# Geometry

Mrs. Tilus

# Unit 12: Surface Area and Volume of Solids

**Priority Standard:** G-GMD: Use volume formulas for cylinders, pyramids, cones and spheres to solve problems

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**Unit 8 "I can" Statements:**

1. I can identify solids
2. I can find the surface area of prisms and cylinders
3. I can find the surface area of pyramids and cones
4. I can find the volume of prisms and cylinders
5. I can find the volume of pyramids and cones
6. I can find the surface area and volume of spheres
7. I can use the properties of similar solids to find unknown ratios, corresponding lengths, areas or volumes

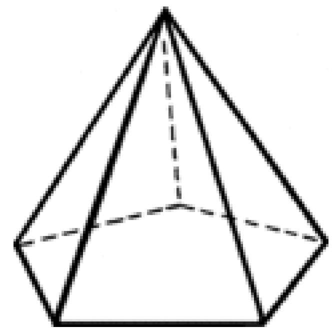
# **Chapter 12.1:** Explore Solids

**Polyhedron:**

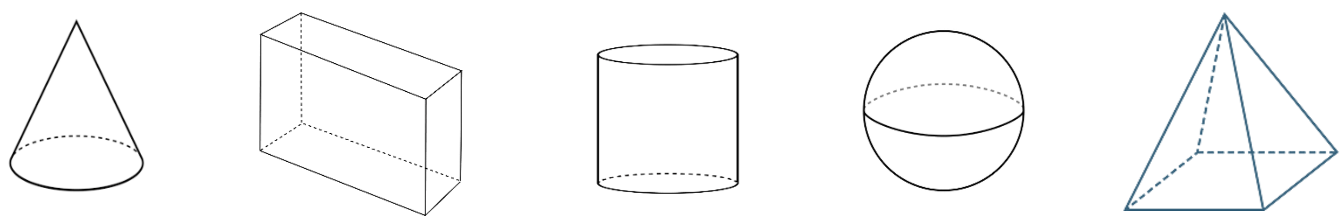
**Face:**

**Edge:**

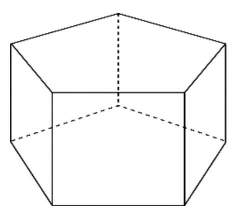
**Vertex:**

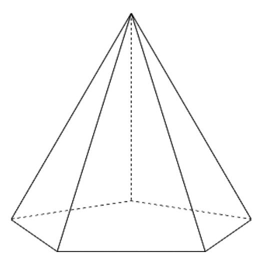


**Types of Solids:** Which solids are polyhedrons?



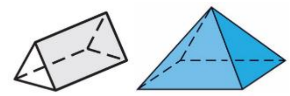
**Classifying Solids:** To name a prism or a pyramid, use the shape of the base.



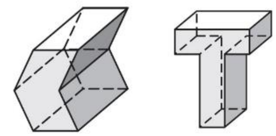


**Regular Polyhedrons:** A polyhedron is regular if all of its ______________________ are ________________regular polygons.

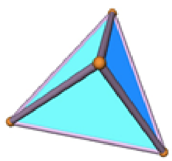
- A polyhedron is **convex:** if any two points on its surface can be connected by a segment that lies entirely ____________________ or on the polyhedron.

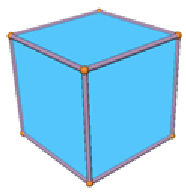


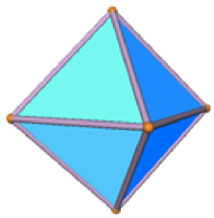
- A polyhedron is **concave:** if two points on its surface is connected by a segment that goes ________________ the polyhedron.

