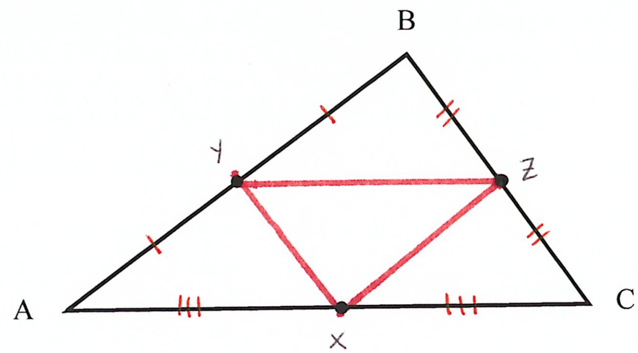
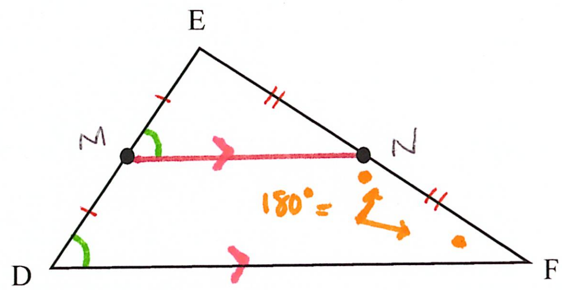
Chapter 5.1: Midsegment Theorem

Midsegment **of a triangle** is a segment that connects the midpoints of two sides of a triangle.

- Points X, Y and Z are midpoints so... $\overline{AY} \cong \overline{BY}$, $\overline{BZ} \cong \overline{CZ}$ and $\overline{AX} \cong \overline{CX}$
- 3 midsegments in each triangle
- Midsegments always lie inside the triangle



Midsegment **Theorem (Theorem 5.1):**



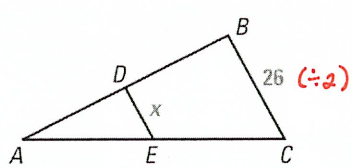
The segment connecting the midpoints of two sides of a triangles is parallel to the third side and is half as long as that side.

What you know...

1. $\overline{DM} \cong \overline{EM}$ and $\overline{EN} \cong \overline{FN}$
2. $\overline{MN} \parallel \overline{DF}$
   - Corresponding Angles ≅ (Example: ∠EMN and ∠EDF)
   - Consecutive Interior Angles (Example: m∠MNF + m∠DFN = 180°)
3. $MN = \frac{1}{2}DF$ or $2(MN) = DF$ (Example: If MN = 6 cm, then DF = 12 cm)

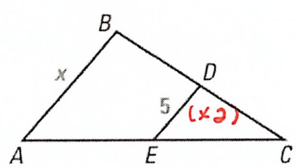
1. $\overline{DE}$ is a midsegment of $\triangle ABC$. Find the value of x.

a.)



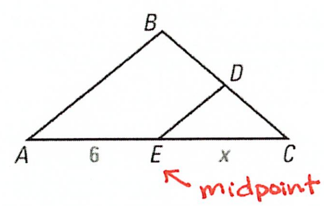
X = 13 units

b.)



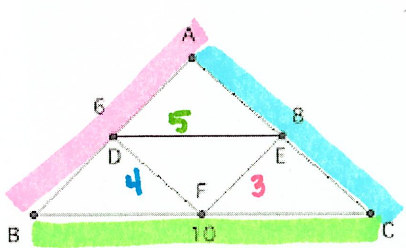
X = 10 units

c.)



X = 6 units

2. Find the perimeter of $\triangle ABC$, then the perimeter of $\triangle DEF$. What do you notice about these values?



Perimeter of △ABC = 6 + 8 + 10

= 24 units

Perimeter of △DEF = 3 + 4 + 5

= 12 units

☆ The perimeter of the triangle made with the midsegments is half of the perimeter of the original triangle ☆

3. In $\triangle JKL, \overline{JR} \cong \overline{RK}, \overline{KS} \cong \overline{SL}$ and $\overline{JT} \cong \overline{TL}$.

