Name: ________________________________

# Geometry

# Unit 4: Congruent Triangles

**Priority Standard:** G-CO.10: Completing proofs involving triangle congruency.

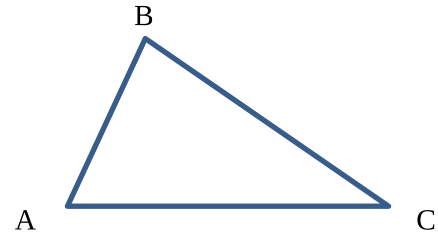
**Unit "I can" statements:**

1. I can classify triangles using their sides.
2. I can classify triangles using their angles.
3. I can apply the Triangle Sum Theorem and Exterior Angles Theorem to find unknown angles in triangles.
4. I can identify congruent figures.
5. I can prove triangles congruent using SSS, SAS, HL, ASA and AAS.
6. I can prove corresponding parts of triangles congruent using CPCTC.
7. I can use properties of isosceles and equilateral triangles to prove triangle congruence.

Common Core State Standards that are addressed in this unit include:
For more information see www.corestandards.org/Math/

# **Chapter 4.1:** Apply Triangle Sum Properties

A **triangle** is polygon with three sides.
A triangle with vertices *A*, *B*, and *C* is called "triangle *ABC*" or $\triangle$ *ABC*



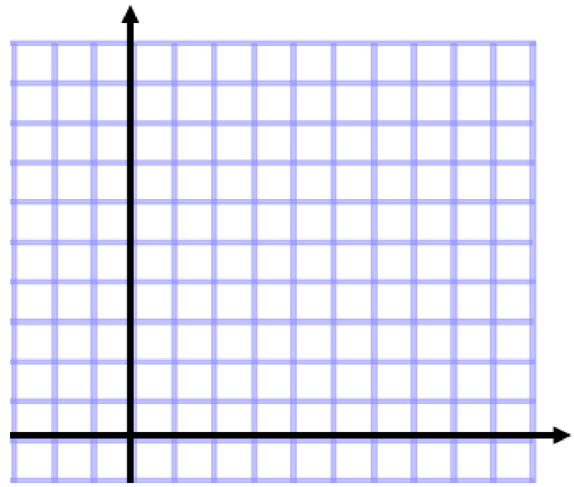

## Classifying Triangles by Sides:

| Term | Definition | Example |
|---|---|---|
| Scalene Triangle | | |
| Isosceles Triangle | | |
| Equilateral Triangle | | |

## Classifying Triangles by Angles:

| Term | Definition | Example |
|---|---|---|
| Acute Triangle | | |
| Right Triangle | | |
| Obtuse Triangle | | |
| Equiangular Triangle | | |

Example #1: Classify △ *PQO* by its sides.
Vertices P(-1, 2); Q(6,3); O(0, 0)

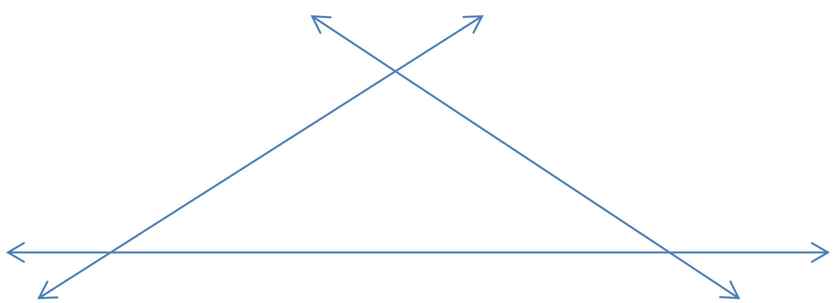


Determine if the triangle is a right triangle.

**Angles:** When the sides of a polygon are extended, other angles are formed. The original angles are the

__________________________. The angles that form linear pairs with the interior angles are the

__________________________.

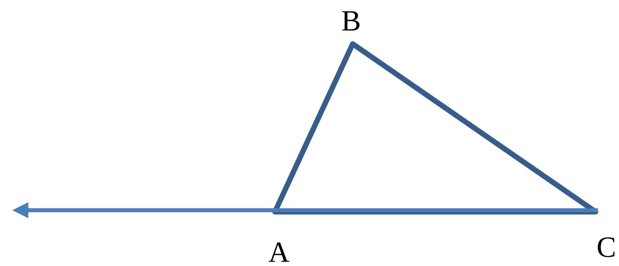


## Triangle Sum Theorem (Theorem 4.1):

The sum of the measures of the interior angles of a triangle is ___________.

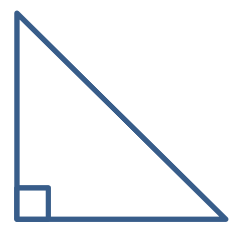
## Exterior Angles Theorem (Theorem 4.2):

The measure of an exterior angles of a triangle is equal to the__________ of the measures of the two ____________________________ interior angles.