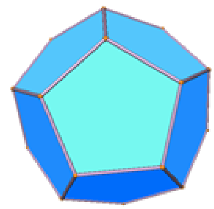


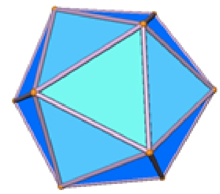
There are 5 regular polyhedra called ______________________________________


Tetrahedron

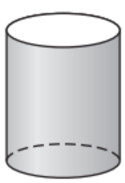

Cube

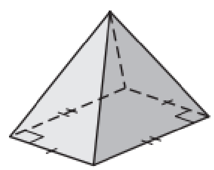

Octahedron

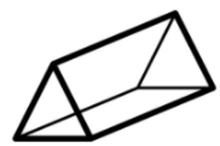

Dodecahedron

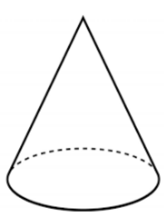

Icosahedron

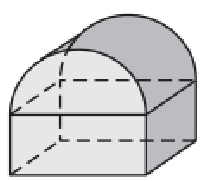
Example #1: Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices and edges.

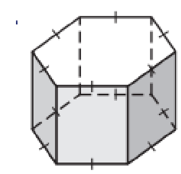
a.) 

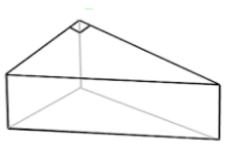
b.) 

c.) 

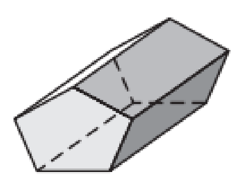
d.) 

e.) 

f.) 

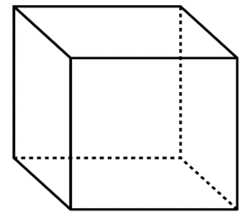
g.) 

h.) 

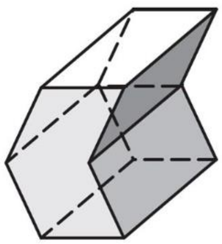
i.) 

## Euler's Theorem (Theorem 12.1):

The number of faces (F), vertices (V) and edges (E) of a Polyhedron are related by the formula…



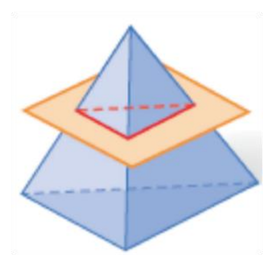
Example #2: Find the number of faces, vertices and edges of the polyhedron shown. Check your answers using Euler's Theorem.



Example #3: Is it possible for a polyhedron to have 16 faces, 34 vertices and 50 edges?

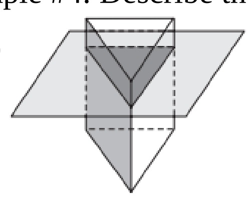
**Cross Section:** the intersection of a ______________________

and a ______________________.

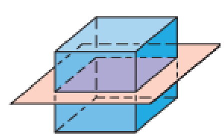


Example #4: Describe the shape formed by the intersection of the plane and the solid.

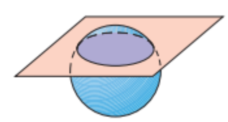
a.)



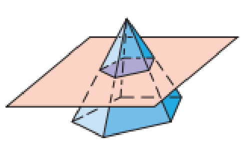
b.)



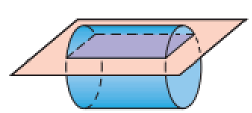
c.)

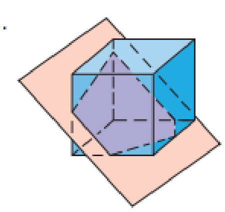


d.)

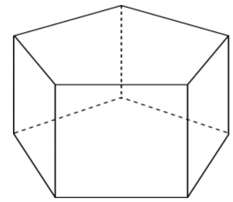


e.)





