Chapter 2.2: Analyze Conditional Statements

Objective: I can write definitions as a conditional statement and analyze conditional statements.

Conditional Statement: a logical statement that contains two parts (Also know as If/then Statements)

- Hypothesis → p
- Conclusion → q

Notation: $p \to q$

Example:

If it meows, then it is a cat

Converse: conditional statement formed by switching the hypothesis and Conclusion

Notation: $q \to p$

Example:

If it is a cat, then it meows

Inverse: conditional statement formed by negating both the hypothesis and Conclusion

- Negating: opposite of original statement (NOT) [symbol: ~]

Notation: $\sim p \to \sim q$

Example:

If it does not meows, then it is a cat

Contrapositive: conditional statement formed by negating the Converse statement

Notation: $\sim q \to \sim p$

Example:

If it is not a cat, then it does not meow.

Equivalent Statement: two statements that are both true or both false.

- Conditional (If/then) and Contrapositive
- Converse and inverse

Biconditional Statement (iff → "if and only if" **):** both conditional and converse statements must be true.

** Best definitions are biconditional statements.

Example:

Conditional Statement: If $\underbrace{\text{a figure is a triangle}}_{\text{hyp}}$, then $\underbrace{\text{it has three sides}}_{\text{conclusion}}$. (Always true)

Converse Statement:

If it has three sides, then a figure is a triangle (Always true)

Biconditional Statement: ☆ Do NOT use if/then ☆ use iff ☆

A figure is a triangle iff it has three sides.

Example #1: Rewrite in all four forms and as a biconditional statement if you can:

$$When\ x{=}3,\ then\ x^2 = 9$$

Write the conditional:

If $x=3$, then $x^2=9$ (Always true)

Write the converse.

If $x^2=9$, then $x=3$ (Not always true ⇒ $(-3)^2=9$)

Write the inverse.

If $x \neq 3$, then $x^2 \neq 9$

Write the contrapositive.

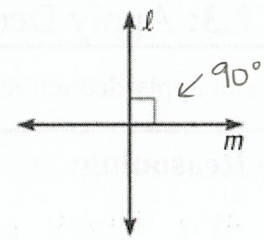
If $x^2 \neq 9$, then $x \neq 3$

If you can, write as a biconditional statement

(Cannot write a biconditional statement since the converse is not ALWAYS true.)

Perpendicular Lines:

If two lines interest to form a right angle, then they are perpendicular.



Notation: line ℓ is perpendicular to line m

$\ell \perp m$

Example #2: Use the following conditional statement to answer question.

If two angles equal 90°, then they are right angles

What is the hypothesis?

Two angles equal 90°

What is the conclusion?

They are right angles.

Write the conditional:

Write the converse.

If two angles are right angles, then they equal 90°

Write the inverse.

If two angles do not equal 90°, then they are not right angles.

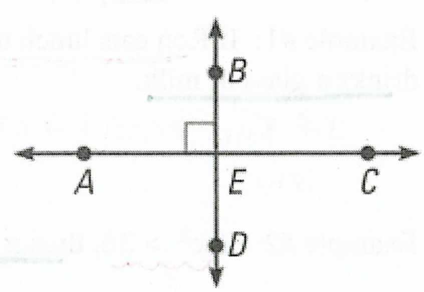
Write the contrapositive.

If two angles are not right angles, then do not equal 90°

If you can, write as a biconditional statement

Two angles equal 90° iff they are right angles.

Example #3: Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a.) $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

True;

b.) $\angle AEB$ and $\angle CED$ are supplementary angles.

True; 90° + 90° = 180°

c.) $\angle AEB$ and $\angle CED$ are a linear pair.

False;

d.) $\angle AEB$ and $\angle CED$ are vertical angles.

True;

e.) $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays

False;

Chapter 2.3: Apply Deductive Reasoning

Objective: I can apply deductive reasoning to form a logical argument.

Deductive Reasoning: using facts, definitions, accepted properties and the laws of logic to form a logical argument.

Inductive Reasoning vs. **Deductive Reasoning**

Inductive Reasoning	Deductive Reasoning
Using patterns and past experiences to form a logical hypothesis	using facts, definitions, theorems, etc. to form a logical argument.

Law of Detachment: In a conditional statement, if the hypothesis is true, then the conclusion is true.

- Example #1: If $\underbrace{\text{two angles have the same measure}}_{\text{hypothesis}}$, then $\underbrace{\text{they are congruent.}}_{\text{conclusion}}$

  So if you know that $\underbrace{m\angle A = m\angle B}_{\text{hypothesis}}$, then… $\underbrace{\angle A \cong \angle B}_{\text{conclusion}}$

- Example #2: Jesse goes to the gym every weekday.

  Today is Monday. Jesse is going to the gym today.

Law of Syllogism:

If hypothesis p, then conclusion q.
If hypothesis q, then conclusion r.
} If these statements are true.

If hypothesis p, then conclusion r. } Then this statement is true.

- Example #1: If Ron eats lunch today, then he will eat a sandwich. If Ron eats a sandwich, then he drinks a glass of milk.

  p q q

  If Ron eats lunch today, then he drinks a glass of milk.

- Example #2: If $x^2 > 36$, then $x^2 > 30$. If $x > 6$, then $x^2 > 36$

  q r p q

  If $x > 6$, then $x^2 > 30$.