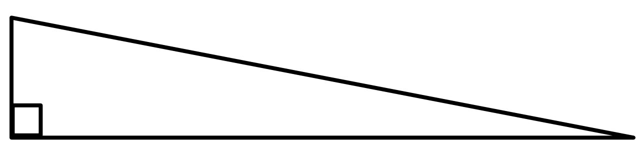



## Corollary to the Triangle Sum Theorem

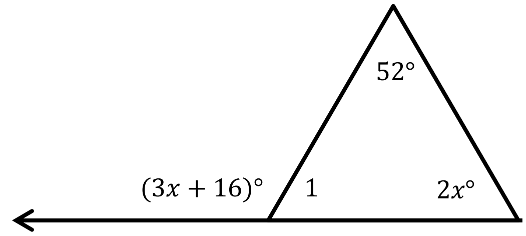
The acute angles of a right triangle are __________________________.



Example #2: The front face of a wheelchair ramp form a right triangle. The measure of one acute angles in the triangle is eight times the measure of the other. Find the measure of each acute angle.



Example #3: Find the value of $x$ and $m\angle 1$.



Example #4: Find the measure of each interior angles of $\triangle ABC$, where $m\angle A = x°$, $m\angle B = 2x°$ and $m\angle C = 3x°$.

## **Chapter 4.2:** Apply Congruence and Triangles

Two geometric figures are **<u>congruent</u>** if they have exactly the ________________________ and shape.

In two ______________________________________, all the parts of one figure are congruent to the

____________________________________of the other figure.


Corresponding Angles: ____________________________________________

Corresponding Sides: ______________________________________________

**Congruent Statements:** When you write a congruence statement for two polygons, always list the corresponding vertices in the same order.

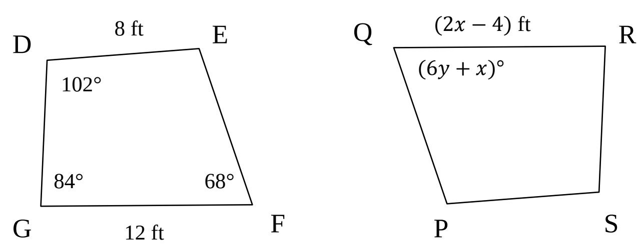
Example Statement: __________________________________________________________

NOT a Congruent Statement: _________________________________________________

Example #1: In the diagram $DEFG \cong SPQR$

a.) Find the value of $x$

b.) Find the value of y

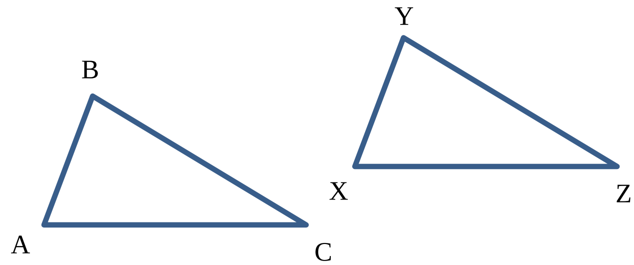


**Third Angles Theorem (Theorem 4.3):**

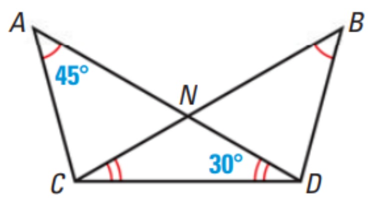
If two angles of one triangle are ___________________

to two angles of another triangle, then the ______________

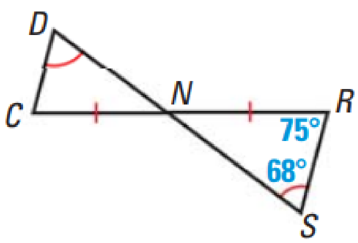
angles are also congruent.




Example #2: Find $m\angle BDC$



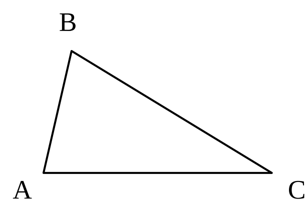

Example #3: In the diagram, what is the $m\angle DCN$?
By the definition of congruence, what additional information is needed to know that $\triangle NDC \cong \triangle NSR$?




**Properties of Congruent Triangles (Theorem 4.4):**

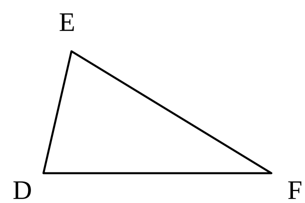
**Reflective Property of Congruent Triangles**

For any triangle ABC, __________________________________________




**Symmetric Property of Congruent Triangles**

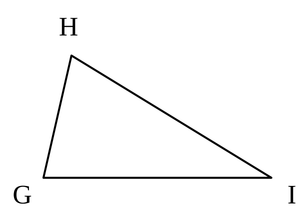
If ____________________________________________________________




**Transitive Property of Congruent Triangles**

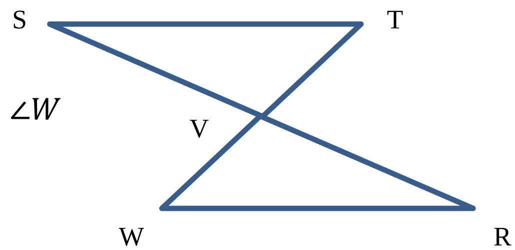
If ____________________________________________________________

________________________________________



Example #4: Prove.