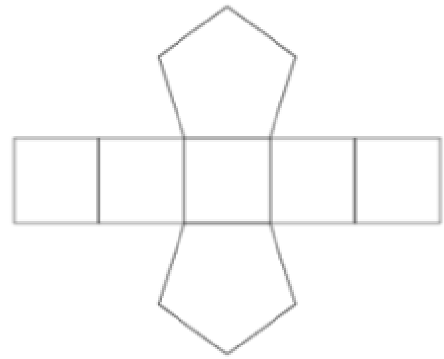
## **Chapter 12.2:** Surface Area of Prisms and Cylinders



**<u>Prism:</u>** Polyhedron with 2 congruent faces (bases) that lie in parallel planes.

**Lateral Faces:** Parallelograms formed by connecting the corresponding vertices of the bases

**Lateral Edge**: Segments connecting the vertices of the base.

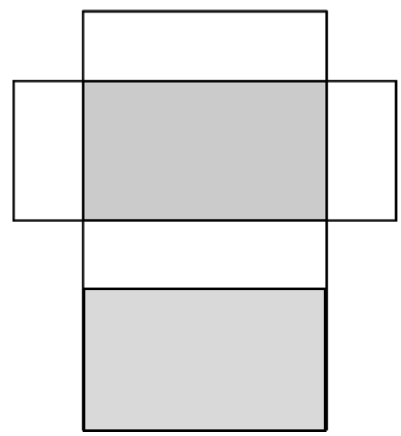


**<u>Net</u>:** two dimensional representations of the faces of a polyhedron.

**<u>Surface Area</u>** of a polyhedron, is the sum of the area of all its faces.

**<u>Lateral Area</u>** of a polyhedron is the sum of the area of the lateral faces.

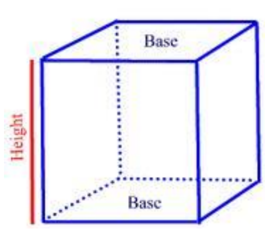
Example #1: Find the surface area of a rectangular prism with height 2cm, length 5 cm and width 6 cm.

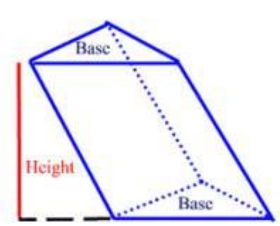


Example #2: Draw a net of a triangular prism.

**Right Prism:** Each lateral edge is perpendicular to both bases.

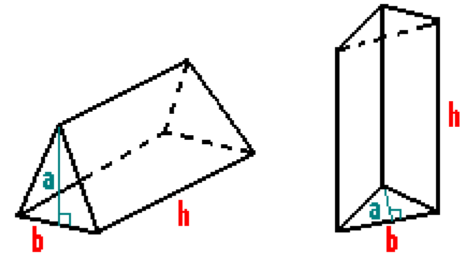
**Oblique Prism:** Each lateral edge is not perpendicular to the bases.



**Surface Area of a Right Prism:**

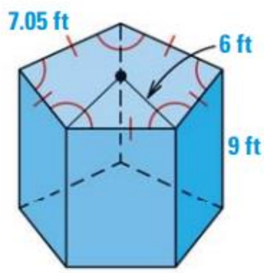
The surface area S of a right prism is



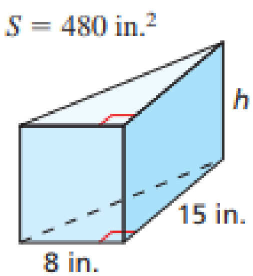

Where *a* is the apothem of the base, *B* is the area of a base,
*P* is the perimeter of a base, and *h* is the height.

Example #3: Find the surface area of a right rectangular prism with height 7 inches, length 3 inches and width 4 inches.

Example #4: Find the surface area of the right pentagonal prism.

