Example #4: Find the measure of each interior angles of $\triangle ABC$, where $m\angle A = x°$, $m\angle B = 2x°$ and $m\angle C = 3x°$.

$m\angle A + m\angle B + m\angle C = 180°$

$x + 2x + 3x = 180$

$\frac{6x}{6} = \frac{180}{6}$

$x = 30°$

$m\angle A = 30°$

$m\angle B = 60°$

$m\angle C = 90°$

Chapter 4.2: Apply Congruence and Triangles

Two geometric figures are **congruent** if they have exactly the Same Size and shape.

In two Congruent figures, all the parts of one figure are congruent to the Corresponding parts of the other figure.

→ Includes Angles, side Lengths, etc

Corresponding Angles: $\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$

Corresponding Sides: $\overline{AB} \cong \overline{EF}$ $\overline{BC} \cong \overline{FG}$ $\overline{CA} \cong \overline{GE}$

→ A statement saying figures are $\cong$

Congruent Statements: When you write a congruence statement for two polygons, always list the corresponding vertices in the same order.

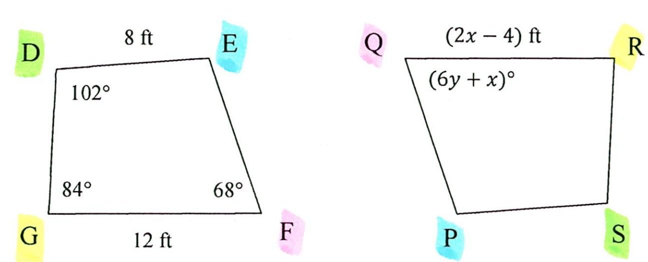
Example Statement: $\triangle ABC \cong \triangle EFG$ OR $\triangle CAB \cong GEF$

NOT a Congruent Statement: $\triangle ABC \cong \triangle GFE$

Example #1: In the diagram $DEFG \cong SPQR$

a.) Find the value of x

b.) Find the value of y



$\angle Q \cong \angle F$

$6y + x = 68$

$6y + 8 = 68$
$-8 \quad -8$

$\frac{6y}{6} = \frac{60}{6}$

$y = 10$

$\overline{GF} \cong \overline{RQ}$

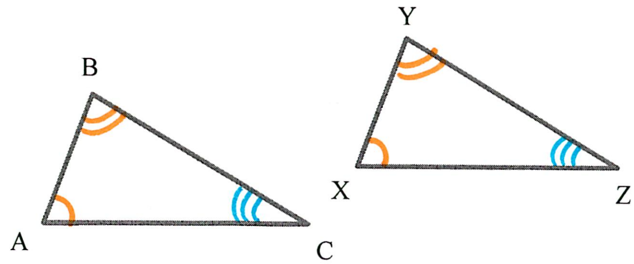
$12 = 2x - 4$
$+4 \quad +4$

$\frac{16}{2} = \frac{2x}{2}$

$x = 8$

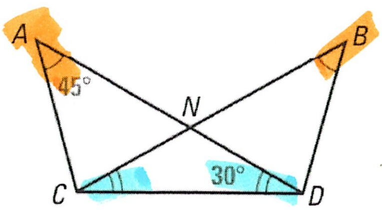
Third Angles Theorem **(Theorem 4.3):**

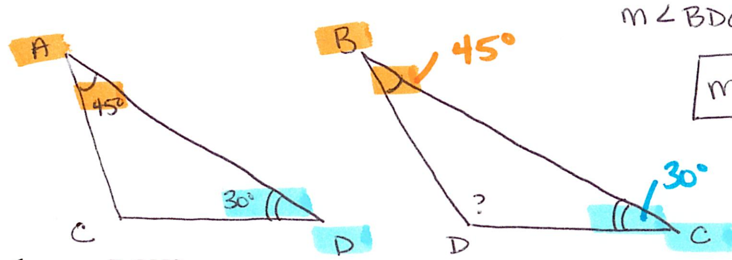
If two angles of one triangle are <u>Congruent</u> to two angles of another triangle, then the <u>third</u> angles are also congruent.