★ Points R, S and T are midpoints

∴ $\overline{RS}$, $\overline{ST}$ and $\overline{TR}$ are midsegments

a.) $\overline{RS} \parallel$ $\overline{JL}$

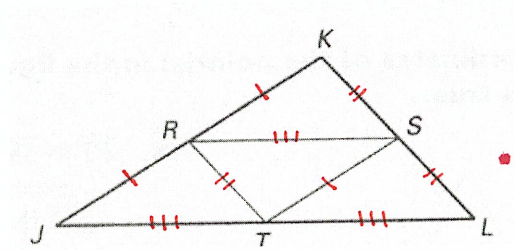
b.) $\overline{ST} \parallel$ $\overline{KJ}$

c.) $\overline{KL} \parallel$ $\overline{RT}$

d.) $\overline{SL} \cong$ $\overline{SK}$ $\cong$ $\overline{RT}$

e.) $\overline{JR} \cong$ $\overline{KR}$ $\cong$ $\overline{ST}$

f.) $\overline{RS} \cong$ $\overline{JT}$ $\cong$ $\overline{TL}$



4. Use $\triangle GHJ$, where D, E and F are midpoints of the sides.

a.) If $DE = 4x + 5$ and $GJ = 3x + 25$, what is DE?

$2(DE) = GJ \Rightarrow 2(4x+5) = 3x+25$

$8x + 10 = 3x + 25$

$-3x \quad -10 \quad -3x \quad -10$

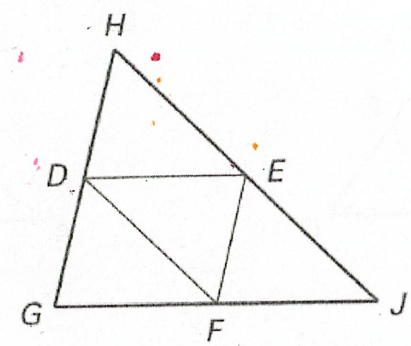
$5x = 15$

$\frac{5x}{5} = \frac{15}{5}$

$x = 3$

$DE = 4(3) + 5$

$\boxed{DE = 17 \text{ units}}$



b.) If $GD = 2x + 7$ and $GH = 5x - 1$, what is EF?

$2(GD) = GH \Rightarrow 2(2x+7) = 5x - 1$

$4x + 14 = 5x - 1$

$-4x \quad +1 \quad -4x \quad +1$

$15 = x$

★ $GD = EF$ ★

$GD = 2(15) + 7$

$GD = 37$

$\boxed{EF = 37 \text{ units}}$

c.) If $HE = 8x - 7$ and $DF = 2x + 11$, what is HJ?

$HE = DF \Rightarrow 8x - 7 = 2x + 11$

$-2x \quad -2x$

$6x - 7 = 11$

$+7 \quad +7$

$6x = 18$

$\frac{6x}{6} = \frac{18}{6}$

$x = 3$

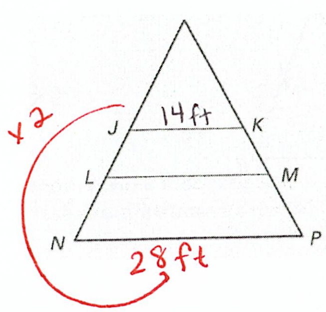
$HJ = 2(HE)$

$HJ = 2(8(3) - 7)$

$HJ = 2(17)$

$\boxed{HJ = 34 \text{ units}}$

5. In an A-frame house, the floor of the second level, labeled $\overline{LM}$, is closer to the first floor, $\overline{NP}$, then midsegment $\overline{JK}$. If $\overline{JK}$ is 14 feet long, can $\overline{LM}$ be 12 feet long? 14 feet long? 20 feet long? 24 feet long? 30 feet long?



Since $\overline{LM}$ is between $\overline{JK}$ and $\overline{NP}$, its length must be greater than 14ft (length of $\overline{JK}$), but less than 28ft (length of $\overline{NP}$). Therefore the possible lengths $\overline{LM}$ could be are 20 ft or 24 ft.