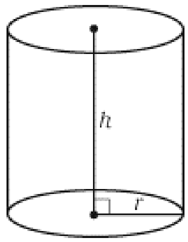


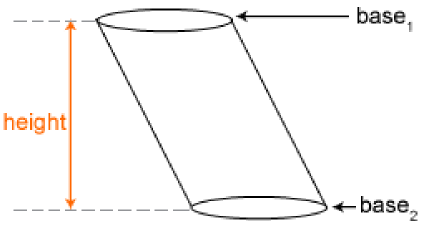

Example #5: Find the height of the right prism



**Cylinder:** A solid with congruent, circular, parallel bases.

**Right Cylinder:** Segment joining the centers of the bases is perpendicular to the bases.

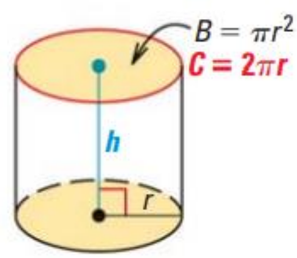



**Cylinder Net:**

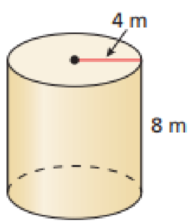
**Surface Area of a Right Cylinder**:

The surface area $S$ of a right cylinder is

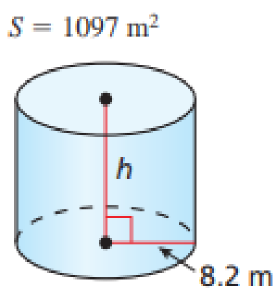



Where $B$ is the area of a base, $C$ is the circumference of a base, $r$ is the radius of a base and $h$ is the height.

Example #5: Find the surface area of the right cylinder.




Example #6: Find the height of the right cylinder.

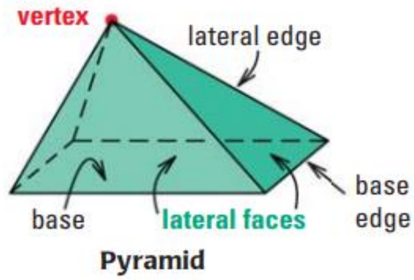


# **Chapter 12.3:** Surface Area of Pyramids and Cones

**Pyramid:** a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex.

Lateral Edge:

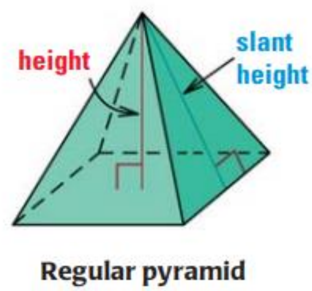
Base Edge:




**Regular Pyramid:** has a ________________________________

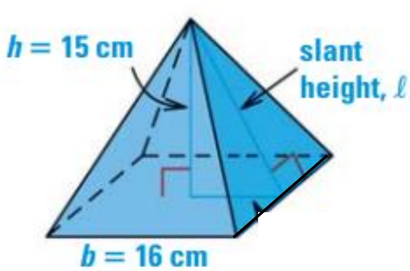
for a base and the segment joining the vertex and the center of the base is __________________________________ to the base.

Lateral Faces are



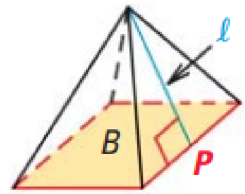

**Slant Height:** the height of a lateral face of the regular pyramid (A nonregular pyramid does not have aslant height)

Example #1: A regular square pyramid has height of 15 cm and a base edge length of 16 cm. Find the area of each lateral face of the pyramid.



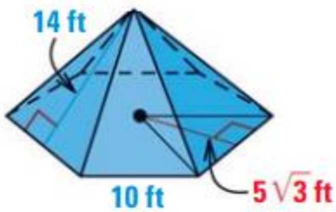
**Surface Area of a Regular Pyramid** **(Theorem 12.4):**

The surface area S of a regular pyramid is

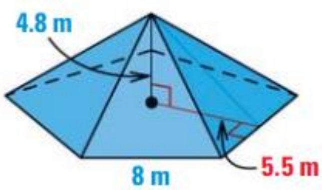



where *B* is the area of the Base, *P* is the perimeter of the base, and
*l* is the slant height.

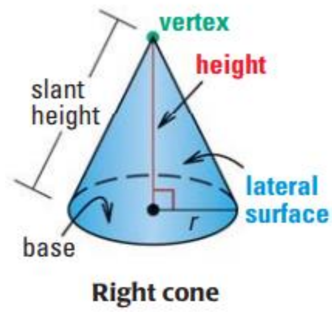
Example #2: Find the surface area of the regular hexagonal pyramid.




