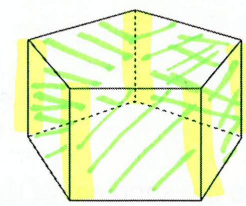
Chapter 12.2: Surface Area of Prisms and Cylinders

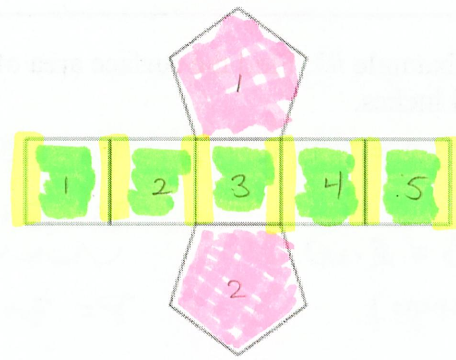
Prism: Polyhedron with 2 congruent faces (bases) that lie in parallel planes.



Lateral Faces: Parallelograms formed by connecting the corresponding vertices of the bases

Lateral Edge: Segments connecting the vertices of the base.

Net: two dimensional representations of the faces of a polyhedron.



Surface Area of a polyhedron, is the sum of the area of all its faces.

Example: 2 bases + 5 Lateral Faces

Lateral Area of a polyhedron is the sum of the area of the lateral faces.

Example: ONLY the 5 lateral faces

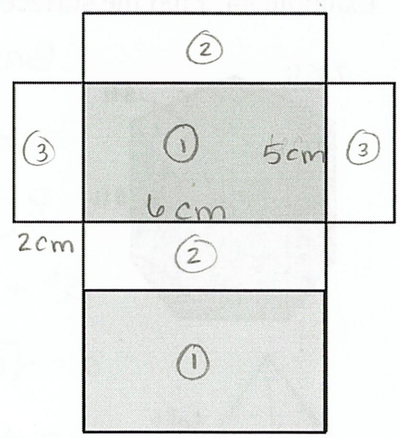
Example #1: Find the surface area of a rectangular prism with $\boxed{\text{height 2cm}}$, length 5 cm and width 6 cm.

① 5cm 6cm $A = (5)(6) = 30 \times 2 = 60\ cm^2$

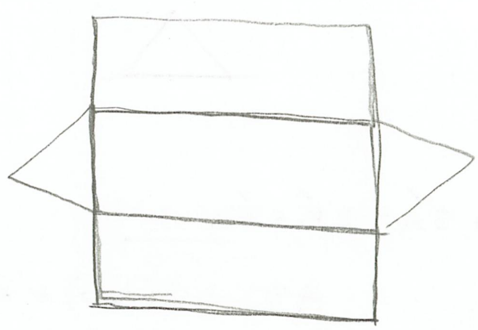
② 2cm 6cm $A = (6)(2) = 12 \times 2 = 24\ cm^2$

③ 5cm 2cm $A = (5)(2) = 10 \times 2 = 20\ cm^2$

Total Area $= 104\ cm^2$

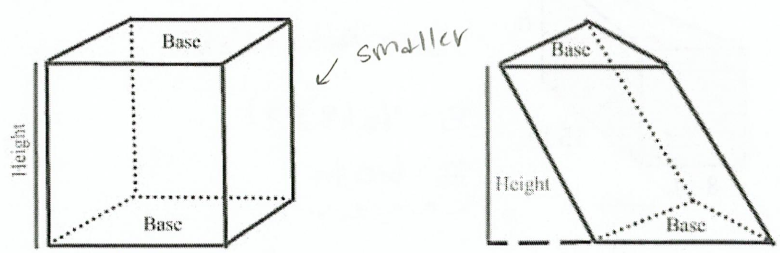


Example #2: Draw a net of a triangular prism.



Right Prism: Each lateral edge is perpendicular to both bases.

Oblique Prism: Each lateral edge is not perpendicular to the bases.

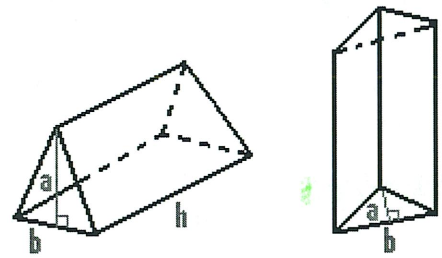


Surface Area of a Right Prism:

The surface area S of a right prism is

$\underbrace{SA = 2B}_{\text{Area of Bases}} + \underbrace{Ph}_{\text{Area of Lateral faces}}$ ← this one can be used with ANY shape

$SA = \overbrace{aP}^{} + \overbrace{Ph}^{}$ ← Short cut for regular polygons.