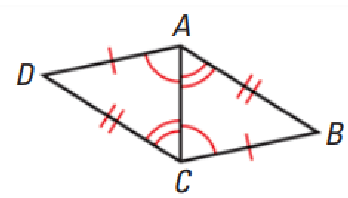
**Given:** $\overline{TV} \cong \overline{WV}$, $\overline{ST} \cong \overline{RW}$, point V is the midpoint of $\overline{SR}$, $\angle T \cong \angle W$
**Prove**: $\triangle STV \cong \triangle RWV$




| Statement | Reason |
|---|---|
| 1. ______ ______ | 1. ______ |
| 1. ______ ______ | |
| 2. ______ | 2. ______ |
| 3. ______ | 3. ______ |
| 4. ______ | 4. ______ |
| 5. ______ | 5. ______ |

Example #5: Prove.

**Given:** $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$, $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$
**Prove**: $\triangle ACD \cong \triangle CAB$




| Statement | Reason |
|---|---|
| 1. ______ ______ | 1. ______ |
| 1. ______ ______ | |
| 2. ______ | 2. ______ |
| 3. ______ | 3. ______ |
| 4. ______ | 4. ______ |

## **Chapter 4.3:** Prove Triangles Congruent by SSS

**Side-Side-Side (SSS) Congruence Postulate (Postulate 19):**

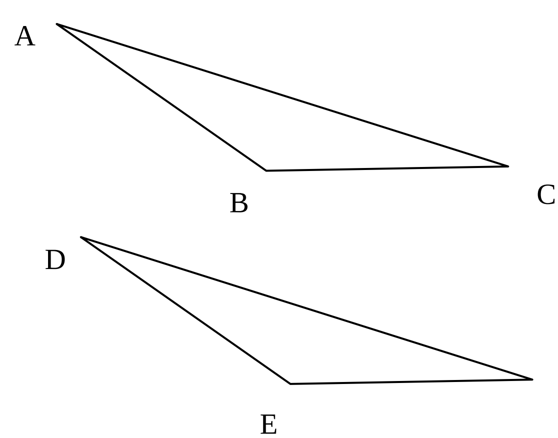
If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are ____________________.

If ______________________________

______________________________

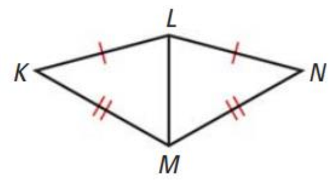
______________________________

Then ___________________________



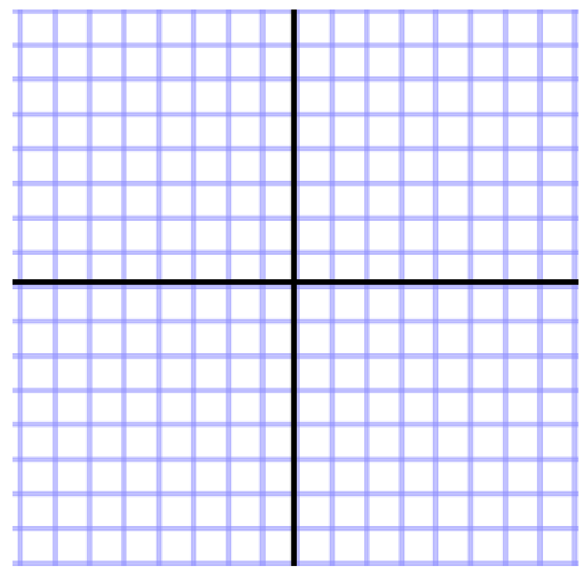
Example #1: Prove.

**Given:** $\overline{KL} \cong \overline{NL},\ \overline{KM} \cong \overline{NM}$
**Prove:** $\triangle KLM \cong \triangle NLM$



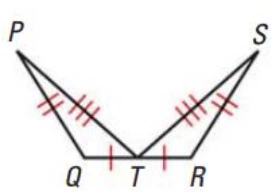

| Statement | Reason |
|---|---|
| 1. ____________________ ____________________ | 1. ______________________________________ |
| 2. ________________________________________ | 2. ______________________________________ |
| 3. ________________________________________ | 3. ______________________________________ |

Example #3: Triangle DFG has vertices D(-2, 4), F(4, 4), and G(-2, 2). Triangle LMN has vertices L(-3, -3), M(-3, 3) and N(-1, -3). Graph the triangles in the same coordinate plane and show that they are congruent.

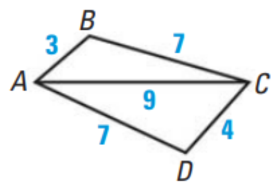


Example #2: Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.

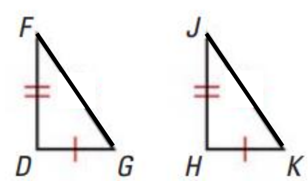
a.)




b.)




c.)