Example #3: Find the surface area of the regular pentagonal pyramid shown



**Cone:** a solid with a ______________________________ and a ____________________ that is not in the same plane as the base.

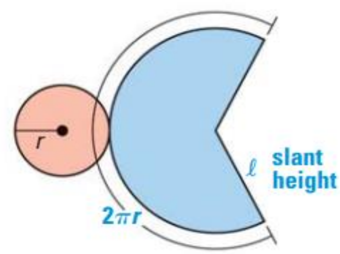



Radius:

Height:

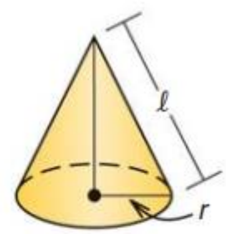
In a ______________________________, the segment joining the vertex and the center of the base is ______________________ to the base, and the ________________________________ is the distance between the vertex and a point of the base edge.

**Lateral Surface:**



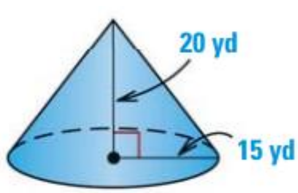

**Surface Area of a Right Cone (Theorem 12.5):**

The surface area S of a right cone is

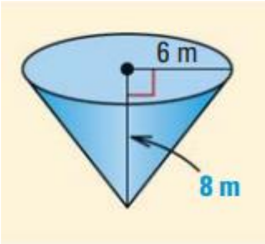



where *B* is the area of the Base, *C* is the circumference of the base,
*r* is the radius of the base, and *l* is the slant height.

Example #4: Find the lateral area of the right cone.

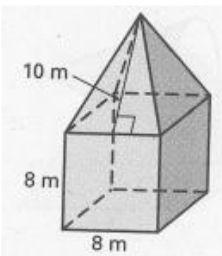


Example #5:  Find the surface area of the right cone.




Example #6: Find the surface area of the solid.  The pyramids are regular and the cones are right.  Round your answer to the nearest hundredth, if necessary.

a.)

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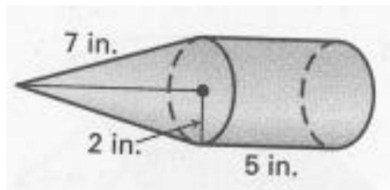
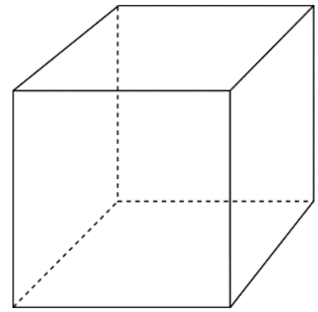


# **Chapter 12.4:** Volume of Prisms and Cylinders

**Volume:**



**Volume of a Cube (Postulate 27):**

The volume of a cube is the cube of the length of its side.

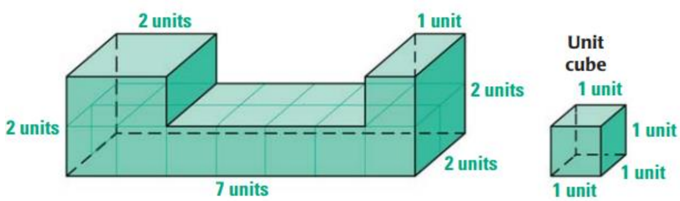
**Volume Congruence Postulate (Postulate 28):**

If two polyhedral are congruent, then they have the same volume.

**Volume Addition Postulate (Postulate 29):**

The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

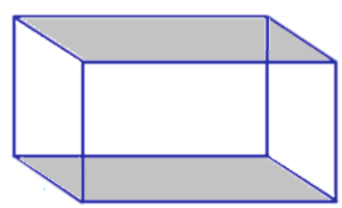
Example #1: Find the volume of the puzzle piece in cubic units.