If $\angle A \cong \angle X$ and $\angle B \cong \angle Y$ then $\angle C \cong \angle Z$

Example #2: Find $m\angle BDC$

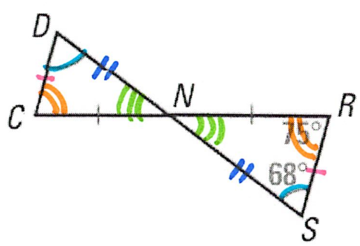




$m\angle BDC = 180° - 45° - 30$

$\boxed{m\angle BDC = 105°}$

Example #3: In the diagram, what is the $m\angle DCN$?
By the definition of congruence, what additional information is needed to know that $\triangle NDC \cong \triangle NSR$?



$m\angle DCN = 75°$

$m\angle DNC \cong m\angle SNR$

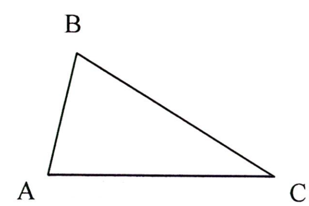
$\overline{CD} \cong \overline{RS}$

$\overline{DN} \cong \overline{SN}$

Properties of Congruent Triangles **(Theorem 4.4):**

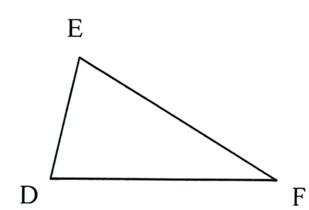
Reflective Property of Congruent Triangles

For any triangle ABC, <u>$\triangle ABC \cong \triangle ABC$</u>



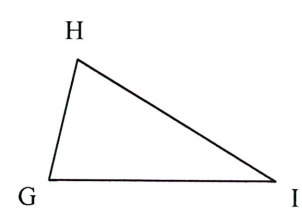
Symmetric Property of Congruent Triangles

If <u>$\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$</u>



Transitive Property of Congruent Triangles

If <u>$\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle GHI$</u>
<u>then $\triangle ABC \cong \triangle GHI$</u>



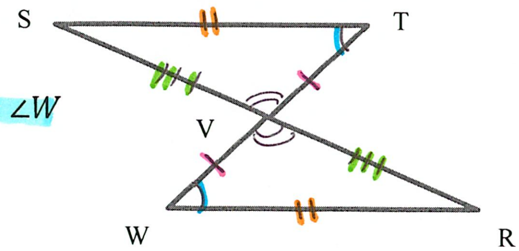
Example #4: Prove.

Why do they give us this info?

Given: $\overline{TV} \cong \overline{WV}$, $\overline{ST} \cong \overline{RW}$, point P is the midpoint of $\overline{SR}$, $\angle T \cong \angle W$

Prove: $\triangle STV \cong \triangle RWV$

↳ Therefore $\overline{SV} \cong \overline{RV}$

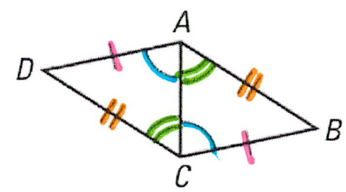


Statement	Reason
1. $\overline{TV} \cong \overline{WV}$ $\overline{ST} \cong \overline{RW}$	1. Given
1. V is the midpt of $\overline{SR}$ $\angle T \cong \angle W$	
2. $\overline{SV} \cong \overline{RV}$	2. Def^{n} of a midpoint
3. $\angle TVS \cong \angle WVR$	3. Vertical Angles Th^{rm}
4. $\angle S \cong \angle R$	4. Third Angles Th^{rm}
5. $\triangle STV \cong \triangle RWV$	5. Def^{n} of Congruent Triangles

Example #5: Prove.

Given: $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$, $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$

Prove: $\triangle ACD \cong \triangle CAB$



Statement	Reason
1. $\overline{AD} \cong \overline{CB}$ $\overline{DC} \cong \overline{BA}$	1. Given
1. $\angle ACD \cong \angle CAB$ $\angle CAD \cong \angle ACB$	
2. $\overline{AC} \cong \overline{CA}$	2. Reflexive Property
3. $\angle D \cong \angle B$	3. Third Angles Th^{rm}
4. $\triangle ACD \cong \triangle CAB$	4. Def^{n} of Congruent Triangles.

Chapter 4.3: Prove Triangles Congruent by SSS

Side-Side-Side (SSS) Congruence Postulate (Postulate 19):

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are Congruent.

If S: $\overline{AB} \cong \overline{DE}$

S: $\overline{BC} \cong \overline{EF}$

S: $\overline{CA} \cong \overline{FD}$

Then $\triangle ABC \cong \triangle DEF$

Congruence Statement