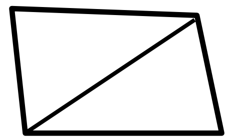


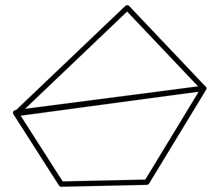
## Stability in Structures:

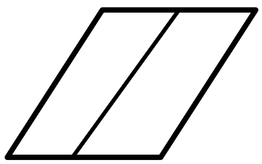
A diagonal support form triangles with fixed side lengths.  By SSS, these triangles cannot change shape.  A structure without a diagonal support is not stable because there are many possible quadrilaterals with the given side length.

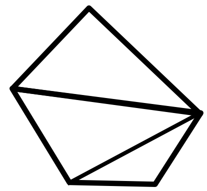
Example #4: Determine whether the figure is stable.

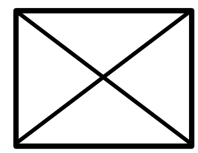
a.) 

b.) 

c.) 

d.) 

e.) 

f.) 

# **Chapter 4.4:** Prove Triangles Congruent by SAS and HL

### Side-Angle-Side (SAS) Congruence Postulate (Postulate 20):

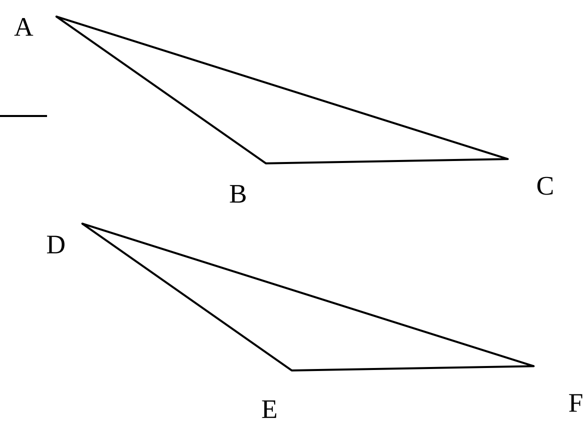
If two sides and the ________________________ angles of one triangle are congruent to two sides and the included of a second triangle, then the two triangles are ____________________

If ________________________________

________________________________

________________________________

Then ____________________________



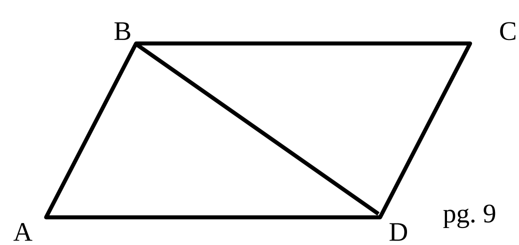

Example #1: Use the diagram to name the included angle between the given pair of sides.

a.) $\overline{AB}$ and $\overline{BC}$

b.) $\overline{BC}$ and $\overline{CD}$

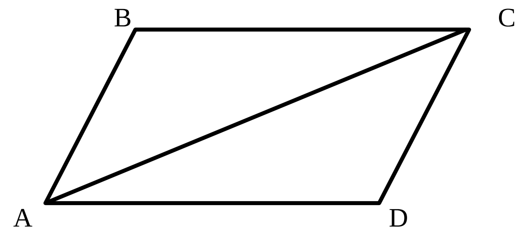
c.) $\overline{AB}$ and $\overline{BD}$

d.) $\overline{BD}$ and $\overline{DA}$



Example #2: Prove.

**Given:** $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$,
**Prove:** $\triangle ABC \cong \triangle CDA$



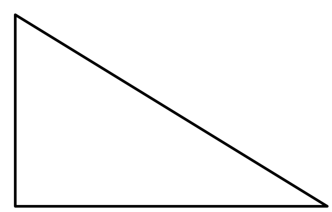

| Statement | Reason |
| --- | --- |
| 1. ____________________ ____________________ | 1. ______________________________________ |
| 2. _________________________________________ | 2. ______________________________________ |
| 3. _________________________________________ | 3. ______________________________________ |
| 4. _________________________________________ | 4. ______________________________________ |

In general, if you know the lengths of two sides and the measure of an angle that is *not included* between them, you can create two different triangles.

Example:


Therefore, SSA is *NOT* a valid method for proving that triangles are congruent, although there is a special case for right triangles.

**Right Triangles:** In a right triangle the sides adjacent to the right angle are called the ____________. The side opposite the right angles is called the ______________________



## Hypotenuse-Leg (HL) Congruence Theorem (Theorem 4.5):

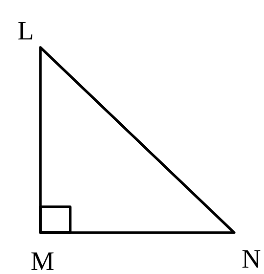
If the _____________________ and a __________ of a right triangles are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are ________________.

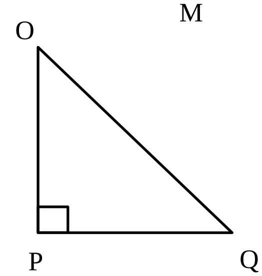
If _______________________________________________

_______________________________

_______________________________

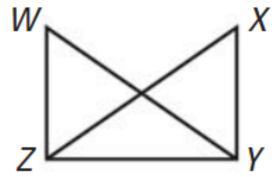
Then ___________________________



Example #3: Prove.