**Volume of a Prism (Theorem 12.6):**

The volume $V$ of a prism is ____________________________

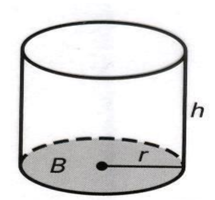
where $B$ is the area of a base and $h$ is the height.



**Volume of a Cylinder (Theorem 12.7):**

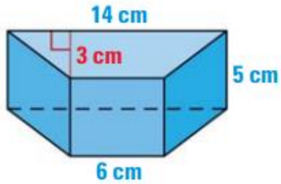
The volume $V$ of a cylinder is _____________________________________

Where $B$ is the area of a base, $h$ is the height, and $r$ is the radius of a base.



Example #2: Name each solid then find the volume. Round your answer to two decimal places, if necessary.

a.)

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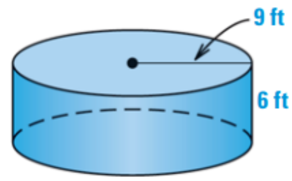
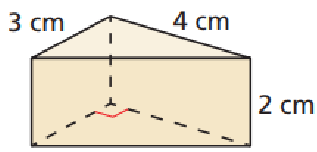
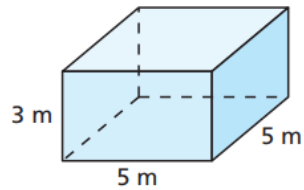


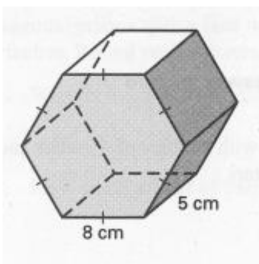

c.)



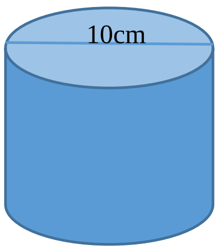

d.)




e.)



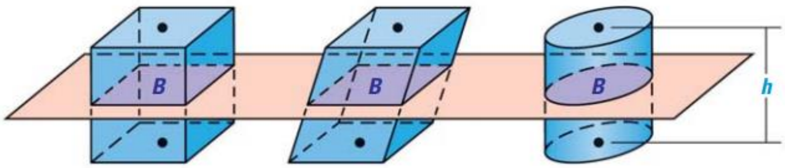

Example #3: The volume of the right cylinder is $200\pi$ cm$^3$. Find the height.



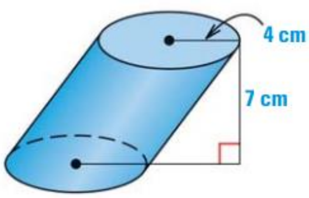
**Cavalieri's Principle** **(Theorem 12.8):**

If two solids have the same ________________ and the same cross-sectional area at every level, then they have the same ____________________

Which means…

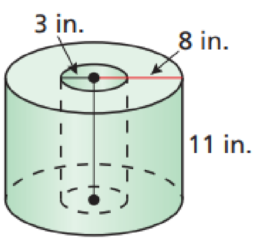



Example #4: Find the volume of the oblique cylinder. Round answers to the nearest hundredth.

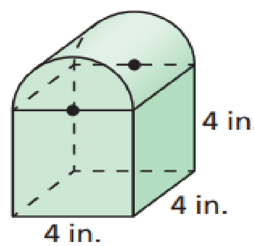



Example #5: Find the volume of each solid. Round answers to the nearest hundredth.

a.)



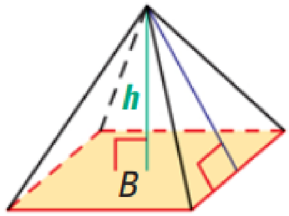

b.)



# **Chapter 12.5:** Volume of Pyramids and Cones

## Volume of a Pyramid (Theorem 12.9):

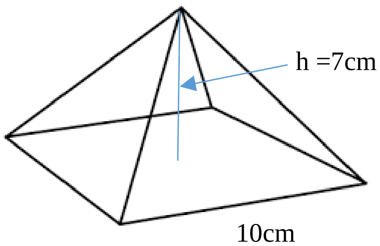
The volume $V$ of a pyramid is



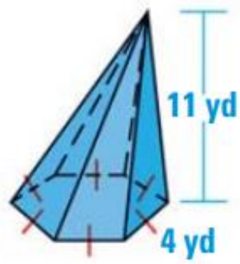

where $B$ is the area of the base and $h$ is the height.

