length of 23 meters and another length of 17 meters. Describe the possible lengths of the third side.

Chapter 5.6: Hinge Theorem

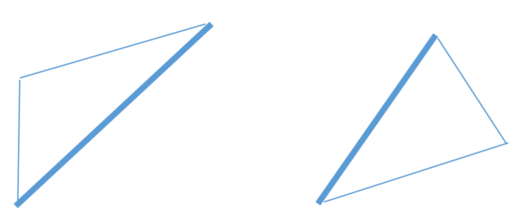
Hinge Theorem (Theorem 5.13):

If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is ___________________ than the third side of the second.



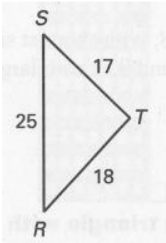
Converse of the Hinge Theorem (Theorem 5.14):

If two sides of one triangle are congruent to two sides of another triangle, and third side of the first is longer than the third side of the second, then the included angle of the first is ___________________ than the included angle of the second.

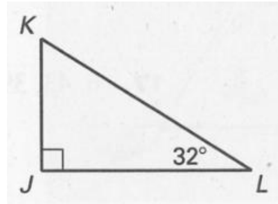


5. List the sides and the angles in order from smallest to largest.

a.)

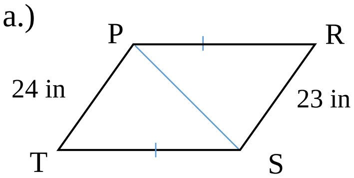


b.)

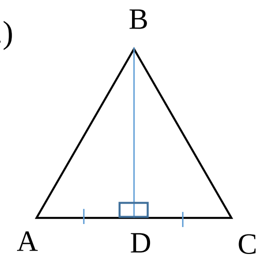


6. Complete with a <, >, = Given that $\overline{ST} \cong \overline{PR}$, how does $\angle PST$ compare (<, >, =) to $\angle SPR$?

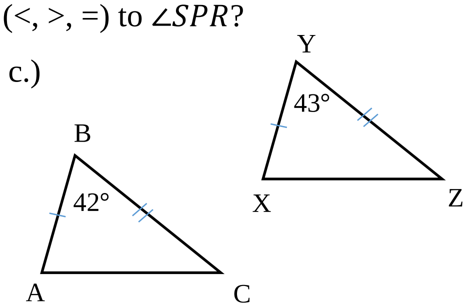
a.)



b.)



c.)



a.) $\angle PST$______ $\angle SPR$

b.) $\overline{AB}$ ______ $\overline{CB}$

c.) $\overline{AC}$ ______ $\overline{XY}$