**Given:** $\overline{WY} \cong \overline{XZ}, \overline{WZ} \perp \overline{ZY}, \overline{XY} \perp \overline{ZY}$
**Prove:** $\triangle WYZ \cong \triangle XZY$




| Statement | Reason |
|---|---|
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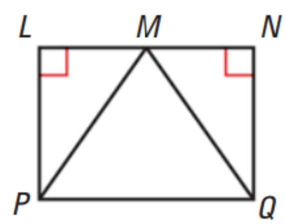
| 2. ________________ | 2. ________________ |
| 3. ________________ | 3. ________________ |
| 4. ________________ | 4. ________________ |
| 5. ________________ | 5. ________________ |
| 6. ________________ | 6. ________________ |

Example #4: Prove.

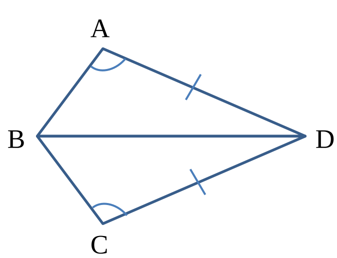
**Given:** Point M is the midpoint of $\overline{LN}$
$\triangle$PMQ is an isosceles triangle with $\overline{MP} \cong \overline{MQ}$
$\angle L$ and $\angle N$ are right angles
**Prove:** $\triangle LMP \cong \triangle NMQ$



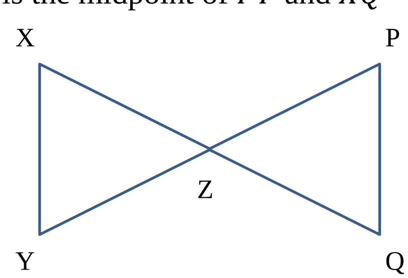

| Statement | Reason |
|---|---|
| 1. ________________ | 1. ________________ |
| 2. ________________ | 2. ________________ |
| 3. ________________ | 3. ________________ |
| 4. ________________ | 4. ________________ |
| 5. ________________ | 5. ________________ |
| 6. ________________ | 6. ________________ |

Example #5: Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.

a.)

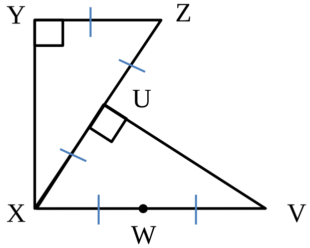



b.) Z is the midpoint of $\overline{PY}$ and $\overline{XQ}$

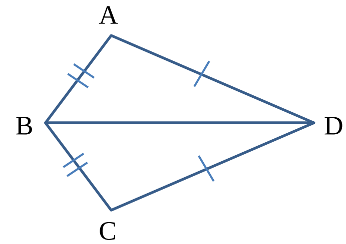


Example #5: Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.

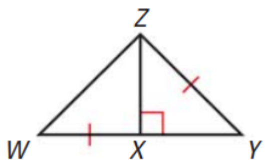
c.)



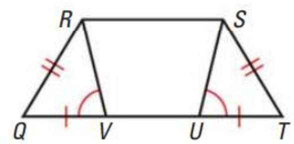

d.)



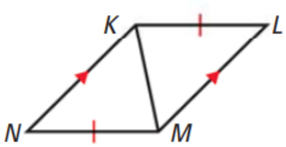

e.)




f.)




g.)

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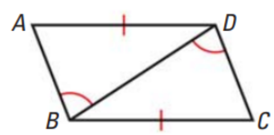
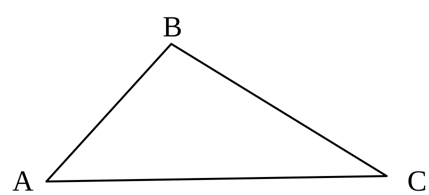



## **Chapter 4.5:** Prove Triangles Congruent by ASA and AAS

### Angle- Side-Angle (ASA) Congruent Postulate **(Postulate 21):**

If two angles and the ____________________ side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are _____________________.

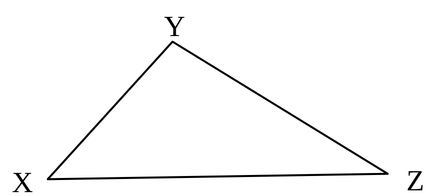



If _______________________________

_______________________________

_______________________________

Then ___________________________



**Angle-Angle-Side (AAS) Congruent Theorem (Theorem 4.6):**

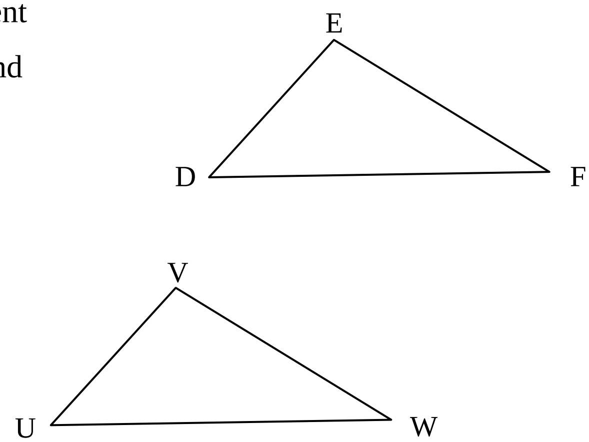
If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are ____________________.