Example #1: Find the volume of the pyramid with the regular base. Round answers to the nearest hundredth.

a.)

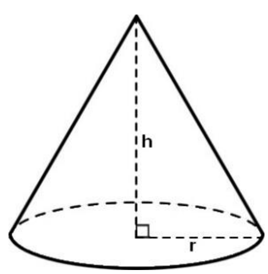



b.)




## Volume of a Cone (Theorem 12.10):

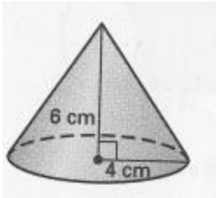
The volume $V$ of a cone is



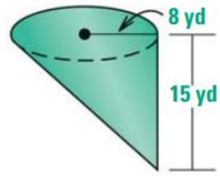

where $B$ is the area of the base, $h$ is the height, and $r$ is the radius of the base.

Example #2: Find the volume of each cone. Round answers to the nearest hundredth.

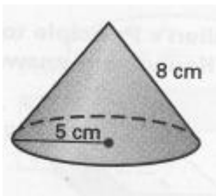
a.)



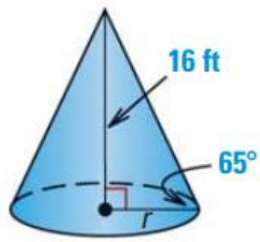

b.)




c.)

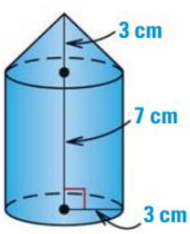



d.)

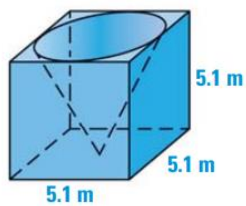



Example #3: Find the volume of the solid shown. Round answers to the nearest hundredth.

a.)

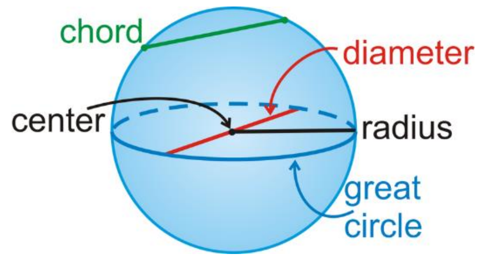



b.)



# **Chapter 12.6:** Surface Area and Volume of Spheres

A ________________ is the set of all points in space equidistant from a given point.




**Center of a Sphere:** the given point from which all points on the sphere is ______________________.

**Radius of a Sphere:** a segment from the ________________ to any point on the sphere

**Chord of a Sphere:** a segment whose ________________are on the sphere.

**Diameter of a Sphere:** a ________________ that contains the ________________ of the sphere.

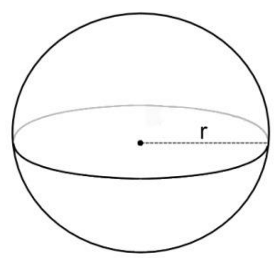
**Great Circle:** the ________________ of a sphere and plane that contains the ________________ of the sphere.

**Hemisphere:** one of the congruent ________________ of a sphere.

## Surface Area of a Sphere (Theorem 12.11):

The surface area *S* of a sphere is

where *r* is the radius of the sphere.



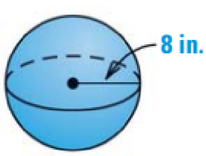

## Volume of a Sphere (Theorem 12.12):

The volume *V* of a sphere is

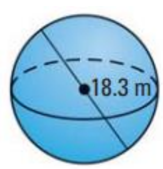
where *r* is the radius of the sphere.