Where a is the apothem of the base, B is the area of a base, P is the perimeter of a base, and h is the height.

The base area formula used will depend on the shape of base.

Example #3: Find the surface area of a right rectangular prism with height 7 inches, length 3 inches and width 4 inches.

$SA = 2B + Ph$

$B = l \cdot w$

(Rectangle)

$B = (3)(4)$

$B = 12 in^2$

$P = 3+4+3+4$

$P = 14$ in

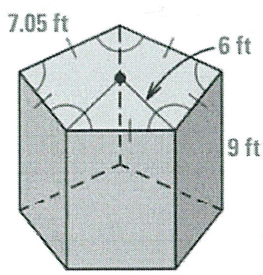
$h = 7$in

$SA = 2(12) + 14(7)$

$SA = 24 + 98$

$\boxed{SA = 122 in^2}$

Example #4: Find the surface area of the right pentagonal prism. $SA = 2B + Ph$ → $h = 9ft$



Base Area $= \frac{1}{2}aP$ → $B = \frac{1}{2}(4.86)(35.25)$

$P = 7.05(5)$

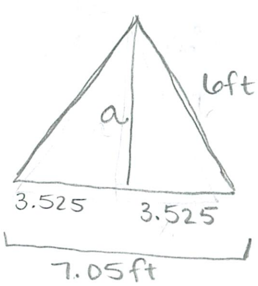
$P = 35.25$

$B \approx 85.66 ft^2$

$a^2 + 3.525^2 = 6^2$

$a = \sqrt{6^2 - 3.525^2}$

$a \approx 4.86 ft$

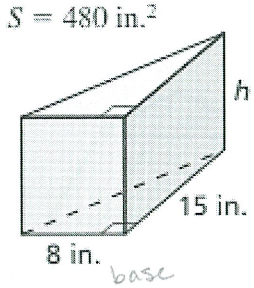


$SA = 2(85.66) + 35.25(9)$

$\boxed{SA = 488.57 ft^2}$

Example #5: Find the height of the right prism → $SA = 2B + Ph$

$S = 480$ in.^{2}



$SA = 480 in^2$

Base Area $= \frac{1}{2}bh$

$B = \frac{1}{2}(8)(15)$

$B = 60$ in

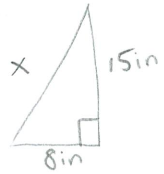
$480 = 2(60) + 40h$

$480 = 120 + 40h$

$-120 \quad -120$

$\frac{360}{40} = \frac{40h}{40}$

$\boxed{h = 9 \text{ in}}$



$15^2 + 8^2 = x^2$

$x = \sqrt{15^2 + 8^2}$

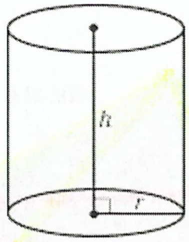
$x = 17$ in

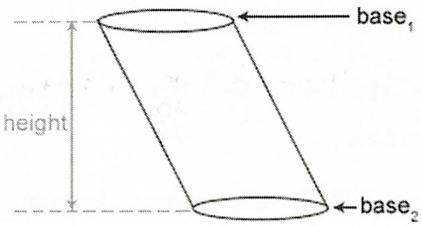
Perimeter of Base $= 15 + 8 + 17$

$= 40$in

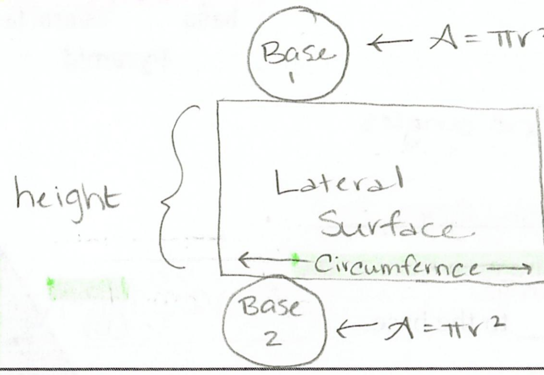
Cylinder: A solid with congruent, circular, parallel bases.

Right Cylinder: Segment joining the centers of the bases is perpendicular to the bases.





Cylinder Net:



← $A = \pi r^2$

← Lateral Area (Rectangle) $= b \cdot h$

$= C(h)$

$= (2\pi r)h$

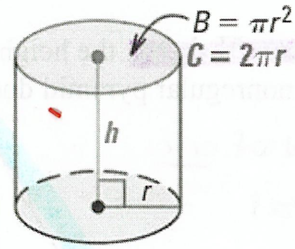
← $A = \pi r^2$

Surface Area of a Right Cylinder:

The surface area S of a right cylinder is

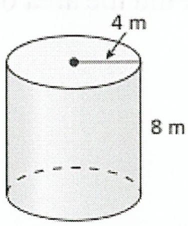
$SA = 2B + Ch \rightarrow SA = \underbrace{2(\pi r^2)}_{\text{Areas of bases}} + \overbrace{(2\pi r)h}^{\text{Area of Lateral Surface}}$

Base Area $= \pi r^2$ Circumference $= 2\pi r$



Where B is the area of a base, C is the circumference of a base, r is the radius of a base and h is the height.