If ______________________________

______________________________

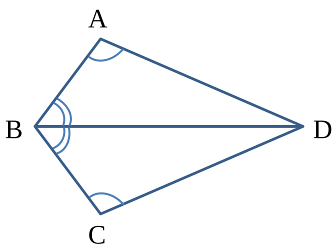
______________________________

Then __________________________

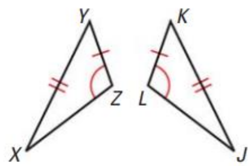



Example #1: Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

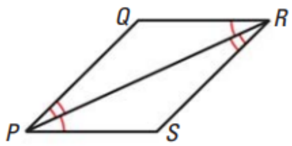
a.)



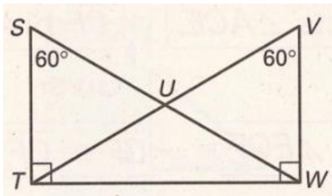

b.)



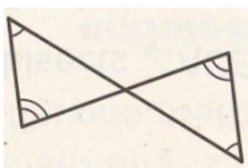

c.)



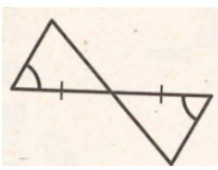

d.)




e.)

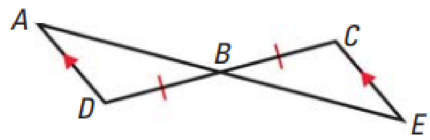


f.)



Example #2: Prove.

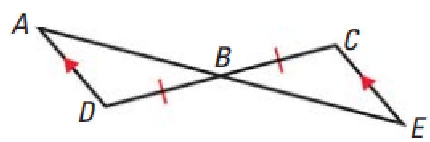
**Given:** $\overline{BD} \cong \overline{BC}, \overline{AD} \parallel \overline{EC}$
**Prove:** $\triangle ABD \cong \triangle EBC$




| Statement | Reason |
|---|---|
| 1. | 1. |
| 2. | 2. |
| 3. | 3. |
| 4. | 4. |
| 5. | 5. |

Example #2: Prove.

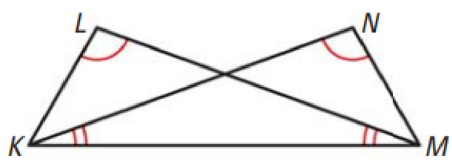
**Given:** $\overline{BD} \cong \overline{BC}, \overline{AD} \parallel \overline{EC}$
**Prove:** $\triangle ABD \cong \triangle EBC$




| Statement | Reason |
|---|---|
| 1. | 1. |
| 2. | 2. |
| 3. | 3. |
| 4. | 4. |
| 5. | 5. |

Example #3: Prove.

**Given:** $\angle NKM \cong \angle LMK, \angle L \cong \angle N$
**Prove:** $\triangle NMK \cong \triangle LKM$



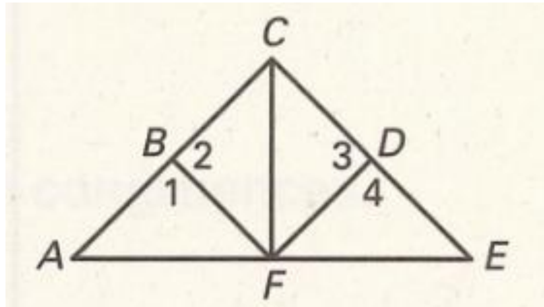

| Statement | Reason |
|---|---|
| 1. | 1. |
| 2. | 2. |
| 3. | 3. |
| 4. | 4. |

Example #4: Prove.

**Given:** $\angle 1 \cong \angle 4$

$\overline{CF}$ bisects $\angle ACE$

**Prove:** $\triangle CBF \cong \triangle CDF$



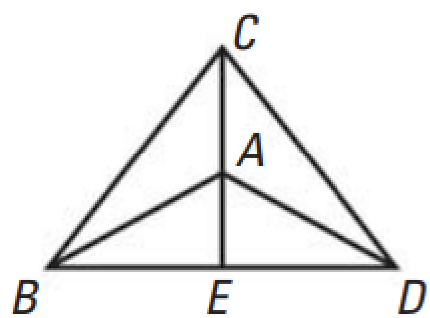

| Statement | Reason |
|---|---|
| 1. ______________________ | 1. ______________________ |
| 2. ______________________ | 2. ______________________ |
| 3. ______________________ | 3. ______________________ |
| 4. ______________________ | 4. ______________________ |
| 5. ______________________ | 5. ______________________ |
| 6. ______________________ | 6. ______________________ |
| 7. ______________________ | 7. ______________________ |

Example #5: Prove.