Example #1: Find the surface area and volume of the sphere. Round answers to the nearest hundredth.

a.)



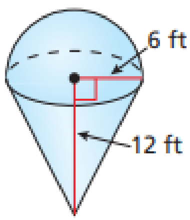

b.)



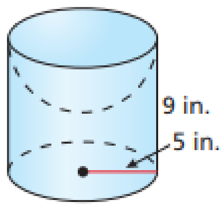
Example #2: The surface area of a sphere is $110.25\pi$ ft$^2$. Find the diameter of the sphere. Round answers to the nearest hundredth.

Example #3: Find the volume of the composite solid. Round answers to the nearest hundredth.

a.)



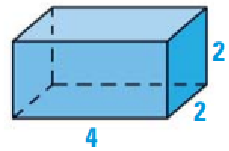

b.)



## **Chapter 12.7:** Explore Similar Solids

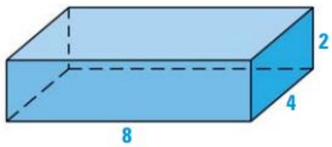
**Similar Solids:** Two Solids of same type with equal ratios of corresponding linear measures.

**Scale Factor:** common ratio to go from one solid to the other.

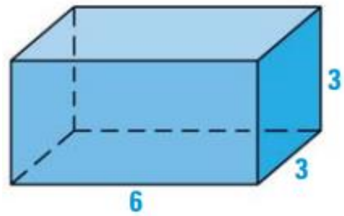



Example #1: Tell whether the given right rectangular prism is similar to the right rectangular prisms shown below.

a.)

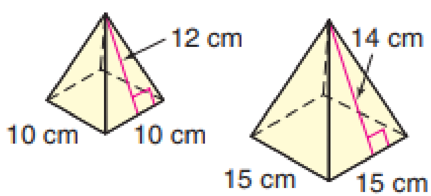



b.)

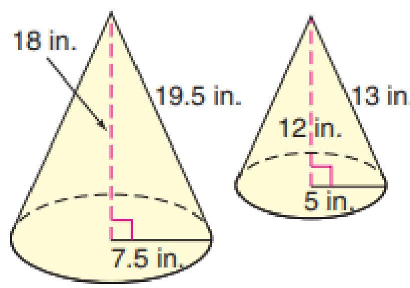



Example #2: Tell whether the pair of solids is similar.

a.)




b.)




**Similar Solids Theorem (Theorem 12.13):**

If two similar solids have a scale factor of __________,

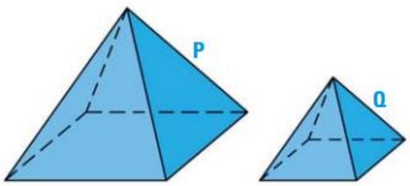
then corresponding areas have a ratio of __________,

and corresponding volumes have a ratio of __________.

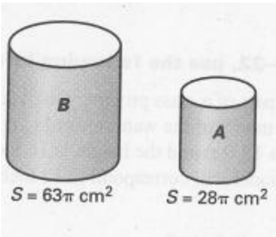
Example #2: Fill in the chart

| Ratio of perimeter/corresponding lengths (scale factor) | Ratio of Areas (surface area) | Ratio of Volumes |
|---|---|---|
| 3:4 | | |
| | 49:36 | |
| | | 1:125 |
| 24:3= | | |
| | | $27\pi:125\pi$ |

Example #3: The pyramids are similar. Pyramid P has a volume of 1000 $in^3$ and Pyramid Q has a volume of 216 $in^3$. Find the scale factor of Pyramid P to Pyramid Q.




Example #4: The two cylinders are similar. Find the scale factor of Cylinder A to Cylinder B.




Example #5: Cones A and B are similar with a scale factor of 5:2. Find the surface area of Cone B given that the surface area of Cone A is 2356.2 $cm^2$. Round your answer to the nearest hundredth.

Find the volume of Cone B given that the volume of Cone A is 7450.9 $cm^3$. Round your answer to the nearest hundredth.