Example #5: Find the surface area of the right cylinder.



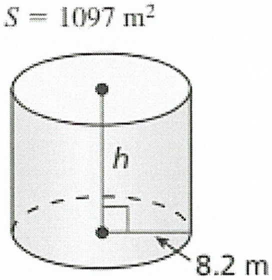
$SA = 2(\pi \cdot 4^2) + (2\pi \cdot 4)8$

$SA = 32\pi + 64\pi$

$\boxed{\begin{array}{l} SA = 96\pi \text{ m}^2 \leftarrow \text{Exact Surface Area} \\ SA \approx 301.59 \text{ m}^2 \leftarrow \text{Approx Surface Area} \end{array}}$

Example #6: Find the height of the right cylinder.

$S = 1097$ m^2



$1097 = 2(\pi \cdot 8.2^2) + (2\pi \cdot 8.2)h$

$1097 = 422.48 + 51.52h$

$-422.48 \quad -422.48$

$\dfrac{674.52}{51.52} = \dfrac{51.52h}{51.52}$

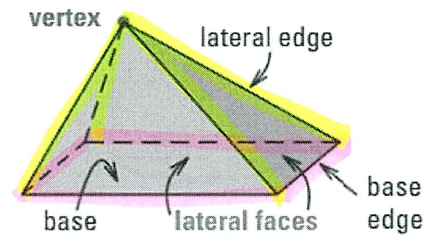
$\boxed{h \approx 13.09 \text{ m}}$

Chapter 12.3: Surface Area of Pyramids and Cones

Pyramid: a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex.

Lateral Edge: Intersection of two lateral faces.

Base Edge: Intersection of the base and a lateral face.



Pyramid

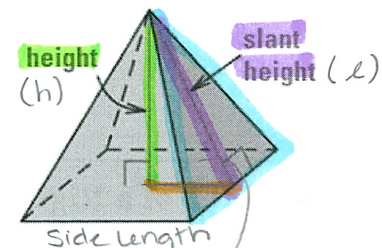
All sides and angles are ≅

Regular Pyramid: has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base.

height of pyramid.

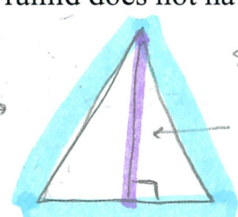
Lateral Faces are all congruent isosceles triangle

Slant Height: the height of a lateral face of the regular pyramid (A nonregular pyramid does not have aslant height)

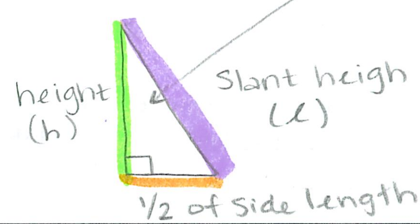


Regular pyramid

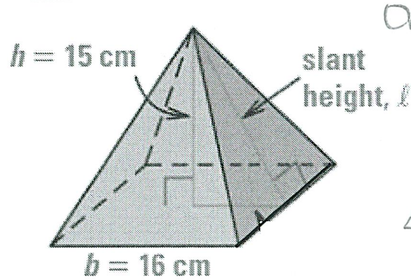
veiw of one lateral face →

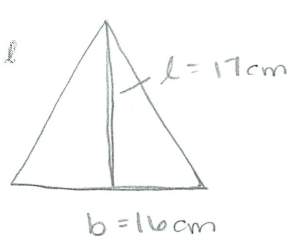


Slant height (ℓ) is the height of the 2-D lateral face



Example #1: A regular square pyramid has height of 15 cm and a base edge length of 16 cm. Find the area of each lateral face of the pyramid.



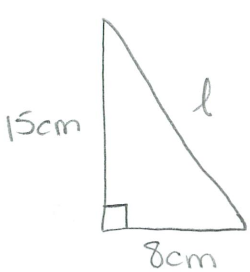


One Lateral Face Area = $\frac{1}{2}bh$

$= \frac{1}{2}b\ell$

$= \frac{1}{2}(16)(17)$

$\boxed{A = 136\text{ cm}^2}$



$15^2 + 8^2 = \ell^2$

$\ell = \sqrt{15^2 + 8^2}$

$\ell = 17\text{cm}$