**Given:** $\overline{CE} \perp \overline{BD}$

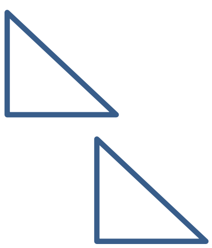
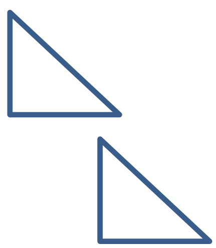
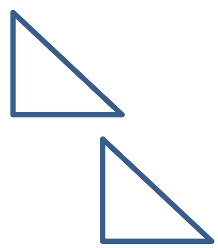
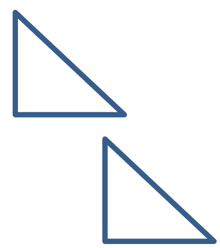
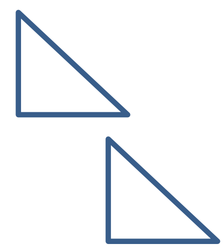
$\angle CAB \cong \angle CAD$

**Prove:** $\triangle ABE \cong \triangle ADE$



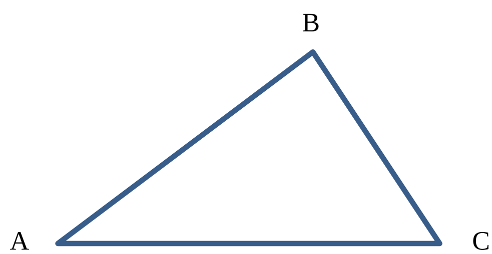

| Statement | Reason |
|---|---|
| 1. ______________________ | 1. ______________________ |
| 2. ______________________ | 2. ______________________ |
| 3. ______________________ | 3. ______________________ |
| 4. ______________________ | 4. ______________________ |
| 5. ______________________ | 5. ______________________ |
| 6. ______________________ | 6. ______________________ |
| 7. ______________________ | 7. ______________________ |
| 8. ______________________ | 8. ______________________ |

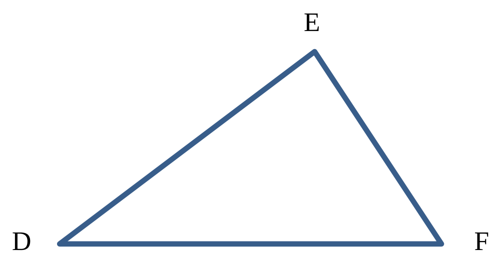
Review:

| Triangle Congruence Postulates and Theorems | | | | |
|---|---|---|---|---|
| | | | | |
| | | | | |
|  |  |  |  |  |

By definition (from section 2):

**Corresponding Parts of Congruent Triangles are Congruent**

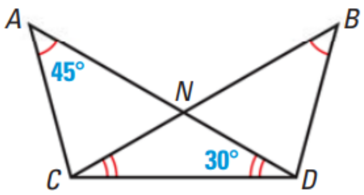



Congruence Statement: ______________________________ because of this you know...

IMPORTANT!! Before we can prove corresponding sides or angles are congruent, we MUST prove that the triangles are congruent using

________, ________, ________, ________, ________, Example #1: $\triangle ADC \cong \triangle BCD$ by ____________. What other parts of the triangles are congruent by CPCTC??



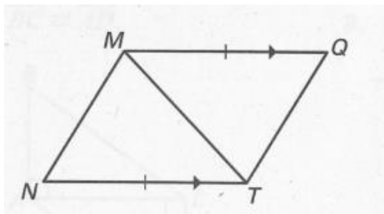

__________ $\cong$ ___________

__________ $\cong$ ___________

__________ $\cong$ ___________

Example #2: Prove.

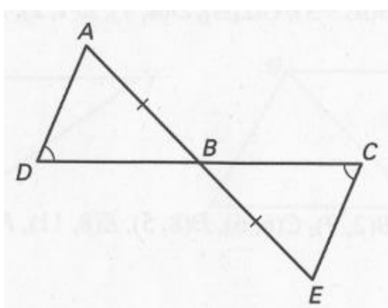
**Given:**
**Prove:** $\overline{MN} \cong \overline{TQ}$




| Statement | Reason |
| --- | --- |
| 1. ______________________ | 1. ______________________ |
| 2. ______________________ | 2. ______________________ |
| 3. ______________________ | 3. ______________________ |
| 4. ______________________ | 4. ______________________ |
| 5. ______________________ | 5. ______________________ |
| 6. ______________________ | 6. ______________________ |

Example #3: Prove.

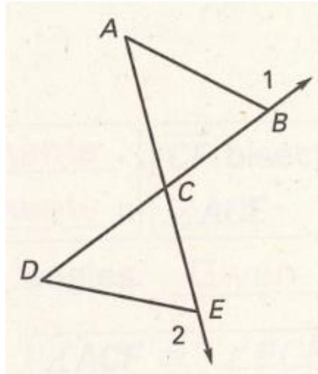
**Given:**
**Prove:** $\overline{DB} \cong \overline{CB}$




| Statement | Reason |
| --- | --- |
| 1. ______________________ | 1. ______________________ |
| 2. ______________________ | 2. ______________________ |
| 3. ______________________ | 3. ______________________ |
| 4. ______________________ | 4. ______________________ |
| 5. ______________________ | 5. ______________________ |

Example #4: Prove.

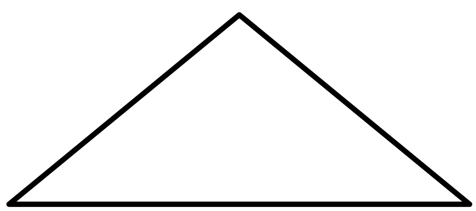
**Given:** $\angle 1 \cong \angle 2$, $\overline{AB} \cong \overline{DE}$
**Prove:** $\overline{DC} \cong \overline{AC}$




| Statement | Reason |
|---|---|
| 1. ______ | 1. ______ |
| 2. ______ | 2. ______ |
| 3. ______ | 3. ______ |
| 4. ______ | 4. ______ |
| 5. ______ | 5. ______ |
| 6. ______ | 6. ______ |
| 7. ______ | 7. ______ |

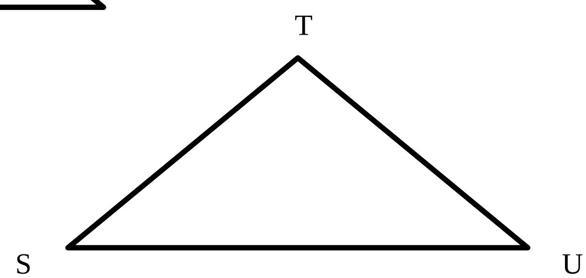
## **Chapter 4.7:** Use Isosceles and Equilateral Triangles

**Isosceles Triangle:**



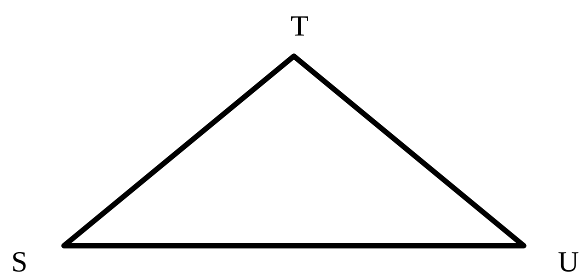
**Base Angles Theorem** **(Theorem 4.7):**

If two sides of a triangle are congruent, then the angles ______________________ them are congruent.




**Converse of Base Angles Theorem** **(Theorem 4.8):**

If two angles of a triangle are congruent, then the sides ______________________ them are congruent.

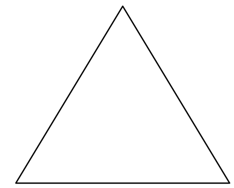


## Corollary to the Base Angles Theorem

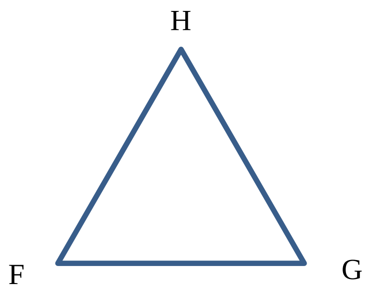
If a triangle is equailateral, then it is equiangular.

## Corollary to the Converse Base Angles Theorem

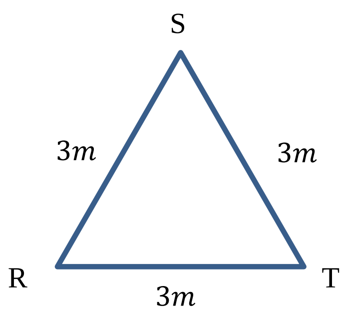
If a triangle is equiangular, then it is equilateral



Example #1: In $\triangle FGH$, $\overline{FH} \cong \overline{GH}$. Name two congruent angles.

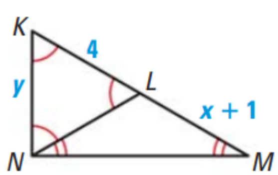



Example #2: Find the measures of $\angle R$, $\angle S$, and $\angle T$

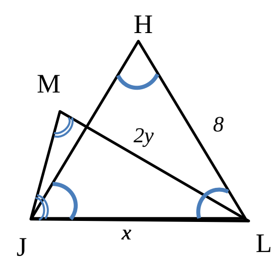



Example #3: Find the values of *x* and *y* in the diagram.

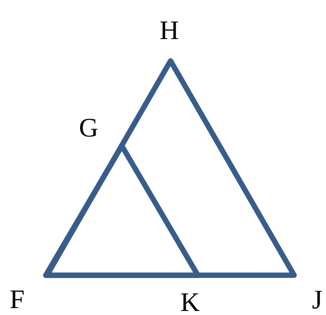
a.)




b.)




Example #4: Complete the statements




If $\overline{FH} \cong \overline{FJ}$, then, $\angle$_____ $\cong \angle$_____

If $\triangle FGK$ is equiangular and $FG = 15$, the $GK$= _______